The conference is supported by the National Laboratory for Health Security project RRF-2.3.1-21-2022-00006

Global stability for price models with delay

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This is a joint work with Tibor Krisztin. We consider the delay differential equation

$$\dot{x}(t) = a \int_0^r x(t-s)\, d\eta(s) - g(x(t)) \tag{1}$$

and the neutral differential equation

$$\dot{y}(t) = a \int_0^r \dot{y}(t-s)\, d\mu(s) - g(y(t)), \tag{2}$$

where $a > 0$, $ug(u) > 0$ for all $u \in \mathbb{R} \setminus \{0\}$, and some further conditions hold. Both equations can be interpreted as price models. Global asymptotic stability of $y = 0$ is obtained, in case $a \in (0,1)$, for (2) by using a Lyapunov functional. Then this result is applied to get global asymptotic stability of $x = 0$ for (1) provided $a \in (0,1)$. As particular cases, two related global stability conjectures are solved, with an affirmative answer.

Homoclinic orbits for the limiting case of the Mackey–Glass equation

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We consider the Mackey–Glass equation and let $n \to \infty$ to obtain the limiting case, namely,

$$x'(t) = -ax(t) + bf(x(t-1)), \tag{1}$$

where $f(\xi) = \xi$ for $\xi \in [0,1)$, $f(1) = 1/2$, and $f(\xi) = 0$ for $\xi > 1$.

In a previous work, we established the existence of *complicated* looking, orbitally asymptotically stable periodic orbits for (1) utilizing rigorous numerics. Now, we present how those techniques can be extended to localize an unstable periodic orbit p and show the existence of a homoclinic orbit to p.

This is a joint work with Gabriella Vas and Tibor Krisztin.

Global dynamics of a compartmental model for the spread of Nipah virus

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Nipah virus, originated in South-East Asia is a bat-borne virus causing Nipah virus infection in humans. This emerging infectious disease has become one of the most alarming threats to public health due to its periodic outbreaks and extremely high mortality rate. We establish and study a novel SIRS model to describe the dynamics of Nipah virus transmission, considering human-to-human as well as zoonotic transmission from bats and pigs. We determine the basic reproduction number which can be obtained as the maximum of three threshold parameters corresponding to various ways of disease transmission and determining in which of the three species the disease becomes endemic. By constructing appropriate Lyapunov functions, we completely describe the global dynamics of our model depending on these threshold parameters. Numerical simulations are shown to support our theoretical results and assess the effect of various intervention measures. Joint work with Attila Dénes.

Coupled delayed negative feedback loops: Stability and oscillations

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The production of mammalian blood cells involves an intertwined network of physiological processes, with nonlinear, delayed feedback control mechanisms. For example, erythrocytes (red blood cells) and thrombocytes (platelets) while each having their own main regulatory hormone, erythropoietin and thrombopoietin respectively, are also intercating, specially in pathological conditions. We consider the simplified model

$$\begin{cases} x'(t) & = -\alpha x(t) + f(x(t-\tau_1), y(t-\tau_2)) \\ y'(t) & = -\beta y(t) + g(x(t-\tau_1), y(t-\tau_2)) \end{cases}$$

with f and g appropriate Hill functions for the coupled regulation of these two cell lines to study how the interaction of the control mechanisms may influence the dynamics. Equilibrium solutions are determined, their stability established and the nature of the oscillations when instability occurs are investigated. The mathematical part of the analysis revolves around a transcendental characteristic equation of second order with two delays and a Centre manifold analysis at the change of stability of equilibria.

Periodic and connecting orbits for delay differential equations

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We consider a class of nonlinear differential equations with delayed feedback. Numerical results suggest that these can generate complex dynamics. An example is the famous *Mackey–Glass equation* modeling physiological processes in which time lag plays a significant role. The nonlinearity in the equation allows to model the so-called Allee effect in population dynamics.

First, a limiting version of the equation is introduced with discontinuous nonlinearity. A combination of analytical and verified numerical tools gives the existence of an orbitally asymptotically stable periodic orbit. Then it is shown that near this periodic orbit the original equation has a periodic orbit, as well.

For nonlinearities, allowing to model the Allee effect, an additional unstable equilibrium occurs. From this equilibrium point, and from small amplitude periodic orbits close to this equilibrium there exist connecting orbits to the stable periodic orbit obtained in the first step.

This is a joint work with Tibor Krisztin and Robert Szczelina.

Processed numerical methods for the SCIR epidemic model

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In mathematics there are several problems which can be described by differential equations with a particular, highly complex structure. For example, the epidemic models, which play an important role, as these models help in the analysis of the behaviour of diseases. Nowadays, the analysing of these models is becoming more and more important, due to the Covid-19 pandemic in previous years. Most of the time, we cannot produce the exact solution of these problems, therefore we approximate them numerically by using some numerical method. In this talk we analyse some methods, based on operator splitting, which approximate the exact solution of original ODE systems well while having a low computational complexity. The two most popular methods include the sequential splitting (SS) and the Strang–Marchuk (SM) splitting. We analyse the relationship between these methods and at the same time we discuss the properties of processed integrator methods (PIM). Then we generalize these methods and introduce the new extended processed methods (EPM) and the economic extended processed methods (EEPM) with low computational complexity. We test these methods in epidemic models. We solve the SCIR model numerically and compare the runtimes and errors.

Cell population models with explicit cell cycle length

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Cells are mesoscopic discrete entities within a multicellular organism, existing on a scale much larger than the molecules they are composed of, yet much smaller than the organism itself. Our expectations for mathematical models are that they can describe a wide range of cell numbers, including extremely low counts, enable the investigation of various phenotypes, and account for the cell cycle.

The cell cycle refers to the series of events that occur in a cell as it grows, replicates its DNA, and divides into two daughter cells. It is a highly regulated process that ensures the accurate duplication and distribution of genetic material. The cell cycle consists of several distinct phases, and its duration can vary depending on the cell type. Typically, the length of the cell cycle is measured in hours.

In this presentation, we will introduce novel stochastic and deterministic models that allow for easy modeling of the impact of the cell cycle on population dynamics. This is a joint work with Gergely Röst.

The smallest bimolecular mass-action systems admitting Andronov–Hopf bifurcation

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We systematically address the question of which small bimolecular reaction networks endowed with mass-action kinetics are capable of Hopf bifurcation. It is easily shown that any such network must have at least 3 species and at least 4 reactions, and its rank is at least 3. Terming the class of n-species, m-reaction, rank-r networks as (n, m, r) networks, we are able to fully classify bimolecular $(3, 4, 3)$ networks: with the extensive help of computer algebra, we divide these networks into those which forbid Hopf bifurcation and those which admit Hopf bifurcation.

Beginning with 14670 bimolecular $(3, 4, 3)$ networks which admit positive equilibria, we show that the great majority of these are incapable of Hopf bifurcation. At the end of this process, we are left 138 networks with the potential for Hopf bifurcation. These fall into 87 distinct classes, up to a natural equivalence. Out of the 87 classes we find that 86 admit nondegenerate Hopf bifurcation (supercritical, subcritical, or both). The remaining exceptional class robustly admits a vertical Hopf bifurcation.

Finally, we can use the results on bimolecular $(3, 4, 3)$ networks, along with previously developed theory on inheritance, to predict the occurrence of Hopf bifurcation in networks with more species and/or reactions. Thus, in fact, finding all small networks with the capacity for Hopf bifurcation greatly expands our knowledge of which reaction networks, not necessarily small, admit Hopf bifurcation.

The talk is based on a recent joint paper with Murad Banaji:

`https://iopscience.iop.org/article/10.1088/1361-6544/acb0a8/pdf`

Time delays, Hopf bifurcation and synchronization

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We consider networks of oscillator nodes with time delayed, global circulant coupling. We first study the existence of Hopf bifurcations induced by coupling time delay, and then use symmetric Hopf bifurcation theory to determine how these bifurcations lead to different patterns of phase-locked oscillations. We apply the theory to a variety of systems inspired by biological neural networks to show how Hopf bifurcations can determine the synchronization state of the network. Finally we show how interaction between two Hopf bifurcations corresponding to different oscillation patterns can induce complex torus solutions in the network.

Discrete asymptotic dynamics with a parabolic critical point

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The aim of this work is the study of the asymptotic dynamical behaviour, of solutions that approach parabolic fixed points in difference equations. In one dimensional difference equations, we present the asymptotic development for positive solutions tending to the fixed point. For higher dimensions, through the study of two families of difference equations in the two and three dimensional case, we take a look at the asymptotic dynamic behaviour. To show the existence of solutions we rely on the parametrization method.

The mathematical analysis of transmission dynamics of varicella

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Varicella is a very contagious disease occurring mostly among children. As a childhood disease it is reasonable to use age structure to model the transmission dynamics. Since the school year plays an important role in the contact rates among children, our aim is to build this effect in our model. That is, we use a realistic age structured model, which means that the transmission dynamics of the infection is handled as a continuous process during the school year, but students age into the next group at the end of the school year.

In my presentation I will show how to define the basic reproduction number for such models and give a method to its practical calculation. Finally, the method will be illustrated on a simple SIR system, and also applied for varicella including different vaccination strategies.

Joint work with Gergely Röst.

On continuously structured epidemic and ecological models

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The basic reproduction number $\mathcal{R}_0$ is defined, in an epidemic model, as the expected number of new infections produced by an infected individual in a fully susceptible population. In an ecological model, the basic reproduction number $\mathcal{R}_0$ is defined as the expected number of offspring that an individual has thoughout its life. It is generally calculated as the spectral radius of the so-called *next-generation operator*. However, when considering continuously structured populations defined in a Banach lattice X with concentrated states at birth it is not possible to define the next-generation operator in X. We will present an approach to compute $\mathcal{R}_0$ of such models as the limit of the basic reproduction number of a sequence of models for which $\mathcal{R}_0$ can be computed as the spectral radius of the next-generation operator. We will present some examples, in particular an epidemic model with asymptomatic transmission for which we will also discuss the final infection size.

Herpes and Chlamydia co-infection in humans

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A mathematical model was formulated to investigate *Chlamydia trachomatis* (*C. trachomatis*) and herpes simplex virus (HSV) co-infection in a population; taking into account that HSV can induce a viable but non-cultivable state of *C. trachomatis* in a host. A complete description of the global dynamics is given for the sub-systems where only one of the diseases is present. For the co-infection model, we show that the extinction or persistence of HSV is determined solely by the basic reproduction number of HSV, regardless of *C. trachomatis* prevalence. On the contrary, *C. trachomatis* can not always invade a HSV-endemic population even when it could invade a susceptible one, and this is determined by a newly introduced threshold parameter. By a limiting system approach, the existence of a co-infection steady state is shown when all reproduction numbers are greater than one. Applying the theory of asymptotically autonomous systems, we prove global stability results for the disease free and the boundary equilibria. Finally, we calibrate the model to estimate the prevalence of both diseases in the population, and compare it with epidemiological observations. Joint work with Gergely Röst.

Delayed pattern formation in two dimensional domains

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Our study focuses on the complex interactions between gene expression time delay and domain size in a reaction-diffusion system, which play crucial roles in the development of intricate patterns over time. To investigate these phenomena, we utilize an advanced version of the Schnakenberg model called the LI (ligand internalisation) model. On a one-dimensional domain, a linear relationship has been observed between the gene expression time delay and the time it takes for patterns to form. We extend the model to the two dimensional domain, and confirm that similar relationship holds there as well. However, our exploration reveals a non-monotonic correlation between domain size and the time required for pattern emergence, and identifies critical domain sizes that optimize the efficiency of pattern formation. In our attempt to unravel this complex dynamics, we expand our analysis by considering a diverse range of initial conditions, including random perturbations of the spatially homogenous steady state as well as initial functions from its unstable manifold, to observe how pattern formation unfolds. Finally, we compute a two-parameter chart of patterns, with respect to time delay and domain size.

