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A short route to Rényi's parking constant

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A classical parking problem due to A. Rényi [2] asks for the expected number of cars of unit length that can be parked randomly in a street of a given length. The problem leads to a delay differential equation, and when the length goes to infinity, the mean density of the cars approaches Rényi's parking constant

$$C = \int_0^\infty \exp\left(-2\int_0^u \frac{1-\mathrm{e}^{-t}}{t}\,\mathrm{d}t\right)\,\mathrm{d}u \approx 0.7475979.$$

We propose an alternative derivation of this constant, which is inspired by N. G. de Bruijn's analysis of the Buchstab function in number theory [1]. It is shorter and more elementary than Rényi's original approach, and relies on the duality between differential equations with delayed and advanced arguments.

[1] N. G. de Bruijn, On the number of uncancelled elements in the sieve of Eratosthenes, *Indag. Math.*, **12**(1950), 247-256.

[2] A. Rényi, On a one-dimensional problem concerning random space filling, *Publ. Math. Inst. Hung. Acad. Sci.*, **3**(1958), 109–127.