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On critical cases in stability for neutral functional differential equations – An approach suggested by certain engineering applications

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It is known from standard textbooks on differential equations with deviated argument that, in the case of the neutral functional differential equations (NFDEs), a necessary condition for asymptotic stability is asymptotic stability of the difference operator associated to it. On the other hand, there are known several applications arising from mechanical and hydraulic engineering whose mathematical models display NFDEs having the difference operator in critical cases i.e. marginally stable (Lyapunov stable but not asymptotically stable. The natural challenge is how to solve such critical cases connected to stability or synchronization problems in engineering. We shall not refer to the direct approach of this challenge meaning emphasizing the resulting asymptotic behavior but take the approach of reformulating the models. This last approach is motivated by the so called stability postulate of N. G. Četaev, stating that "stability as a general phenomenon has to appear somehow in the basic natural laws". Stimulated by his ideas, we included inherent stability as a complementary model validation besides standard well posedness conditions. Consequently such an approach legitimates modifications of the basic models to achieve inherent stability. This modification is usually made in the sense of introducing "forgotten" (or neglected) energy dissipation terms. For e.g. hydraulic engineering energy dissipation is increased by introducing additional hydraulic dissipators. In the case of the mechanical engineering applications (various elastic beams in oilwell drilling, manipulator or crane dynamics, Huygens synchronization of mechanical oscillators) the stability improvement is achieved by taking into account elastic strains as in the case of the statically undetermined systems. As a consequence of these model improvements, the difference operators of the associated difference equations become asymptotically stable. The paper illustrates these aspects through three applications from water hammer of Hydraulic Engineering, oilwell drillstring vibrations and Huygens synchronization.