The conference is supported by the National Laboratory for Health Security project RRF-2.3.1-21-2022-00006

Example for a dynamically unstable ESS and a periodic orbit in matrix games under time constraints

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Matrix games under time constraints are natural extensions of matrix games. The consequence of a pairwise interaction is not only a strategy-dependent payoff described by a payoff matrix but a strategy-dependent waiting time which is to be waited by the contestants before the following interaction. The two values determine the fitness of the individual together. Then one can investigate the behaviour of the relating replicator dynamics. One of the fundamental theorems of evolutionary matrix games (without time constraints) asserts that the state corresponding to an evolutionarily stable strategy is an asymptotically stable rest point of the replicator equation [1, 2, 3]. In some particular cases ([4] and [5]), the theorem remains true under time constraints too. We show, however, that the theorem does not hold in general under time constraints. Namely, we give an example which has an evolutionary stable strategy such that the corresponding point of the replicator dynamics is unstable. Moreover, we point out through the rock-scissor-paper game that arbitrary small differences between waiting times can destabilize the rest point corresponding to an ESS. It is also shown that a stable limit cycle can arise around the unstable rest point in a supercritical Hopf bifurcation.

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