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A geometric method for computer assisted proofs in delay differential equations

Robert Szczelina

Jagiellonian University, Poland
robert.szczelina@uj.edu.pl

A covering relation is a tool to express a concept that a given map $f\colon \mathbb{R}^n \to \mathbb{R}^n$ stretches in a proper fashion one set over another. Similarly to one-dimensional interval coverings, covering relations can be used to obtain coding for the orbits of the system, which is generally referred to as symbolic dynamics. Due to their geometric nature and open conditions, covering relations can be rigorously checked with the computer assistance. We will present one possible extension of the finite-dimensional covering relations to infinite dimensional systems in Banach space X, with functions $f\colon X \to X$ being compact.

A recently developed high-order Lohner-type rigorous algorithm [1] can be used to get enclosures of solutions to systems of delay differential equations (DDEs) of quality good enough for various computer assisted proofs. We apply this method to get enclosures on images of some Poincaré maps f in the (subspace) of the phase space $C^0([-\tau, 0], \mathbb{R}^d)$ of DDEs to show covering relations in computer assisted proofs of several unstable periodic solutions to Mackey–Glass equation in the chaotic regime of parameters, and to prove persistence of symbolic dynamics (semiconjugacy to a subshift on two symbols) in a chaotic ODE perturbed with a delayed term with a relatively long delay.

The method in [1] is quite general and does not impose severe restrictions on the kind of solutions it can track, i.e. the integration time does not need to be a multiple of the basic time lag nor the solutions need not to be of a specific class, e.g. periodic.

[1] R. Szczelina, P. Zgliczyński, High-order Lohner-type algorithm for rigorous computation of Poincaré maps in systems of Delay Differential Equations with several delays, *J. Found. Comp. Math.*, accepted (2023), https://arxiv.org/abs/2206.13873