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Multiple solutions for one-dimensional billiard table with time-changing boundary

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This is a joint work with Jorge Rodríguez-López (Universidade de Santiago de Compostela).

We investigate a Dirichlet problem for an ODE of the second order with state-dependent impulses in the form

$$\begin{aligned} &x''(t) = f(t, x(t)) \quad \text{for a.a. } t \in [0,T],\ x(t) \in (\alpha(t), \beta(t)),\\ &x'(s+) - \gamma'(s) = \gamma'(s) - x'(s-), \quad \text{if } x(s) = \gamma(s),\ \gamma \in \{\alpha, \beta\},\\ &x(0) = A \in (\alpha(0), \beta(0)), \quad x(T) = B \in (\alpha(T), \beta(T)), \end{aligned}$$

where $\alpha, \beta \in W^{2,1}([0,T])$, $\alpha < \beta$, $f : [0,T] \times \mathbb{R} \to \mathbb{R}$ satisfies Carathéodory conditions on the set $\{(t,x) \ : \ 0 \le t \le T,\ \alpha(t) \le x \le \beta(t)\}$, $T > 0$. The impulse conditions correspond with the absolutely elastic impact of the ball at the boundary of the "billiard table". We give existence and multiplicity result for solutions with prescribed number of impacts.