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Periodic solutions of simple differential delay equations

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We discuss the problem of existence of periodic solutions in simple form scalar differential delay equations. Historically one of the first and simplest equations is the famous Hutchinson–Wright equation [2, 7]. Many named delay equations were studied since then, some of which are the well-known Mackey-Glass [4], Wazewska–Lasota [6], Nicholson [5], and a few other equations.

Recently and over the previous years scalar essentially nonlinear differential delay equations were proposed as mathematical models of various real world phenomena (e.g. physiological processes [1, 3]). Though observed numerically the existence of periodic solutions in many of such models was not rigorously proved. We show that the slowly oscillating periodic solutions exist when the equations exhibit the negative feedback property and the unique equilibrium is linearly unstable. Several approaches can be used to prove the existence; among them is an extension of the well established techniques of the ejective fixed point theorem.

We also discuss the periodicity problem for similar equations with periodic coefficients.

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