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# Gronwall inequalities with superlinear growth

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We will discuss some types of integral type Gronwall inequalities for a non-negative function  $u$ . One inequality allows terms to include fractional powers  $u^{k/m}$  where  $k < m$ , as well as the important linear term, a second allows fractional powers with  $k > m$ . The most applicable situation in the second case is when  $k/m = 2$ , quadratic growth. Such an inequality is impossible in general so we have an extra condition. When the result is applied to the study of positive solutions of second order ODEs such as  $v'' + f(t, v, v') = 0$  when  $f \geq 0$ , the inequality is applied with  $u = |v'|$ , that is, we allow quadratic growth with respect to the derivative term provided (the extra condition in this case) that there is a known a priori bound on  $|v|$ .