Joint work with Bornali Das, István Balázs, and Gergely Röst.

Numerical analysis for structured population models

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Population dynamics can be described by taking into account individuals traits, e.g., age, size and spatial movement [3, 4]. These models, to which we refer as structured population models, are often formulated as (integro-)partial differential equations with nonlocal boundary conditions. From the dynamical system point of view, as a result, their evolution is considered on abstract spaces. In order to assess local stability of equilibria or other invariants, one is typically led to investigate the spectrum of linear operators acting between infinite-dimensional vector spaces, a target that can rather be achieved analytically. In this talk I will present a general numerical approach based on collocation for approximating those spectra in the case of two (physiological or spatial) structures. Convergence has been rigorously investigated for some important problems, numerical tests confirm the general validity of the approach and applications are provided [1, 2].

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[2] D. Breda, S. De Reggi, R. Vermiglio, A numerical method for the stability analysis of linear age-structured models with nonlocal diffusion, arxiv.org/abs/2304.10835. Submitted.

[3] P. Magal, S. Ruan, *Structured population models in biology and epidemiology*, Lect. Notes Math., Springer, 2008.

[4] J.A. Metz, O. Diekmann, *The dynamics of physiologically structured populations*, Springer, 1986.

Global dynamics of a compartmental model to assess the effect of transmission from deceased

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During several epidemics, transmission from deceased people significantly contributed to disease spread, but mathematical analysis of this transmission has not been seen in literature numerously. Transmission of Ebola during traditional burials was the most well-known example, however, there are several other diseases such as hepatitis, plague or Nipah virus that can potentially be transmitted from disease victims. This is especially true in the case of serious epidemics when healthcare is overwhelmed and the operative capacity of the health sector is diminished, such as it could be seen during the COVID-19 pandemic. We present a compartmental model for the spread of a disease with an imperfect vaccine available, also considering transmission from deceased infected in general. The global dynamics of the system are completely described by constructing appropriate Lyapunov functions. We perform numerical simulations to assess the importance of transmission from the deceased considering the data collected from three infectious disease Ebola virus disease, COVID-19, and Nipah fever to support our analytical results. Joint work with Saumen Barua.

Oscillation and asymptotic properties of second order delay equations: New trends

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Delays, arising in nonoscillatory and stable ordinary differential equations, can induce oscillation and instability of their solutions. That is why the traditional direction in the study of nonoscillation and stability of delay equations is to establish a smallness of delay, allowing delay differential equations to preserve these convenient properties of ordinary differential equations with the same coefficients. In this talk, we find cases in which delays, arising in oscillatory and asymptotically unstable ordinary differential equations, induce nonoscillation and stability of delay equations. We demonstrate that, although the ordinary differential equation $x''(t) + c(t)x(t) = 0$ can be oscillating and asymptoticaly unstable, the delay equation $x''(t) + a(t)x(t - \tau(t) - b(t)x(t - \theta(t)) = 0$, where $c(t) = a(t) - b(t)$, can be nonoscillating and exponentially stable. Results on nonoscillation and exponential stability of delay differential equations are obtained. On the basis of these results on nonoscillation and stability, the new possibilities of non-invasive (non-evasive) control, which allow us to stabilize a motion of single mass point, are proposed. Stabilization of this sort, according to common belief requires damping term in the second order differential equation. Results obtained in this paper refutes this delusion.

It is demonstrated that, although the solutions of the delay differential equation

$$x^{''}(t) + \sum_{i=1}^{m} p_i(t)x(t - \tau_i(t)) = 0 \text{ for } t \in [0, \infty), \quad \text{where } x(\xi) = \varphi(\xi) \text{ for } \xi < 0,$$

considered for the zero intitial functions $\varphi(\xi) = 0$ can be oscillating with amplitudes tending to infinity, there can exist such initial functions $\varphi(\xi)$ that amplitudes of its oscillation solutions "started" with such φ tend to zero when $t \to \infty$. The fact of tending solutions to zero on the semiaxis is obtained through their "fast" oscillation, i.e. lenght of distance between adjacent zeros. The basis of this property is in the folowing: small distance between zeros does

not allow amplitudes of oscillating solutions to increase. Even more, these amplitudes can tend to zero when $t \to \infty$. Results of this sort were considered as impossible. The exact estimates of this distances between zeros are proposed through estimates of the spectral radii of corresponding compact operators.

Unbounded solutions for differential equations with mean curvature operator

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This is the joint work with Mauro Marini and Serena Matucci (University of Florence).

We present necessary and sufficient conditions for the existence of unbounded positive solutions for nonlinear differential equations

$$(a(t)\Phi_E(x'))' + b(t)F(x) = 0, \quad t \geq t_0,$$

where Φ_E denotes the Euclidean mean curvature operator

$$\Phi_E(u) = \frac{u}{\sqrt{1+u^2}}\,.$$

We use a fixed point result for operators defined in a Fréchet space, which does not require the explicit form of the fixed point operator and reduces the solvability of a boundary value problem for nonlinear equations to the solvability of an associated boundary value problem for a linear equation.

The results illustrate the asymptotic proximity of such solutions with those of an auxiliary linear equation on the threshold of oscillation. A new oscillation criterion for equations with mean curvature operator, extending Leighton criterion for linear Sturm–Liouville equation, is also derived.

Shear-induced chaos via stochastic forcing: A tale of finding positive Lyapunov exponents

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We discuss the phenomenon of shear-induced chaos, coined by Wang and Young about twenty years ago and referring to chaotic behavior as a result of shear being magnified by some forcing, in the context of stochastic perturbations. As a latest result, we show the positivity of Lyapunov exponents for the normal form of a Hopf bifurcation, perturbed by additive white noise, under sufficiently strong shear strength. This completes a series of related results for simplified situations which we can exploit by studying suitable limits of the shear and noise parameters. Some general ideas concerning conditioned random dynamics, computer-assisted proofs and continuity of Lyapunov exponents will be highlighted along the way.

Nonstandard finite difference method and its application

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A model of a physical system corresponds to the construction of an approximate mathematical representation of the system, incorporating certain important and essential features of the system, while ignoring everything else. We consider the dicretization of the Cauchy problem for the ordinary differential equation. Our aim is construct such discrete models which result in convergent numerical solutions, and the discrete solutions preserve the main qualitaive properties of the solution of the continuous model. In our talk we introduce the nonstandard finite difference method (NSFD) and investigate its consistency and convergence. We also consider the qualitative propeties of this method. We show that this combined method does not only preserve the consistency order and convergence of the base ERK method but also have many other good features: it is both absolute stable and unconditionally nonnegativity preserving. We demonstrate our theoretical results on the extended Ross model for malaria propagation.

Asymptotic behaviour for nonautonomous delay differential systems

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The global dynamics of a family of nonautonomous systems of delay differential equations is studied. This family includes structured systems inspired in mathematical biology models, with either discrete or distributed delays in both the linear and nonlinear terms. Sufficient conditions for the persistence and permanence are established. For periodic systems, criteria for the existence of positive periodic solutions are also given. The results are illustrated with applications.

Partially based on joint work with R. Figueroa (Univ. Santiago de Compostela).

A structured population model with distributed recruitment on a space of measures

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In this talk we will formulate a physiologically structured population model with a distributed recruitment on the space of non-negative Radon measures. We will show how to apply the theory of strongly continuous positive semigroups in this setting. In particular, we will carry out a detailed spectral analysis to study the asymptotic behaviour of solutions.

Joint work with Anna Marciniak-Czochra and Piotr Gwiazda.

Linear multistep methods and Richardson extrapolation

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In this talk, we study the application the classical Richardson extrapolation (RE) technique to accelerate the convergence of sequences resulting from linear multistep methods (LMMs) for solving initial-value problems of systems of ordinary differential equations numerically. The advantage of the LMM-RE approach is that the combined method possesses higher order and favorable linear stability properties in terms of A- or $A(\alpha)$-stability, and existing LMM codes can be used without any modification.

This is a joint work with Lajos Lóczi (ELTE Eötvös Loránd University, Hungary and BME Budapest University of Technology and Economics, Hungary). The main results are based on the paper [1] and on an ongoing research project.

Acknowledgments The author was supported by the ÚNKP-22-5 New National Excellence Program of the Ministry for Culture and Innovation from the source of the National Research, Development and Innovation Fund.

[1] I. Fekete, L. Lóczi, Linear multistep methods and global Richardson extrapolation, *Appl. Math. Lett.*, **133**(2022), 108267.

Time reversible attractors of parabolic PDEs: A meandering tale of three noses

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As a paradigm of irreversibility, we consider dissipative parabolic semilinear "heat equations"

$$u_t = u_{xx} + f(x, u, u_x) \tag{PDE}$$

on the unit interval with Neumann boundary. By results of Angenent, Brunovský, Fusco, Henry, Matano, Rocha, Wolfrum, and myself, among others, the global attractors are determined by the shooting meanders for the ODE boundary value problem $0 = v'' + f(x, v, v')$. The celebrated 1974 Chafee-Infante case of cubic f is characterized by meanders with two noses.

As a natural next step, we investigate the heteroclinic connection graphs for meanders with three noses, i.e. innermost arcs to p, q, and $p + q$ nested meander arcs, respectively. Such meanders arise in the PDE context if, and only if, $p = r(q + 1)$, for some integer r. Surprisingly, all their connection graphs are *time reversible*: a rather non-intuitive bijection of equilibria *reverses* all heteroclinic directions.

We also explore the case $q = r(p - 1)$, which leads to the absurdity of negative "unstable dimensions" ranging from $1 - r$ to -1. Only after $r - 1$ unstable double cone suspensions, such meanders first provide parabolic attractors again. Much to our surprise, their connection graphs then seem to coincide with their time-reversible cousins above.

For general 3-nose meanders however, i.e. for general co-prime $p - 1$ and $q + 1$, a full classification remains elusive – after almost 50 years.

All this is joint work with Carlos Rocha.

Absence of small solutions and existence of Morse decomposition for a cyclic system of delay differential equations

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We consider the unidirectional cyclic system of nonautonomous delay differential equations

$$\dot{x}^i(t) = g^i(x^i(t), x^{i+1}(t - \tau^i), t), \qquad 0 \le i \le N,$$

where the indexes are taken modulo $N+1$, with $N \in \mathbb{N}_0$, $\tau := \sum_{i=0}^{N} \tau^i > 0$, and for all $0 \le i \le N$, the feedback functions $g^i(u,v,t)$ are continuous in $t \in \mathbb{R}$ and C^1 in $(u,v) \in \mathbb{R}^2$, and each of them satisfies either a positive or a negative feedback condition in the delayed term.

In the theory of infinite dimensional dynamical systems, the question of existence of small solutions (i.e. nonzero solutions that converge to an equilibrium faster than any exponential function) have a crucial role, since only in the absence of such solutions can one describe asymptotically the solutions from the stable manifold by the associated linear equation.

We show that all components of a small solution have infinitely many sign-changes on any interval of length τ. As a corollary we obtain that if a pullback attractor exists, then it does not contain any small solutions. In the autonomous case we also prove that the global attractor possesses a Morse decomposition that is based on an integer valued Lyapunov function.

