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Lambert W function method in investigating asymptotic properties of fractional delay differential equations

LUDEK NECHVATAL

Brno University of Technology, Czech Republic

nechvatal@fme.vutbr.cz

Typically, stability/asymptotic issues connected with the linear delay differential equation

$$x'(t) = \lambda x(t - \tau), \quad \lambda \in \mathbb{C}, \tau > 0, \quad (1)$$

can be treated by the Lambert W function method. Under some restrictions, this approach can still be used in the case of a fractional counterpart of the mentioned equation (i.e., when the first-order derivative on the left-hand side is replaced by a fractional derivative $D^\alpha x$ of a suitable order α). The key step of our considerations consists in the fact that the Lambert W function (despite it is a complex function) can be, in some sense, manipulated in the real domain only. Then, we are very easily able to rediscover the known “iff” condition on λ for the asymptotic stability of the zero solution to the fractional version of (1). In addition, a precise description of the decay/growth rate of the solutions can be obtained.