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On the probability of existence of limit cycles

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The goal of this talk is the study of the probability of existence of limit cycles for the family of random vector fields

$$\dot{x} = Af(x) + Bg(y), \quad \dot{y} = Cf(x) + Dg(y),$$

where f and g are fixed smooth functions such that $f(0) = g(0) = 0$ and A, B, C and D are independent and identically distributed random variables with $N(0, 1)$ distribution. Notice that this family is a natural extension of planar linear random vector fields where $f(x) \equiv x$ and $g(y) \equiv y$.

To achieve our goals we first develop several results of non-existence, existence, uniqueness and non-uniqueness of limit cycles for the deterministic version of this family, $\dot{x} = af(x) + bg(y)$, $\dot{y} = cf(x) + dg(y)$, where a, b, c and d are real constants. These results are obtained by studying some Abelian integrals, via degenerate Adronov–Hopf bifurcations or by using the Bendixson–Dulac criterion.

To the best of our knowledge, this is the first time that the probability of existence of limit cycles for a non-trivial family of planar systems is obtained analytically. In particular, we give vector fields for which the probability of having limit cycles is positive, but as small as desired.

Most of the presented results are obtained in collaboration with Bartomeu Coll and Rafel Prohens.