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Multiscale dynamics: From finite to infinite

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Systems with multiple time scales appear in a wide variety of applications. Yet, their mathematical analysis is challenging already in the context of ordinary differential equations (ODEs), where about four decades were needed to develop a more comprehensive theory based upon invariant manifolds, geometric desingularization, asymptotic analysis, and many other techniques that span across the mathematical sciences. This framework has become known as geometric singular perturbation theory (GSPT). Yet, for partial differential equations (PDEs) progress has been extremely slow due to many obstacles in generalizing several ODE methods. In my talk, I shall first provide an introduction to multiple time scale dynamics. Then I am going to outline several recent advances for fast-slow PDEs: (1) the extension of slow manifold theory for unbounded operators driving the slow variables, (2) the design of blow-up methods for PDEs, where normal hyperbolicity is lost and (3) amplitude/modulation theory for slowly-driven pattern forming systems. These advances provide one (of many) needed building blocks to understand the dynamics of multiscale PDEs.