Finally we show some analogoues partial results for the case of state-dependent delay.

Our results generalize some former results by Mallet-Paret and Polner. Partially joint work with István Balázs, Ferenc Á. Bartha and Tibor Krisztin.

On the dynamics of an electronic circuit in moderate dimensions

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We consider the cyclic system of differential equations

$$\dot{x}_n = -x_n + \alpha\sigma(x_{n-1}) + \beta\sigma(x_{n+1}), \quad n = 1, 2, \dots, N \tag{1}$$

with parameters in

$$\mathcal{R} = \Big\{(\alpha,\beta) \in \mathbb{R}^2 \mid \alpha > 0 \ \text{ and } \ \beta \in (0,\alpha]\Big\}$$

and the saturated, piecewise linear sigmoid nonlinearity

$$\sigma(x) = \frac{1}{2}(|x+1| - |x-1|) \quad \text{for each } x \in \mathbb{R} \tag{2}$$

modeling a Chua–Yang ring of N electrical oscillators with two–sided nearest neighbor couplings.

Comments on periodic and heteroclinic orbits are made. Numerical aspects are not trivial, too. Equilibrium patterns are discussed by using generalized Fibonacci–Lucas polynomials.

This is ongoing joint work with Miklós Koller

On the probability of existence of limit cycles

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The goal of this talk is the study of the probability of existence of limit cycles for the family of random vector fields

$$\dot{x} = Af(x) + Bg(y), \quad \dot{y} = Cf(x) + Dg(y),$$

where f and g are fixed smooth functions such that $f(0) = g(0) = 0$ and A, B, C and D are independent and identically distributed random variables with $N(0, 1)$ distribution. Notice that this family is a natural extension of planar linear random vector fields where $f(x) \equiv x$ and $g(y) \equiv y$.

To achieve our goals we first develop several results of non-existence, existence, uniqueness and non-uniqueness of limit cycles for the deterministic version of this family, $\dot{x} = af(x) + bg(y)$, $\dot{y} = cf(x) + dg(y)$, where a, b, c and d are real constants. These results are obtained by studying some Abelian integrals, via degenerate Adronov–Hopf bifurcations or by using the Bendixson–Dulac criterion.

To the best of our knowledge, this is the first time that the probability of existence of limit cycles for a non-trivial family of planar systems is obtained analytically. In particular, we give vector fields for which the probability of having limit cycles is positive, but as small as desired.

Most of the presented results are obtained in collaboration with Bartomeu Coll and Rafel Prohens.

Attracting period 3 implies all natural periods for multidimensional maps

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We present the method from [1] for finding a wide variety of periodic orbits for multidimensional maps with an attracting n-periodic orbit. The set of periods is induced by the Sharkovskii ordering '$\lhd$' of natural numbers:

$$3 \lhd 5 \lhd 7 \lhd \cdots \lhd 2 \cdot 3 \lhd 2 \cdot 5 \lhd \cdots \lhd 2^2 \cdot 3 \lhd 2^2 \cdot 5 \lhd \cdots \lhd 2^k \lhd 2^{k-1} \lhd \cdots \lhd 2^2 \lhd 2 \lhd 1.$$

As an example, we prove the existence of n-periodic orbits for all $n \in \mathbb{N}$ in the Rössler system with a 3-periodic orbit, the existence of n-periodic orbits for all $n \in \mathbb{N} \setminus \{3\}$ in a similar system with a 5-periodic orbit, *etc.* We also expect that this method works for DDEs (joint work in progress with R. Szczelina). The proofs are computer-assisted with the use of CAPD library for C++.

[1] A. Gierzkiewicz, P. Zgliczyński, From the Sharkovskii theorem to periodic orbits for the Rössler system, *J. Differential Equations*, **314**(2022), 733–751.

Extension of the Lanchester combat model to the Russian invasion of Ukraine

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The Russian invasion of Ukraine began on 24 February 2022. A unique feature of this war is that the Ukrainian armed forces not only use their own resources to fight, but also gather and deploy intact weapons left behind by their enemy. According to military experts, the capture of Russian weapons was paramount for Ukraine to achieve victory in the battles of Kharkiv and Lyman.

During the First World War, Lanchester constructed the first differential equation models to describe and quantify the losses of armies in war and the factors that influenced the outcome of battles. In Lanchester's model, the sign of a quadratic invariant determines which army will win the battle and which one will perish.

In this presentation we extend Lanchester's model to include capture of weapons from the enemy. For the extended model, we derive a new invariant, which generalizes the quadratic law, but has a much more complicated form. We analyze the role of capture in the outcome of battles, and parametrize the model for the Russian invasion using a database of documented military losses.

This is a joint work with Gergely Röst.

One-dimensional reduction of abstract renewal equations describing population dynamics

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Despite their relevance in mathematical biology, there are, as yet, few general results about the asymptotic behaviour of measure valued solutions of renewal equations on the basis of assumptions concerning the kernel. We characterise, via their kernels, a class of renewal equations whose measure-valued solution can be expressed in terms of the solution of a scalar renewal equation. The asymptotic behaviour of the solution of the scalar renewal equation, is studied via Feller's classical renewal theorem and, from it, the large time behaviour of the solution of the original renewal equation is derived.

The talk is based on joint work with Eugenia Franco and Odo Diekmann.

Stability preserving in parameter dependent systems

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In this talk the asymptotic stability of equilibrium solutions and the orbital stability of periodic solutions of parameter dependent differential equations are studied. We assume that the considered solution has some type of stability property with a certain value of the parameters and investigate the question of how much the parameters can be changed in order to guarantee the preservation of the stability.

On the role of the basic reproduction number in continuous and discrete systems modelling disease propagation

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This talk is about dynamical properties of an SIS model without vertical transmission. In the first part we give an overview, how the basic reproduction number $\mathcal{R}_0$ can be calculated in continuous, in discrete time, resp. in reaction-diffusion systems. Then the algorithm will be applied in these three cases. In the second part we examine the stability properties of a disease free equilibrium of a continuous time model and show that forward transcritical bifurcation takes place: a new locally asymptotically stable interior equilibrium emerges. After that we discretize the system by explicit Euler and a non-standard method of Mickens type and give conditions for the step size under which the dynamics of the discretized version resembles that of their continuous counterpart. Joint work with Sándor Kovács and Szilvia György.

On numerical approximation of functional differential equations with impulses using equations with piecewise constant arguments

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In this talk we study numerical approximation of several classes of functional differential equations with the help of equations with piecewise constant arguments. After summarizing some earlier work in this field, we study two recent results. First we consider a scalar linear delay equation with constant delay associated with an impulsive self-support condition. We define a numerical approximation scheme using a sequence of approximate delay equations with piecewise constant arguments, and we discuss its theoretical convergence. We present numerical examples to illustrate the applicability of the method, and we also observe existence of periodic solutions of the impulsive delay equation using numerical studies. In the second part of the talk we investigate uniform approximation of a nonautonomous delayed CNN-Hopfield-type impulsive system on the half-line $[0, \infty)$ with an associated impulsive differential system where a partial discretization is introduced with the help of piecewise constant arguments.

Heterogeneous carrying capacities and global extinction in metapopulations

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The lattice Nagumo equation (reaction-diffusion equation with bistable nonlinearity) may serve as a model of neuron transmissions, image processing or spatial population growth. This equation is well known for its rich structure of bounded stationary patterns and traveling wavefront.

In this talk we consider a simple two-patch Nagumo system with heterogeneous capacities. Two-patch models may serve as the simplest case of discrete-space models and reveal new phenomena. Moreover, they are directly connected to the construction of periodic solutions of lattice equations.

In contrast to the Nagumo two-patch system with the same capacities, which naturally provides two asymptotically stable and one nonstable stationary solutions for any combination of diffusion and reaction rates, we show that heterogeneous capacities may cause the disappearance of all nonzero equilibria. Zero solution is then the unique stationary solution and straightforwardly corresponds to the extinction of populations. We give a simple condition for capacities parameters which establishes that global extinction does not occur. On the other hand, we prove a sufficient condition for existence of the unique zero solution. Consequently, we obtain a nonexistence of traveling wave for Nagumo equation on periodic lattices.

Stability and periodic solutions for a price model with delay

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We consider the delay differential equation:

$$x'(t) = a\left[x(t) - x(t-1)\right] - \left|x(t-\tau)\right| x(t-\tau), \tag{1}$$

with $a > 0$, $\tau > 0$. This is a modification of a model for exchange rates introduced by Brunovský, Erdélyi, Walther (2004) with $\tau = 0$. If $a \in (0,1)$ and $\tau = 0$ then $x = 0$ is globally attracting (Balázs, Krisztin, 2019). If $a > 1$ and $\tau = 0$ then there is a stable slowly oscillating periodic solution (Brunovský, Erdélyi, Walther, 2004).

If $\tau = \frac{1}{4n}$ for some $n \in \mathbb{N}$ then equation (1) has a $\frac{1}{n}$-periodic solution and global attractivity of $x = 0$ is not satisfied. We estimate the region of attraction

$$A\left(a,\tau\right) = \left\{\varphi \in C\left(\left[-1,0\right], \mathbb{R}\right) :\ x^{\varphi}\left(t\right) \to 0 \text{ as } t \to \infty\right\}$$

of 0 in case $a \in (0,1)$, and show that $A\left(a,\tau\right)$ approaches $A\left(a,0\right) = C\left(\left[-1,0\right], \mathbb{R}\right)$ as $\tau \to 0^{+}$. Joint work with Tibor Krisztin.

The smallest bimolecular mass-action system with a vertical Andronov–Hopf bifurcation

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We present a three-dimensional differential equation, which robustly displays a degenerate Andronov–Hopf bifurcation of infinite codimension, leading to a center, i.e., an invariant two-dimensional surface that is filled with periodic orbits surrounding an equilibrium. The system arises from a three-species bimolecular chemical reaction network consisting of four reactions. In fact, it is, up to a natural equivalence, the only such mass-action system that admits a center via an Andronov–Hopf bifurcation.

The talk is based on a recent joint paper with Murad Banaji and Boros Balázs: https://www.sciencedirect.com/science/article/pii/S0893965923001039

Threshold delay differential equations

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Threshold delays arise naturally in a wide variety of dynamical systems, including maturation and transport processes. When the speed of the process depends on the state of the system the delay is state-dependent. Moreover, even though the delay may be discrete, its evaluation depends on the solution over the whole delay interval. These can therefore be thought of as distributed delay problems, or functional differential equations, and standard off the shelf numerical methods for discrete delays cannot be applied directly. They can also be thought of as delay differential algebraic equations, by differentiating the threshold condition.

Hal Smith showed that when the (maturation) speed is strictly positive, the (threshold) delayed time is a strictly monotonically increasing function of the current time. This allows for a time rescaling to reduce the equation to a constant delay problem. Perhaps for this reason, these equations have not received much attention, however the time rescaling does not result in a constant delay when there are multiple delays, and since threshold delays are ubiquitous the dynamics of these equations is worthy of study. We will present the dynamics of a system of equations describing mRNA production and an idealised scalar DDE reduction of the model, and discuss some of the issues that arise in numerical and analytical treatments.

