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Coupled delayed negative feedback loops: Stability and oscillations

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The production of mammalian blood cells involves an intertwined network of physiological processes, with nonlinear, delayed feedback control mechanisms. For example, erythrocytes (red blood cells) and thrombocytes (platelets) while each having their own main regulatory hormone, erythropoietin and thrombopoietin respectively, are also intercating, specially in pathological conditions. We consider the simplified model

$$\begin{cases} x'(t) &= -\alpha x(t) + f(x(t-\tau_1), y(t-\tau_2)) \\ y'(t) &= -\beta y(t) + g(x(t-\tau_1), y(t-\tau_2)) \end{cases}$$

with f and g appropriate Hill functions for the coupled regulation of these two cell lines to study how the interaction of the control mechanisms may influence the dynamics. Equilibrium solutions are determined, their stability established and the nature of the oscillations when instability occurs are investigated. The mathematical part of the analysis revolves around a transcendental characteristic equation of second order with two delays and a Centre manifold analysis at the change of stability of equilibria.