
**The conference is supported by the National Laboratory for Health Security project RRF-2.3.1-21-2022-00006**

# Lambert $W$ function method in investigating asymptotic properties of fractional delay differential equations

Luděk Nechvátal

Brno University of Technology, Czech Republic
nechvatal@fme.vutbr.cz

Typically, stability/asymptotic issues connected with the linear delay differential equation

$$x'(t) = \lambda x(t-\tau), \quad \lambda \in \mathbb{C},\ \tau > 0, \tag{1}$$

can be treated by the Lambert $W$ function method. Under some restrictions, this approach can still be used in the case of a fractional counterpart of the mentioned equation (i.e., when the first-order derivative on the left-hand side is replaced by a fractional derivative $D^\alpha x$ of a suitable order $\alpha$). The key step of our considerations consists in the fact that the Lambert $W$ function (despite it is a complex function) can be, in some sense, manipulated in the real domain only. Then, we are very easily able to rediscover the known "iff" condition on $\lambda$ for the asymptotic stability of the zero solution to the fractional version of (1). In addition, a precise description of the decay/growth rate of the solutions can be obtained.