Széchenyi Terv Plusz Építsük együtt Magyarországot!
Magyarország Kormánya
Az Európai Unió finanszírozásával NextGeneration EU

The conference is supported by the National Laboratory for Health Security project RRF-2.3.1-21-2022-00006

The smallest bimolecular mass-action systems admitting Andronov–Hopf bifurcation

Balázs Boros

Department of Mathematics, University of Vienna, Austria

borosbalazs84@gmail.com

We systematically address the question of which small bimolecular reaction networks endowed with mass-action kinetics are capable of Hopf bifurcation. It is easily shown that any such network must have at least 3 species and at least 4 reactions, and its rank is at least 3. Terming the class of n-species, m-reaction, rank-r networks as (n, m, r) networks, we are able to fully classify bimolecular $(3, 4, 3)$ networks: with the extensive help of computer algebra, we divide these networks into those which forbid Hopf bifurcation and those which admit Hopf bifurcation.

Beginning with 14670 bimolecular $(3, 4, 3)$ networks which admit positive equilibria, we show that the great majority of these are incapable of Hopf bifurcation. At the end of this process, we are left 138 networks with the potential for Hopf bifurcation. These fall into 87 distinct classes, up to a natural equivalence. Out of the 87 classes we find that 86 admit nondegenerate Hopf bifurcation (supercritical, subcritical, or both). The remaining exceptional class robustly admits a vertical Hopf bifurcation.

Finally, we can use the results on bimolecular $(3, 4, 3)$ networks, along with previously developed theory on inheritance, to predict the occurrence of Hopf bifurcation in networks with more species and/or reactions. Thus, in fact, finding all small networks with the capacity for Hopf bifurcation greatly expands our knowledge of which reaction networks, not necessarily small, admit Hopf bifurcation.

The talk is based on a recent joint paper with Murad Banaji:

https://iopscience.iop.org/article/10.1088/1361-6544/acb0a8/pdf