Dynamics of chronic myelogenous leukaemia cell population with logistic growth and cell division delay

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We develop a nonlinear delay differential equation model to study the dynamics of chronic myelogenous leukemia (CML) cell concentration in the resting phase. In our model, we assume that cells leave the resting phase and enter a proliferation phase of duration τ at a rate that is smoothly dependent on the present concentration and is modelled by a logistic function. We derive a formula for σ from the characteristic equation of the system, which determines the stability of the zero and positive steady states. The study shows that the delay τ can cause stability switches, and the model undergoes a Hopf bifurcation at certain threshold values of τ and exhibits symmetric patterns as the time delay increases. The model is shown to be permanent when a certain condition is met, and numerical simulations are presented to illustrate the model's rich dynamics. The study concludes that when the delay exceeds a certain threshold value, the positive equilibrium vanishes, resulting in the decay of the cancer cells. The graphical representation diagram of the model's dynamics makes it easier to understand and interpret the results. Overall, the model proposed in this article provides insight into the dynamics of CML cancer cell concentration in the resting phase and sheds light on the role of time delay in cancer growth or decay. Joint work with Attila Dénes and Gergely Röst.

Periodic solutions of simple differential delay equations

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We discuss the problem of existence of periodic solutions in simple form scalar differential delay equations. Historically one of the first and simplest equations is the famous Hutchinson–Wright equation [2, 7]. Many named delay equations were studied since then, some of which are the well-known Mackey-Glass [4], Wazewska–Lasota [6], Nicholson [5], and a few other equations.

Recently and over the previous years scalar essentially nonlinear differential delay equations were proposed as mathematical models of various real world phenomena (e.g. physiological processes [1, 3]). Though observed numerically the existence of periodic solutions in many of such models was not rigorously proved. We show that the slowly oscillating periodic solutions exist when the equations exhibit the negative feedback property and the unique equilibrium is linearly unstable. Several approaches can be used to prove the existence; among them is an extension of the well established techniques of the ejective fixed point theorem.

We also discuss the periodicity problem for similar equations with periodic coefficients.

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Unexpected linearity: a first-order drug response curve observed and explained for SARS-CoV-2 in a hybrid mathematical model

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We observe, analyze and explain a pharmacokinetical phenomenon of extraordinary simplicity. Specifically, we show that the probability of infection extinction in a complex, non-deterministic hybrid mathematical model is a linear function of the virus removal rate under rather general circumstances. Joint work with Ferenc Bartha, Sadegh Marzban, Renji Han and Gergely Röst.

A class of difference equations modeling state-dependent delay

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We consider difference equations of the form

$$x_{k+1} = x_k - ag(x_k)x_k - a(1 - g(x_{k-1}))x_{k-1}, \quad a > 0, \quad g\colon \mathbb{R} \to [0, 1].$$

We view g as determining the share of a feedback response that occurs “quickly" versus “slowly"; thus the above equation can be viewed as a simple discrete-time model incorporating state-dependent delay. We present some preliminary results and pose several open questions.

Dissipative lattice dynamical systems

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Lattice dynamical systems (LDS) are essentially infinite dimensional systems of ordinary differential equations and are formulated as ordinary differential equations on Banach spaces of bi-infinite sequences. There have been many generalisations to include delayed, random and stochastic terms as well as multi-valued terms. LDS arise in a wide range of applications with intrinsic discrete structures such as chemical reaction, pattern recognition, image processing, living cell systems, cellular neural networks, etc. Sometimes they are derived as spatial discretisations of models based on partial differential equations, but they need not arise in this way.

There is an extensive number of papers on lattice dynamical systems. During the 1990s there was a strong emphasis on patterns and travelling waves in such systems. In recent decades attention focused on attractors with results summarised in the monograph [1].

This talk focuses on dissipative lattice dynamical systems and their attractors of various forms such as autonomous, nonautonomous and random. The existence of such attractors is established by showing that the corresponding dynamical system has an appropriate kind of absorbing set and is asymptotically compact in some way.

The main ideas and techniques are discussed and typical examples are presented including the approximation of Heaviside switching functions in LDS by sigmoidal functions.

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Nonlinear semigroups for nonlocal conservation laws

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We investigate a class of nonlocal conservation laws in several space dimensions, where the continuum average of weighted nonlocal interactions are considered over a finite horizon. We establish well-posedness for a broad class of flux functions and initial data via semigroup theory in Banach spaces and, in particular, via the celebrated Crandall-Liggett Theorem. We also show that the unique mild solution satisfies a Kružkov-type nonlocal entropy inequality. Similarly to the local case, we demonstrate an efficient way of proving various desirable qualitative properties of the unique solution. Finally, we also prove some perturbation results.

This is a joint work with M. A. Vághy (Pázmány Péter Catholic University).

On the conjecture of Miklós Farkas: Bifurcation of time-periodic pattern in reaction diffusion systems

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One of the fascinating properties of reaction-diffusion systems is the variety of special types of solutions they exhibit. Certain systems of this type can have for example travelling waves or rotating wave solutions, furthermore via bifurcation analysis one can show new class of solutions. This talk is about the possibility the occurrence of time periodic solution (time periodic pattern) of reaction-diffusion systems when the kinetic system, i.e. the system without the diffusion term exhibits periodic solution, as well. We look into the question whether the usual result regarding Turing bifurcation can be replaced by some other one.

Existence and multiplicity of solutions of Stieltjes differential equations

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This is a joint work with Alberto Cabada and F. Adrián F. Tojo (Departamento de Estatística, Análise Matemática e Optimización, Universidade de Santiago de Compostela).

We investigate a first order non-linear boundary value problem

$$u_g'(t) + b(t)u(t) = f(t, u(t)), \quad t \in [0, T]$$

with linear boundary conditions

$$u(0) = u(T) + k\, B(u),$$

where k is a constant and B is a linear functional. Note that boundary conditions extend the periodic case.

We use techniques from Stieltjes calculus and fixed point index theory to show the existence and multiplicity of solution.

Genesis of ectosymbiotic features based on commensalisic syntrophy

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The symbiogenetic origin of eukaryotes with mitochondria is considered a major evolutionary transition. The initial interactions and conditions of such symbiosis, along with the phylogenetic affinity of the host, are widely debated. We focus on a possible evolutionary path toward an association of individuals of two species based on uni-directional syntrophy. With the backing of a theoretical model, a commensalistic system based on the syntrophy hypothesis is considered in the framework of coevolutionary dynamics (density-dependent dynamical system) and mutant invasion (evolutionary substitution) into a monomorphic resident system. We investigate the ecological and evolutionary stability of the consortium (or symbiotic merger), with vertical transmissions playing a crucial role. The dynamics of the population densities of the involved species are represented using a set of ordinary differential equations, and the growth rates of each species are represented using novel Malthusian functions based on a branching process. The ecological fixation of the ectocommensalistic association is modeled in terms of the local asymptotic stability (linearization) of certain fixed points corresponding to the coevolutionary dynamical system. We find that the transmission of symbionts and the additional costs incurred by the mutant determine the conditions of fixation of the consortia.

Multiscale dynamics: From finite to infinite

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Systems with multiple time scales appear in a wide variety of applications. Yet, their mathematical analysis is challenging already in the context of ordinary differential equations (ODEs), where about four decades were needed to develop a more comprehensive theory based upon invariant manifolds, geometric desingularization, asymptotic analysis, and many other techniques that span across the mathematical sciences. This framework has become known as geometric singular perturbation theory (GSPT). Yet, for partial differential equations (PDEs) progress has been extremely slow due to many obstacles in generalizing several ODE methods. In my talk, I shall first provide an introduction to multiple time scale dynamics. Then I am going to outline several recent advances for fast-slow PDEs: (1) the extension of slow manifold theory for unbounded operators driving the slow variables, (2) the design of blow-up methods for PDEs, where normal hyperbolicity is lost and (3) amplitude/modulation theory for slowly-driven pattern forming systems. These advances provide one (of many) needed building blocks to understand the dynamics of multiscale PDEs.

Hopf bifurcation in a chronological age-structured SIR epidemic model

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It is known that the endemic equilibrium in a chronological age-structured SIR epidemic model is not always asymptotically stable, even if it uniquely exists when the basic reproduction number is greater than 1. In this study, we consider a special case where the transmission rate depends only on the age of the infective population and it is given by a shifted exponential function. We show that if the distance between the force of infection in the endemic equilibrium and the removal rate is sufficiently small, then there always exists a critical value such that a Hopf bifurcation occurs and the endemic equilibrium is destabilized when the bifurcation parameter reaches the critical value. This work is done in collaboration with Prof. Hisashi Inaba in Tokyo Gakugei University.

Mean square asymptotic stability characterisation of perturbed linear stochastic functional differential equations

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In this talk we present necessary and sufficient conditions that ensure various types of stability of the mean square of the following perturbed linear stochastic functional differential equation,

$$dX(t) = \left(f(t) + \int_{[-\tau,0]} X(t+s)\nu(ds) \right) dt + \left(g(t) + \int_{[-\tau,0]} X(t+s)\mu(ds) \right) dB(t),$$

where ν and μ are finite Borel measures. In particular we are concerned with when the mean square of solutions tend to zero, when this convergence to zero is exponentially fast and when the mean square lies in $L^1(\mathbb{R}_+)$. We find convergence to zero of the mean square of the unperturbed equation to be essential for sharp results, along with certain "interval average" conditions on the functions f and g. All conditions can be formulated in terms of the problem data. Additionally we highlight how the conditions on the forcing functions used

in our results can also be used to characterise the solution space of deterministic perturbed Volterra integrodifferential equations.

This is joint work with John Appleby (Dublin City University).

The effect of age-dependent toxicity of twig segments

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We use two types of age-structured population models to study how the distribution of toxins among segments of the twigs of woody plants, by affecting the feeding behavior of snowshoe hares, might affect snowshoe hare population dynamics. In the first model a twig has N discrete toxin containing segments joined end to end. Depending on species these segments vary tremendously in length. Preferred browse species such as the deciduous shrub birch Betula glandulosa have a small number of long twig segments and defend only the youngest segments near the twig tip. Hares counter this defense by biting off a twig at an older segment, eating only the older segments and rejecting younger, more toxic attached segments. Twigs of less preferred foods, such as the juvenile developmental stages of the evergreen spruces Picea glauca and P. mariana, have an arrangement of toxin producing resin ducts along their twig's long axis that is best modeled using a large number of short segments. We also propose a continuous model as an alternative to the N-segment model in the case when N is large. For each model we determine completely the conditions for linear stability of the hare-extinct equilibrium. An important implication is that the most effective defense against hares is to defend twig segments of all diameters that a hare can eat, as does spruce. Numerical simulations of both models confirm and enhance our understanding of the dynamics of the interaction between woody plants and snowshoe hares.

Optimal subsets in the stability regions of multistep methods

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We study the linear stability regions of linear multistep or multiderivative multistep methods for initial value problems by using techniques that are straightforward to implement in modern computer algebra systems. In many applications, one is interested in (i) checking whether a given subset of the complex plane (e.g., a sector, disk, or parabola) is included in the stability region of the numerical method, (ii) finding the largest subset of a certain shape

contained in the stability region of a given method, or (iii) finding the numerical method in a parametric family of multistep methods whose stability region contains the largest subset of a given shape. First, we describe a simple procedure to exactly calculate the stability angle α in the definition of $A(\alpha)$-stability. As an illustration, we exactly compute the stability angles for the k-step BDF methods and for the k-step second-derivative multistep methods of Enright. Next, we determine the exact value of the stability radius in the BDF family for each k, that is, the radius of the largest disk in the left half of the complex plane, symmetric with respect to the real axis, touching the imaginary axis and lying in the stability region of the corresponding method. Finally, we demonstrate how some Schur–Cohn-type theorems of recursive nature and not relying on the root locus curve method can be used to exactly solve some optimization problems within infinite parametric families of multistep methods. As an example, we identify the unique method in a two-parameter family of implicit-explicit (IMEX) methods having the largest stability angle, then we find the unique method whose stability region contains the largest parabola.

