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The global dynamics of enharmonic oscillators

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Discrete-time delays often arise in real-world problems, such as maturation times in population dynamics, time-delayed feedback loops in laser devices, and heat transfer lags in atmospheric models. Mathematically, delay differential equations (DDEs) produce dynamics in an infinite-dimensional phase space and contain complicated dynamics. In the talk, I will discuss the dynamics of the enharmonic oscillator, a scalar DDE of the special form

$$\dot{x}(t) = f(x(t), x(t-1)) := -\Omega\left(x(t)^2 + x(t-1)^2\right) x(t-1).$$

Here Ω is a positive nonlinear frequency function and f is assumed to decrease monotonically in the delayed component, i.e., $\partial_2 f < 0$.

Surprisingly, the structure of the maximal compact invariant set $\mathcal{A}$ can be described in detail. More precisely, $\mathcal{A}$ possesses a graph structure whose vertices correspond to periodic or stationary solutions of the delay equation and whose edges represent dynamic transition states connecting the vertices. We conclude that the connection graph is encoded in the frequency Ω, providing an explicit method to construct examples.