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Threshold delay differential equations

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Threshold delays arise naturally in a wide variety of dynamical systems, including maturation and transport processes. When the speed of the process depends on the state of the system the delay is state-dependent. Moreover, even though the delay may be discrete, its evaluation depends on the solution over the whole delay interval. These can therefore be thought of as distributed delay problems, or functional differential equations, and standard off the shelf numerical methods for discrete delays cannot be applied directly. They can also be thought of as delay differential algebraic equations, by differentiating the threshold condition.

Hal Smith showed that when the (maturation) speed is strictly positive, the (threshold) delayed time is a strictly monotonically increasing function of the current time. This allows for a time rescaling to reduce the equation to a constant delay problem. Perhaps for this reason, these equations have not received much attention, however the time rescaling does not result in a constant delay when there are multiple delays, and since threshold delays are ubiquitous the dynamics of these equations is worthy of study. We will present the dynamics of a system of equations describing mRNA production and an idealised scalar DDE reduction of the model, and discuss some of the issues that arise in numerical and analytical treatments.