The global dynamics of enharmonic oscillators

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Discrete-time delays often arise in real-world problems, such as maturation times in population dynamics, time-delayed feedback loops in laser devices, and heat transfer lags in atmospheric models. Mathematically, delay differential equations (DDEs) produce dynamics in an infinite-dimensional phase space and contain complicated dynamics. In the talk, I will discuss the dynamics of the enharmonic oscillator, a scalar DDE of the special form

$$\dot{x}(t) = f(x(t), x(t-1)) := -\Omega\left(x(t)^2 + x(t-1)^2\right) x(t-1).$$

Here Ω is a positive nonlinear frequency function and f is assumed to decrease monotonically in the delayed component, i.e., $\partial_2 f < 0$.

Surprisingly, the structure of the maximal compact invariant set $\mathcal{A}$ can be described in detail. More precisely, $\mathcal{A}$ possesses a graph structure whose vertices correspond to periodic or stationary solutions of the delay equation and whose edges represent dynamic transition states connecting the vertices. We conclude that the connection graph is encoded in the frequency Ω, providing an explicit method to construct examples.

A quantification of long transient dynamics

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The stability of equilibria and asymptotic behaviors of trajectories are often the primary focuses of mathematical modeling. However, many interesting phenomena that we would like to model, such as the "honeymoon period" of a disease after the onset of mass vaccination programs, are transient dynamics. Honeymoon periods can last for decades and can be important public health considerations. In many fields of science, especially in ecology, there is growing interest in a systematic study of transient dynamics. In this work we attempt to provide a technical definition of "long transient dynamics" such as the honeymoon period and explain how these behaviors arise in systems of ordinary differential equations. We define a transient center, a point in state space that causes long transient behaviors, and derive some of its properties. We also define reachable transient centers, which are transient centers that can be reached from initializations that do not need to be near the transient center.

Modeling evolutionary diversification of plant-pollinator trophic networks

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Mutualism is recognized as a major driver of biodiversity by enabling widespread cospeciation in terrestrial communities. An important case is that of flowering plants and their pollinators, where the convergent selection of plant and pollinator traits combines with their divergent selection, which minimizes niche overlap within each group. In this article, we study cospeciation in communities structured trophically: plants are primary resource producers, required by primary consumers, the servicing pollinators. We model the natural selection of traits affecting resource consumption, competition between pollinators, and competition between plants. We show that species diversification is favored by broad plant niches, suggesting that bottom-up trophic control leads to cospeciation. We demonstrate that mutualistic generalism, i.e., tolerance of trait differences, promotes plant speciation, but it is also unfavorable for pollinator speciation. Evolved communities display skewed distributions of interaction effects, with proportional dominance of weak effects. Overall, we conclude that the trophic hierarchy of plant–pollinator communities is of utmost importance for the evolution of complex, richer, and diverse mutualistic networks.

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Period-two solution for a class of distributed delay differential equations

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We study the existence of periodic solutions for differential equations with distributed delay. It is shown that, for a class of distributed delay differential equation, a symmetric period-2 solution can be obtained as a periodic solution of a Hamiltonian system of ordinary differential equations, where the period is twice the maximum delay. This study extends the result of Kaplan and Yorke (J. Math. Anal. Appl., 1974) for a discrete delay differential equation with an odd nonlinear function. We illustrate the result presenting distributed delay differential equations that have periodic solutions expressed in terms of Jacobi elliptic functions.

Dynamical analysis of an HIV infection model including quiescent cells and immune response

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This talk presents a comprehensive analysis of an HIV infection model that incorporates quiescent cells and immune response dynamics within the host. The model, represented by a system of ordinary differential equations, captures the intricate interplay between the host's immune response and the viral infection. The study investigates the fundamental properties of the model, including equilibrium analysis, the computation of the basic reproduction number $\mathcal{R}_0$, stability analysis, bifurcation phenomena, numerical simulations, and sensitivity analysis. An endemic equilibrium, which reflects the persistence of the infection, and a disease-free equilibrium, which represents the possibility of disease control, are both revealed

by the analysis. By applying matrix-theoretical methods, stability analysis confirmed that the disease-free equilibrium is both locally and globally stable for $\mathcal{R}_0 < 1$. The research also reveals a transcritical forward-type bifurcation at $\mathcal{R}_0 = 1$, which denotes a critical threshold that affects the behavior of the system. The temporal dynamics of the model are investigated through numerical simulations, and sensitivity analysis determines the most important variables by examining the effects of parameter changes on the system's behavior.

Lambert W function method in investigating asymptotic properties of fractional delay differential equations

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Typically, stability/asymptotic issues connected with the linear delay differential equation

$$x'(t) = \lambda x(t - \tau), \quad \lambda \in \mathbb{C},\ \tau > 0, \tag{1}$$

can be treated by the Lambert W function method. Under some restrictions, this approach can still be used in the case of a fractional counterpart of the mentioned equation (i.e., when the first-order derivative on the left-hand side is replaced by a fractional derivative $D^\alpha x$ of a suitable order α). The key step of our considerations consists in the fact that the Lambert W function (despite it is a complex function) can be, in some sense, manipulated in the real domain only. Then, we are very easily able to rediscover the known "iff" condition on λ for the asymptotic stability of the zero solution to the fractional version of (1). In addition, a precise description of the decay/growth rate of the solutions can be obtained.

"Mild solutions" and variation of constants formula for delay differential equations

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The method and the formula of variation of constants for ordinary differential equations (ODEs) is a fundamental tool to analyze the dynamics of an ODE near an equilibrium. It is natural to expect that such a formula works for delay differential equations (DDEs), however, it is well-known that there is a conceptual difficulty in the formula for DDEs. In this talk, we discuss the variation of constants formula for DDEs by introducing the notion of a *mild solution*, which is a solution under an initial condition having a discontinuous history function.

Global stability for a nonlinear differential system with infinite delay and applications to BAM neural networks

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In this presentation, we give sufficient conditions for the global asymptotic stability of the following family of functional differential equations with finite delays in the linear terms and infinite delays in the nonlinear terms,

$$x_i'(t) = -a(t)x_i(t-\tau_i(t)) + h_i\Big(t, x(t-\tau_{i1}(t)), \dots, x(t-\tau_{im}(t))\Big) + f_i(t, x_t),$$

with $t \geq 0$ and $i \in \{1, \dots, n\}$.

The main stability criterion depends on the size of the delay on the linear part and the dominance of the linear terms over the nonlinear terms.

The general results are applied to obtain new global asymptotic and global exponential stability of a Bidirectional memory (BAM) neural network type model with delays which generalizes models recently studied. In the same way, some answers are given to open problems left by Berezansky et al. in 2014 [1].

A numerical example is presented to illustrate the effectiveness of the new results.

[1] L. Berezansky, E. Braverman, L. Idels, New global exponential stability criteria for nonlinear delay differential systems with applications to BAM neural networks, *Appl. Math. Comput.*, **243**(2014)

Oscillation criteria for the second-order linear advanced differential equations

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On the half-line $\mathbb{R}_+ = [0, +\infty[$, we consider the second-order linear differential equation with argument deviation

$$u''(t) + p(t)u(\sigma(t)) = 0, \tag{1}$$

where $p\colon \mathbb{R}_+ \to \mathbb{R}_+$ is a locally Lebesgue integrable function and $\sigma\colon \mathbb{R}_+ \to \mathbb{R}_+$ is a continuous function such that $\sigma(t) \geq t$, for $t \geq 0$.

New oscillatory criteria are established for solutions to equation (1). Riccati's technique and suitable estimates of non-oscillatory solutions are used for the proof of the obtained results. The presented criteria, in a certain sense, generalize those known from the theory of ordinary differential equations.

Existence of a homoclinic solution for a delay differential equation

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We consider the delay differential equation

$$y'(t) = -ay(t) + b\begin{cases} y^2(t-1) & \text{if } y(t-1) \in [0,1) \\ 0 & \text{if } y(t-1) \geq 1 \end{cases} \tag{1}$$

with $b > a > 0$. Equation (1) is the limit version (as $n \to \infty$) of the Mackey-Glass type equation $x'(t) = -ax(t) + bx^2(t-1)/[1 + x^n(t-1)]$. Considering a local unstable manifold of the equilibrium point $\xi_* = \frac{a}{b}$ we get a solution $y : \mathbb{R} \to \mathbb{R}$ of Equation (1) such that

$$\lim_{t\to-\infty} y(t) = \xi_*, \quad y(0) = 1, \quad y(s) > 1, \text{ for } s \in [-1, 0).$$

If $\lim\limits_{t\to+\infty} y(t) = \xi_*$ then the solution y of Equation (1) is homoclinic to ξ_*. The transform $u(t) = by(t) - a$ leads to the equation

$$u'(t) = -au(t) + 2au(t-1) + u^2(t-1). \tag{2}$$

There is a unique $b^* > a$ so that $u : [-1, \infty) \to \mathbb{R}$ with $u^*(s) = b^* e^{-a(s+1)} - a$, $-1 \leq s \leq 0$, oscillates. Choosing $b = b^*$ in Equation (1), $\lim_{t \to +\infty} y(t) = \xi_*$ is satisfied if $u^*(t) \to 0$ as $t \to +\infty$. We give a $\rho = \rho(a) > 0$ such that

$$u_t^* \in B_\rho = \{\varphi \in C([-1, 0], \mathbb{R}) : \|\varphi\| < \rho\}$$

for all large t, and B_ρ does not contain periodic orbits. This step requires a careful choice of the exponential dichotomy constants at the equilibrium $u = 0$, and a computer-assisted estimation of u^* on a finite interval. A consequence is that $u^*(t) \to 0$, and therefore $y(t) \to \xi_*$ as $t \to +\infty$. The technique works for $a \in (0, a_*]$ so that near a_* the spectral condition at ξ_*, required for Shilnikov chaos, is satisfied.

This is a joint work with Tibor Krisztin and Mónika Polner.

Global asymptotic stability of nonautonomous master equations

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We consider nonautonomous master equations of finite-state, continuous-time Markovian jump processes with uniformly continuous and bounded transition matrix functions. The Earnshaw–Keener conjecture from 2010 states that if the omega-limit set of the transition matrix function contains at least one matrix which is neither decomposable nor splitting, then the difference of any two probability distribution solutions tends to zero at infinity. The conjecture has been confirmed under the additional assumption that the transition matrix function is almost-automorphic. In this talk, we give a proof of the conjecture in its full generality.

[1] B. A. Earnshaw, J. P. Keener, Global asymptotic stability of solutions of nonautonomous master equations, *SIAM J. Appl. Dyn. Syst.*, **9**(2010), No. 1, 220–237.

[2] M. Pituk, Global asymptotic stability of nonautonomous master equations: A proof of the Earnshaw–Keener conjecture, *J. Differential Equations*, **364**(2023), 456–470.

