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Absence of small solutions and existence of Morse decomposition for a cyclic system of delay differential equations

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We consider the unidirectional cyclic system of nonautonomous delay differential equations

$$\dot{x}^i(t) = g^i(x^i(t), x^{i+1}(t-\tau^i), t), \qquad 0 \le i \le N,$$

where the indexes are taken modulo $N+1$, with $N \in \mathbb{N}_0$, $\tau := \sum_{i=0}^{N} \tau^i > 0$, and for all $0 \le i \le N$, the feedback functions $g^i(u,v,t)$ are continuous in $t \in \mathbb{R}$ and C^1 in $(u,v) \in \mathbb{R}^2$, and each of them satisfies either a positive or a negative feedback condition in the delayed term.

In the theory of infinite dimensional dynamical systems, the question of existence of small solutions (i.e. nonzero solutions that converge to an equilibrium faster than any exponential function) have a crucial role, since only in the absence of such solutions can one describe asymptotically the solutions from the stable manifold by the associated linear equation.

We show that all components of a small solution have infinitely many sign-changes on any interval of length τ. As a corollary we obtain that if a pullback attractor exists, then it does not contain any small solutions. In the autonomous case we also prove that the global attractor possesses a Morse decomposition that is based on an integer valued Lyapunov function.

Finally we show some analogoues partial results for the case of state-dependent delay.

Our results generalize some former results by Mallet-Paret and Polner. Partially joint work with István Balázs, Ferenc Á. Bartha and Tibor Krisztin.