Dynamics of a delay differential SEIR model with test, trace, isolate

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We study an extension of a compartmental SEIR model by including a TTI-strategy (test, trace, isolate), inspired by COVID-19. Here infected individuals are identified with some testing rate, and then they and their contacts are quarantined for a fixed period of time, resulting a system of delay differential equations. We prove the threshold result between extinction and persistence of the disease, and then study the stability of the endemic equilibrium when the reproduction number exceeds unity. For a particular COVID-like set of parameters we observe rich dynamics such as stability switches of the endemic equilibrium, Hopf bifurcations, bistability regions and endemic bubbles. A particularly interesting situation is when a branch of periodic orbits connects a super- and a subcritical Hopf-bifurcation from the endemic steady state.

Joint work with Gabriella Vas and Gergely Röst.

From invariant manifolds to fiber bundles: Numerical dynamics of integrodifference equations

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Integrodifference equations are successful models to describe spatial dispersal and temporary evolution. For this reason they can be understood as a discrete-time counterpart to reaction-diffusion equations and form an interesting class of infinite-dimensional dynamical systems. Nevertheless, for the sake of numerical simulations integrodifference equations require a spatial discretization.

In this talk, we investigate how their full hierarchy of invariant manifolds (stable, center-stable, center, center-unstable, unstable) behaves under the commonly used discretizations methods. We begin with the classical situation near periodic solutions and proceed to a general nonautonomous framework.

[1] F. Lutscher, *Integrodifference Equations in Spatial Ecology*, Interdisciplinary Applied Mathematics, Springer, Cham, 2019.

[2] C. Pötzsche, Numerical dynamics of integrodifference equations: Periodic solutions and invariant manifolds in $C^{\alpha}(\Omega)$, submitted (2021)

[3] C. Pötzsche, Numerical dynamics of integrodifference equations: Hierarchies of invariant bundles of $L^p(\Omega)$, Numer. Funct. Anal. Optimization, accepted (2023)

On critical cases in stability for neutral functional differential equations – An approach suggested by certain engineering applications

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It is known from standard textbooks on differential equations with deviated argument that, in the case of the neutral functional differential equations (NFDEs), a necessary condition for asymptotic stability is asymptotic stability of the difference operator associated to it. On the other hand, there are known several applications arising from mechanical and hydraulic engineering whose mathematical models display NFDEs having the difference operator in critical cases i.e. marginally stable (Lyapunov stable but not asymptotically stable. The natural challenge is how to solve such critical cases connected to stability or synchronization problems in engineering. We shall not refer to the direct approach of this challenge meaning emphasizing the resulting asymptotic behavior but take the approach of reformulating the models. This last approach is motivated by the so called stability postulate of N. G. Četaev, stating that "stability as a general phenomenon has to appear somehow in the basic natural laws". Stimulated by his ideas, we included inherent stability as a complementary model validation besides standard well posedness conditions. Consequently such an approach legitimates modifications of the basic models to achieve inherent stability. This modification is usually made in the sense of introducing "forgotten" (or neglected) energy dissipation terms. For e.g. hydraulic engineering energy dissipation is increased by introducing additional hydraulic dissipators. In the case of the mechanical engineering applications (various elastic beams in oilwell drilling, manipulator or crane dynamics, Huygens synchronization of mechanical oscillators) the stability improvement is achieved by taking into account elastic strains as in the case of the statically undetermined systems. As a consequence of these model improvements, the difference operators of the associated difference equations become asymptotically stable. The paper illustrates these aspects through three applications from water hammer of Hydraulic Engineering, oilwell drillstring vibrations and Huygens synchronization.

Asymptotics of nonlinear fractional differential equations and regular variation

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In the talk, we will consider the fractional differential equation of the form

$$\mathcal{D}^{\alpha+1}y = p(t)\Phi_\gamma(y),$$

$t \in [0,\infty)$, where $\Phi_\gamma(u) = |u|^\gamma \operatorname{sgn} u$, $\gamma \in (0,1)$ (the *sublinearity* condition), p is a continuous function on $[0,\infty)$, positive on $(0,\infty)$, and $\alpha \in (0,1)$; the fractional differential operator is of Caputo type. We will present (unimprovable) conditions guaranteeing that this equation possesses asymptotically superlinear solutions (i.e., the solutions y such that $\lim_{t\to\infty} y(t)/t = \infty$). We will show that all of these solutions are regularly varying and establish precise asymptotic formulae for them. (In the very special case, when the coefficient is asymptotically equivalent to a power function and the order of the equation is 2, our results reduce to the known theorems in their full generality.) In addition to the asymptotically superlinear solutions we will discuss also other (asymptotic) classes of solutions, some of them having no ODE analogy. We will reveal several substantial differences between the integer order and non-integer order case. We will highlight some of the tools used in the proofs such as theory of regular variation (in particular, the fractional Karamata integration theorem) and the fractional generalized L'Hospital which have the great potential to find applications in much wider “fractional” framework.

From Newton to COVID, via Riesz

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A famous anagram of Newton says that it is useful to solve differential equations. In this talk, I give an overview of how differential equations were used to fight COVID-19, by providing vital information to understand which factors are governing the transmission dynamics, to generate forecasts, and to plan rapid pandemic response. Many examples are taken from the work of the Hungarian COVID-19 Epidemiological Analysis and Modelling Task Force, which operated from March 2020 till March 2022, and then transformed into the National Laboratory for Health Security.

F. Riesz was one of the founders of the Bolyai Institute at the University of Szeged, who made fundamental contributions to functional analysis. Interestingly, some of his very theoretical results can give us significant insights into epidemiological dynamics as well, as I will illustrate in a case study.

In the last part, I show some of our results with Gabriella Vas (1983–2021) about the application of delayed relay systems in epidemic control.

The results presented in the talk are joint work with many colleagues.

Principal spectral theory and asynchronous exponential growth for age-structured models with nonlocal diffusion of Neumann type

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We study the principal spectral theory and asynchronous exponential growth for age-structured models with nonlocal diffusion of Neumann type. First, we provide two general sufficient conditions to guarantee existence of the principal eigenvalue of the age-structured operator with nonlocal diffusion. Then we show that such conditions are also enough to ensure that the semigroup generated by solutions of the age-structured model with nonlocal diffusion exhibits asynchronous exponential growth. Compared with previous studies, we prove that the semigroup is essentially compact instead of eventually compact, where the latter is usually obtained by showing the compactness of solution trajectories. Next, following the technique developed in Vo (*Math. Nach.* 2022) we obtain some limit properties of the principal eigenvalue with respect to the diffusion rate and diffusion range. Finally, we establish the strong maximum principle for the age-structured operator with nonlocal diffusion. (Based on H. Kang & S. Ruan, *Math. Ann.* 2022).

Epidemic patterns of emerging variants with dynamical social distancing

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Motivated by the emergence of new variants during the COVID-19 pandemic, we consider an epidemiological model of disease transmission dynamics, where novel strains appear by mutations of the virus. In the considered scenarios, disease prevalence in the population is modulated by social distancing. We study the various patterns that are generated under different assumptions of cross-immunity. If recovery from a given strain provides immunity

against all previous strains, but not against more novel strains, then we observe a very regular sequential pattern of strain replacement where newer strains predominate over older strains. However, if protection upon recovery holds only against that particular strain and none of the others, we find much more complicated dynamics with potential recurrence of earlier strains, and co-circulation of various strains. We compare the observed patterns with genomic analysis we have seen during the COVID-19 pandemic. Joint work with Gergely Röst.

Weak disconjugacy and weak controllability for linear Hamiltonian systems

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In this talk we focus on mutual interrelations between the notions of weak disconjugacy and weak controllability for linear Hamiltonian differential systems. These notions have been used in connection with the study exponential dichotomy, nonoscillation, and dissipative control processes for these systems (e.g. Johnson et al., 2016). We provide characterizations of the weak controllability and weak disconjugacy in terms of properties of certain subspaces arising in the recently introduced theory of genera of conjoined bases for linear Hamiltonian systems. We also present new results regarding the zero value of the maximal order of abnormality of the system in terms of a weak controllability condition, or in terms of a weak disconjugacy condition when the system is nonoscillatory and satisfies the Legendre condition. The talk may be regarded as a clarification of the previous considerations in the literature about the weak disconjugacy and weak controllability for linear Hamiltonian systems (e.g. Fabbri et al., 2011, and Johnson et al., 2016).

Oscillations in neuronal network models

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Starting from the mathematical model of a single neuron, dynamical models of neuronal populations are presented. The well-known integrate and fire model is extended with the adaptation of the neurons. The behaviour of the whole population can be described by the rate model, where the dynamic variables are the firing rates of certain populations of neurons and the quantity characterizing the adaptation. The model consisting of an excitatory and an inhibitory population together with adaptation has been shown recently to describe several phenomena observed in experimental studies. The model is introduced in the talk and some important features of its qualitative behavior is dealt with. The fold and Hopf bifurcation curves can be determined analytically. The existence of periodic orbits is studied numerically, by exploiting the fact that the system is piece-wise linear, hence an implicit formula can be derived for the Poincaré map. It is shown that two stable limit cycles may coexist with a stable steady state. The more detailed investigation of the model is presented in the talk given by Anita Windisch.

Singular Sturmian comparison theorems for even order differential equations and linear Hamiltonian systems

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This work was obtained jointly with Peter Šepitka (Masaryk University, Brno). In 2010, Aharonov and Elias proved a singular comparison theorem for two second order differential equations satisfying the Sturmian majorant condition. In this talk we present how this result can be generalized to two linear Hamiltonian systems. At the same time we do not impose any controllability condition. The results are phrased in terms of the comparative index and the numbers of proper focal points of the (minimal) principal solutions of these systems at both endpoints of the considered interval. The main idea is based on an application of new transformation theorems for principal and antiprincipal solutions at infinity and on new limit properties of the comparative index involving these solutions. The results are new even for completely controllable linear Hamiltonian systems, notably also for even order Sturm–Liouville differential equations. In this way we also obtain an extension of the previous result of Aharonov and Elias.

[1] D. Aharonov, U. Elias, Singular Sturm comparison theorems, *J. Math. Anal. Appl.*, **371**(2010), No. 2, 759–763.

[2] P. Šepitka, R. Šimon Hilscher, Singular Sturmian comparison theorems for linear Hamiltonian systems, *J. Differential Equations* **269**(2020), No. 4, 2920–2955.

[3] P. Šepitka, R. Šimon Hilscher, Sturmian comparison theorems for completely controllable linear Hamiltonian systems in singular case, *J. Math. Anal. Appl.*, **487**(2020), No. 2, 124030.

About stability of state-dependent delay equations

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There are almost no results in mathematical literature about exponential stability of state-dependent delay differential equations. One of the main purposes of this research is to fill this gap. An approach to study of stability of n-th order delay differential equations is proposed. On the basis of these results, new possibilities for stabilization by delay feedback input control are proposed.

Our research is the development of the ideas presented in the paper [1].

[1] A. Domoshnitsky, S. Shemesh, A. Sitkin, E. Yakovi, R. Yavich, Stabilization of third-order differential equation by delay distributed feedback control, *J. Inequal. Appl.* (2018), 341. https://doi.org/10.1186/s13660-018-1930-5

A short route to Rényi's parking constant

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A classical parking problem due to A. Rényi [2] asks for the expected number of cars of unit length that can be parked randomly in a street of a given length. The problem leads to a delay differential equation, and when the length goes to infinity, the mean density of the cars approaches Rényi's parking constant

$$C = \int_0^\infty \exp\left(-2\int_0^u \frac{1-\mathrm{e}^{-t}}{t}\,\mathrm{d}t\right)\mathrm{d}u \approx 0.7475979.$$

We propose an alternative derivation of this constant, which is inspired by N. G. de Bruijn's analysis of the Buchstab function in number theory [1]. It is shorter and more elementary than Rényi's original approach, and relies on the duality between differential equations with delayed and advanced arguments.

[1] N. G. de Bruijn, On the number of uncancelled elements in the sieve of Eratosthenes, *Indag. Math.*, **12**(1950), 247-256.

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Bifurcations of neural fields on the sphere

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Neural Fields are dynamical models which are used to study pattern formation in large groups of neurons. These differential equations combine a diffusion term, modelling gap junctions, with a nonlinear distributed delay term, modelling the synaptic connections. We investigate if there is a connection between the strenght of the diffusion term and synchronous waves of activations, seen in Parkinsonian patients.

A key feature is the spherical domain on which this neural field is defined. Therefore, we look at the periodic orbits which are generated by Hopf bifurcation in the presence of spherical symmetry. For this end, we derive general formulas to compute the normal form coefficients for these bifurcations up to third order and predict the stability of the resulting branches.

Vallée-Poussin theorem for fractional functional differential equations

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An analog of the classical Vallée-Poussin theorem about differential inequality in the theory of ordinary differential equations is developed for fractional functional differential equations. The main results are obtained in a form of a theorem about several equivalent assertions. Among them solvability of two-point boundary value problems with fractional functional differential equation, negativity of Green's function, and its derivatives and existence of a function $v(t)$ satisfying a corresponding differential inequality. Thus the Vallée-Poussin theorem presents one of the possible "entrances" to assertions on nonoscillating properties and assertions about the negativity of Green's functions and their derivatives for various two-point problems. Choosing the function $v(t)$ in the condition, we obtain explicit tests of sign-constancy of Green's functions and their derivatives. It can be stressed that a choice of a corresponding function in the Vallée-Poussin theorem leads to explicit criteria in the form of algebraic inequalities, which, as we demonstrate with examples, cannot be improved.

Semicycles and correlated asymptotics of oscillatory solutions to second-order delay differential equations

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We obtain several new comparison results on the distance between zeros and local extrema of solutions for the second order delay differential equation

$$x''(t) + p(t)x(t - \tau(t)) = 0,\ t \geq s$$

where $\tau : \mathbb{R} \to [0, +\infty)$, $p : \mathbb{R} \to \mathbb{R}$ are Lebesgue measurable and uniformly essentially bounded, including the case of a sign-changing coefficient. We are thus able to calculate upper bounds on the semicycle length, which guarantee that an oscillatory solution is bounded or even tends to zero. Using the estimates of the distance between zeros and extrema, we investigate the classification of solutions in the case $p(t) \leq 0, t \in \mathbb{R}$. Joint work with Elena Braverman and Alexander Domoshnitsky.

Square waves and Bykov T-points in DDEs with large delay

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Dynamical effects in DDEs with large delay have been studied extensively. They hold significance in various fields including optical and opto-electronic systems. These systems exhibit a wide range of intriguing phenomena, such as the generation of square waves and pulse solutions (temporal localized states). The study of square waves in DDE-systems date back to the early stages in the field. By formulating an equation with an advanced argument, we are able to treat square waves like temporal localized states. Using the limit of large delay, temporal localized states can be treated like homoclinic orbits in a desigularized equation. We can employ this approach that allows us to examine bifurcations of square waves supported by the framework of homoclinic bifurcation theory. We demonstrate our results by discussing the unfolding of a heteroclinic bifurcation called Bykov T-point in a model of the Kerr–Gires–Tournois interferometer, based on delay differential algebraic equations, in which square waves are generated.

Qualitative properties of some discrete models of a general SEIR model

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Studying epidemic models plays an important role both in a biological and in a mathematical point of view. In this talk we analyze an SEIR type compartmental model with a constant recruitment and natural death rate. We observe the case when the incidence rate function is given in a general form. This function describes how quickly the disease spreads in a population.

Some qualitative properties of the continuous models have been investigated. It is also important to define such models that preserve qualitative properties of the continuous problem: such as nonnegativity and the preservation of the amount of the members. We investigate these attributes using strong stability preserving (SSP) Runge–Kutta methods.

Propagation reversal for bistable differential equations on trees

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We study traveling wave solutions to the bistable differential equations on infinite k-ary trees in the form

$$\dot{u}_i = d(ku_{i+1} - (k+1)u_i + u_{i-1}) + g(u_i; a),$$

in which $i \in \mathbb{Z}$, $d > 0$ and $g\colon \mathbb{R} \to \mathbb{R}$ is a bistable nonlinearity of the Nagumo type, e.g.,

$$g(s; a) = s(1-s)(s-a), \quad a \in (0,1).$$

In this talk, we discuss how comparison principles and construction of explicit lower and upper solution can be used to obtain information about the dependence of the wave speed $c \in \mathbb{R}$ on the parameters a, d, k.

In particular, we show that for certain range of the detuning parameter a the changes to the diffusion parameter d lead to a reversal of the propagation direction.

Joint work with Hermen Jan Hupkes, Mia Jukić (Mathematisch Instituut, Universiteit Leiden) and Petr Stehlík (Department of Mathematics, Faculty of Applied Sciences, University of West Bohemia)

A geometric method for computer assisted proofs in delay differential equations

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A covering relation is a tool to express a concept that a given map $f\colon \mathbb{R}^n \to \mathbb{R}^n$ stretches in a proper fashion one set over another. Similarly to one-dimensional interval coverings, covering relations can be used to obtain coding for the orbits of the system, which is generally referred to as symbolic dynamics. Due to their geometric nature and open conditions, covering relations can be rigorously checked with the computer assistance. We will present one possible extension of the finite-dimensional covering relations to infinite dimensional systems in Banach space X, with functions $f\colon X \to X$ being compact.

A recently developed high-order Lohner-type rigorous algorithm [1] can be used to get enclosures of solutions to systems of delay differential equations (DDEs) of quality good enough for various computer assisted proofs. We apply this method to get enclosures on images of some Poincaré maps f in the (subspace) of the phase space $C^0([-\tau, 0], \mathbb{R}^d)$ of DDEs to show covering relations in computer assisted proofs of several unstable periodic solutions to Mackey–Glass equation in the chaotic regime of parameters, and to prove persistence of symbolic dynamics (semiconjugacy to a subshift on two symbols) in a chaotic ODE perturbed with a delayed term with a relatively long delay.

The method in [1] is quite general and does not impose severe restrictions on the kind of solutions it can track, i.e. the integration time does not need to be a multiple of the basic time lag nor the solutions need not to be of a specific class, e.g. periodic.

[1] R. Szczelina, P. Zgliczyński, High-order Lohner-type algorithm for rigorous computation of Poincaré maps in systems of Delay Differential Equations with several delays, *J. Found. Comp. Math.*, accepted (2023), https://arxiv.org/abs/2206.13873

Persistence and stability of generalized ribosome flow models

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In this contribution the qualitative dynamical properties of generalized ribosome flow models are presented using a compartmental modeling approach. Compartmental models are used to describe and analyze the transport between different containers, called compartments in various natural and technological systems. The modeled objects (molecules, people, vehicles, etc.) can move between compartments obeying the given constraints such as limits of directions, flow rates, or capacities. The dynamical modeling of the mRNA translation process has been in the focus of research since the second half of the 20th century. The first large scale analysis of gene translation through the so-called ribosome flow model (RFM) was presented in [1], where the applied second order nonnegative compartmental model based on the principle of totally asymmetric exclusion was able to capture the most important dynamical features of the translation process. We generalize the original RFM model class having a tubular or ring-like structure in two ways. Firstly, we assume arbitrary directed graph structure between the compartments and secondly, we allow general time-varying rate functions describing the compartmental transitions. Persistence of the dynamics is shown using the kinetic and the corresponding Petri net representation of the system by efficiently characterizing all siphons in the network. Further, we show the stability of different compartmental structures including strongly connected ones with an entropy-like logarithmic Lyapunov function. The L^1 contractivity of solutions is also studied in the case of periodic reaction rates having the same period. Additionally, it is shown that different Lyapunov functions may be assigned to the same model depending on the factorization of the transition rates. Finally, a so-called port-Hamiltonian representation of the system is constructed both in the original and in the reduced state spaces with clear connection between the structure matrices and the compartmental graph topology.

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Modelling the applicability of voluntary testing methods for COVID-19 transmission

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To prevent the spread of SARS-CoV-2, various measures have been implemented worldwide. One of most effective non-pharmaceutical measure is to conduct regular mass testing on a proportion of the population which can help reduce disease transmission. We develop a compartmental model to study the applicability of a voluntary testing method. The model includes isolated compartments as well, from where individuals become removed. A non-monotonic relationship is shown between the testing rate and the observed epidemic. By collecting real-world data, we aim to compare strategies throughout the pandemic by applying different testing activities. Joint work with Gergely Röst.

Location analysis of complex trinomial roots with respect to the unit circle in the complex plane

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Stability analysis of linear difference equation with constant coefficients is closely related to a location of roots of its characteristic polynomial, particularly with respect to the unit disk in the complex plane. We focus to a particular case of a trinomial with two strictly complex coefficients

$$T_{k,m}(\lambda) = \lambda^k + \mathrm{i}a\lambda^{k-m} + \mathrm{i}b,$$

where $a, b \in \mathbb{R}$ and $k > m$, $k, m \in \mathbb{N}$. Our aim is to analyze a relation between number of roots of the trinomial $T_{k,m}$ with a modulus lower than one, equal to one and greater than one, and a pair of parameters (a, b). The root locus technique is utilized to obtain regions in the (a, b) plane, where a number of roots of $T_{k,m}$ with modulus lower than one is preserved. Several figures will be introduced to demonstrate the above mentioned regions for particular cases of $T_{k,m}$ together with relevant remarks about their properties. It is also possible to use an introduced procedure in another special cases of polynomials. The talk is based on a joint work with Jiří Jánský.

Multiple solutions for one-dimensional billiard table with time-changing boundary

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This is a joint work with Jorge Rodríguez-López (Universidade de Santiago de Compostela).

We investigate a Dirichlet problem for an ODE of the second order with state-dependent impulses in the form

$$\begin{aligned}
&x''(t) = f(t, x(t)) \quad \text{for a.a. } t \in [0, T],\ x(t) \in (\alpha(t), \beta(t)),\\
&x'(s+) - \gamma'(s) = \gamma'(s) - x'(s-), \quad \text{if } x(s) = \gamma(s),\ \gamma \in \{\alpha, \beta\},\\
&x(0) = A \in (\alpha(0), \beta(0)), \quad x(T) = B \in (\alpha(T), \beta(T)),
\end{aligned}$$

where $\alpha, \beta \in W^{2,1}([0,T])$, $\alpha < \beta$, $f : [0,T] \times \mathbb{R} \to \mathbb{R}$ satisfies Carathéodory conditions on the set $\{(t,x) \ : \ 0 \leq t \leq T, \ \alpha(t) \leq x \leq \beta(t)\}$, $T > 0$. The impulse conditions correspond with the absolutely elastic impact of the ball at the boundary of the "billiard table". We give existence and multiplicity result for solutions with prescribed number of impacts.

Rigorous derivation of Michaelis–Menten kinetics in the presence of diffusion

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Reactions with enzymes are critical in biochemistry, where the enzymes act as catalysis in the process. One of the most used mechanisms for modeling enzyme-catalyzed reactions is the Michaelis–Menten (MM) kinetic. In the ODE level, i.e. concentrations are only on time-dependent, this kinetic can be rigorously derived from mass action law using quasi-steady-state approximation. This issue in the PDE setting, for instance when molecular diffusion is taken into account, is considerably more challenging and only formal derivations have been established. In this paper, we prove this derivation rigorously and obtain MM kinetic in the presence of spatial diffusion. In particular, we show that, in general, the reduced problem is a cross-diffusion-reaction system. Our proof is based on improved duality method, heat regularisation and a suitable modified energy function. To the best of our knowledge, this work provides the first rigorous derivation of MM kinetic from mass action kinetic in the PDE setting.

Global stability of a mathematical pandemic model with immigration and loss of immunity

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Population mobility is one factor contributing to the persistence of the COVID-19 pandemic. This work aims to investigate the global stability of a mathematical model of COVID-19. The main features of this model are the restriction of mobility for the infected population, including the immigration of individuals, and considering the individuals who lose their immunity. Hence, we investigate the global stability of SEIR and SEIRS models. As a result,

we proved the global asymptotically stability of the first model via a Lyapunov function and using the Volterra–Lyapunov matrix method for the second model.

Oscillation of first order delay differential equations with nonhomogeneous impulse

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In this work, We have discussed the behaviour of solutions of a class of first order functional differential equations with impulsive effect of the form:

$$\begin{cases} z(s) - q(s)z(s-\sigma)' + p(s)f(z(s-\tau)) = 0 \\ \Delta z(\sigma_k) = \beta_k z(\sigma_k) + d_k. \end{cases}$$

The results are illustrated with examples under suitable fixed moments of impulsive effect. Joint work with Rashmi R. Sahu.

Keywords: Oscillation, nonoscillation, neutral, impulsive differential equation, Schauder's fixed point theorem.

Mathematics Subject Classification (2010): 34K, 34C10, 34K11.

Example for a dynamically unstable ESS and a periodic orbit in matrix games under time constraints

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Matrix games under time constraints are natural extensions of matrix games. The consequence of a pairwise interaction is not only a strategy-dependent payoff described by a payoff matrix but a strategy-dependent waiting time which is to be waited by the contestants before the following interaction. The two values determine the fitness of the individual together. Then one can investigate the behaviour of the relating replicator dynamics. One of the fundamental theorems of evolutionary matrix games (without time constraints) asserts that the state corresponding to an evolutionarily stable strategy is an asymptotically stable rest point of the replicator equation [1, 2, 3]. In some particular cases ([4] and [5]), the theorem remains true under time constraints too. We show, however, that the theorem does not hold in general under time constraints. Namely, we give an example which has an evolutionary stable strategy such that the corresponding point of the replicator dynamics is unstable. Moreover, we point out through the rock-scissor-paper game that arbitrary small differences between waiting times can destabilize the rest point corresponding to an ESS. It is also shown that a stable limit cycle can arise around the unstable rest point in a supercritical Hopf bifurcation.

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Parameter estimation using bifurcation points

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Bistable behaviour and oscillatory patterns have often been observed in experimental studies of some ecological, chemical and mechanical systems. Some examples of such systems are the Belousov–Zhabotinsky reaction, the glycolytic pathway, predator-prey ecosystems and aircraft wings. Mathematical models describing the dynamics of these systems often attribute this behaviour to bifurcations. These bifurcation points can be experimentally measured by slowly varying the experimental controls until a sudden change in behaviour is observed. We propose a novel parameter estimation framework that uses these measured bifurcation points to estimate the parameters of the mathematical model. This approach is particularly useful when time-series data is not available for the usual parameter estimation methods used in mathematical modelling.

Real-time estimation of the effective reproduction number of COVID-19 from behavioral data

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Near-real time estimations of the effective reproduction number are among the most important tools to track the progression of a pandemic and to inform policy makers and the general public. However, these estimations rely on reported case numbers, commonly recorded with significant biases. The epidemic outcome is strongly influenced by the dynamics of social contacts, which are neglected in conventional surveillance systems as their real-time observation is challenging.

Here, we propose a concept using online and offline behavioral data, recording age stratified contact matrices at a daily rate. These contact matrices serve as an input to age structured compartmental models of transmission dynamics, expressed by a high-dimensional system of differential equations. Modeling the epidemic using the reconstructed matrices we dynamically estimate the effective reproduction number during the two first waves of the COVID-19 pandemic in Hungary by the spectral radius of a time-varying next generation matrix. Our results demonstrate how behavioral data combined with differential equation models can be used to build alternative monitoring systems complementing the established public health surveillance. They can identify and provide better signals during periods when official estimates appear unreliable due to observational biases.

This is a joint work with Eszter Bokányi, Júlia Koltai, Gergely Röst, and Márton Karsai.

Unbounded stationary solutions of lattice Nagumo equation

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The lattice Nagumo equation is a discrete-space reaction-diffusion equation which is well known for its rich structure of bounded stationary patterns. We study another class of stationary solutions which is also unique to the lattice version of Nagumo equations – global unbounded stationary solutions. We show that for any bistable cubic nonlinearity and arbitrary diffusion rate there exists a two-parametric set of equivalence classes of generally asymmetric stationary solutions which diverge to infinity.

Stability of Lyapunov exponents and Perron-type theorems

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In this talk, first we briefly review some fundamental notions and results in the spectral theory for systems of differrential equations. We focus on the stability of Lyapunov/Bohl exponents of solutions when a linear system is subject to linear and nonlinear perturbations. In the latter case, the result is referred to as Perron theorem. Then, we discuss some interesting results in the last two decades, which are extensions of Perron theorem to functional differential equations by Pituk (2006), nonautonomous differential equations by Barreira et al. (2015), and differential-algebraic equations by Linh et al. (2022). Finally, we propose two conjectures that would improve and extend Barreira et al.'s result. A preliminary result is also given under the integral separation assumption.

On solution manifolds

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Joint work with Tibor Krisztin: We show that for a system

$$x'(t) = g(x(t - d_1(Lx_t)), \ldots, x(t - d_k(Lx_t)))$$

of n differential equations with k discrete state-dependent delays the associated solution manifold $X_f \subset C^1([-r, 0], \mathbb{R}^n)$, on which the system defines a semiflow of differentiable solution operators, is an almost graph and therby nearly as simple as a graph over the trivial solution manifold X_0 given by $\phi'(0) = 0$. In particular X_f is diffeomorphic to an open subset of the closed subspace X_0. The map L in the system is continuous and linear from $C([-r, 0], \mathbb{R}^n)$ onto a finite-dimensional vectorspace, and g as well as the delay functions d_κ are assumed to be continuously differentiable.

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Gronwall inequalities with superlinear growth

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We will discuss some types of integral type Gronwall inequalities for a non-negative function u. One inequality allows terms to include fractional powers $u^{k/m}$ where $k < m$, as well as the important linear term, a second allows fractional powers with $k > m$. The most applicable situation in the second case is when $k/m = 2$, quadratic growth. Such an inequality is impossible in general so we have an extra condition. When the result is applied to the study of positive solutions of second order ODEs such as $v'' + f(t, v, v') = 0$ when $f \geq 0$, the inequality is applied with $u = |v'|$, that is, we allow quadratic growth with respect to the derivative term provided (the extra condition in this case) that there is a known a priori bound on $|v|$.

Coexisting periodic orbits in a 3-dimensional neuronal firing rate model

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Investigation of a deterministic population based firing rate model of neurons is presented. The system describes the dynamical behavior of an excitatory and an inhibitory population and as a negative feedback of excitatory cells, adaptation current is also added. The nonlinearity of the model is given by a shifted rectified linear unit function which is applied as activation function. The investigation of the model is focused on the effect of the adaptation current and the strength of connections between the two populations. Bifurcation theory is applied to study the dynamical behavior of the model. Our goal is to detect different kind of oscillations which are observed in neurobiological experiments. The piece-wise linear activation function allows us to give explicit formulas for the curves of local bifurcations. Hopf bifurcation is determined which results in birth of periodic orbits and unstable limit cycle is detected in the three-dimensional phase space via Poincaré map. The behavior of the model in the different parameter regions created by the bifurcation curves are described and a few examples for the most interesting phase portraits are shown including coexisting periodic solutions and bistability.

Dynamics of localized structures in DDEs with large delay

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Temporally localized structures arise in various types of applications and are often modelled by DDE systems with large delay. We develop a theory for such states, which appear as periodic solutions with a period close to the large delay of the system. Using the limit of infinite delay, they can be treated as homoclinic solutions of an equation with an advanced argument. Using the Morris–Lecar model with time-delayed feedback as a paradigmatic example, we demonstrate how classical homoclinic bifurcation theory can be used to study the emergence, stability, and bifurcations of such localized states. In particular, we show how a homoclinic orbit flip of a single-pulse solution leads to the destabilization of equidistant multi-pulse solutions and to the emergence of stable pulse packages. It turns out that this transition is induced by a heteroclinic orbit flip in the system without feedback, which is related to the excitability properties of the Morris–Lecar model.

Generic dynamics of nonlinear systems with embedded predator-prey feedback cycles

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Many nonlinear systems with embedded predator-prey interactions have the order-preserving monotonicity with respect to a rank-k cone (with $k > 1$). We show such systems enjoy the limit-set trichotomy properties, and the restricted semiflows on a given ω-set of precompact semiorbit can be characterized by a Lipschitz-continuous flow on the k-dimensional Euclidean space. In the case where $k = 2$, we obtain the generic Poincaré–Bendixson theorem, and the generic periodicity of high-dimensional epidemic systems with non-standard nonlinear instance functions. The talk is based on a series of studies in collaboration with Lirui Feng and Yi Wang, some of these results have been reported in SIAM J. Math. Anal. (2017), J. Differential equations (2021) and SIAM J. Applied Math. (2022), and many more to come.

Construction of deep neural networks using a time-delayed system

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We present a method for folding a deep neural network of arbitrary size into a single system with multiple time-delayed feedback loops [1]. This single-neuron deep neural network contains only a single nonlinearity and appropriately adjusted modulations of the feedback signals. The network states emerge in time as a temporal unfolding of the dynamics. By adjusting the modulations within the feedback loops, we adapt the network's connection weights. These connection weights are determined via a back-propagation algorithm. Our approach can fully represent standard deep neural networks (DNN), encompasses sparse DNNs, and extends the DNN concept toward dynamical systems implementations. The new method, which we call folded-in-time DNN (Fit-DNN), exhibits promising performance in a set of benchmark tasks.

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Mathematical modeling of COVID-19 transmission in the form of system of integro-differential equations

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The model of the spread of the coronavirus pandemic in the form of a system of integro-differential equations is studied. We focus our consideration on the number of hospitalized patients, i.e., on the needs of the system regarding hospital beds that can be provided for hospitalization and the corresponding medical personnel. Traditionally, in such models, the number of places needed was defined as a certain percentage of the number of infected at the moment. This is not quite adequate, since it takes a certain period of time for the development of the disease to the stage at which hospitalization is required. This will be especially evident at the start of new waves of the epidemic, when there is a large surge in the number of infected people, but the need for hospitalization places and additional medical personnel will appear later. Taking this circumstance into account using integral terms in the model allows us to conclude in corresponding additional to existing cases that the wave of disease will attenuate after some time. In others, it will relieve unnecessary panic, because the healthcare system has a certain period to create additional hospitalization places, order medicines and mobilize the necessary medical personnel. We obtain estimates of reproduction number in the case of the model described by a system of integro-differential equations. Results on the exponential stability of this integro-differential system are obtained. It is demonstrated that the condition of the exponential stability coincides with the fact that the reproduction number of the spread of the pandemic is less than one.