ABSTRACTS

Convergence analysis of collocation methods for the computation of periodic solutions of delay systems

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ω-periodic solutions of Renewal Equations (REs), Delay Differential Equations (DDEs) and coupled REs/DDEs systems can be expressed as BVPs in two equivalent – although formally different – ways, both used in literature. Recently, in [1], a general approach was proposed to solve BVPs for neutral Functional Differential Equations numerically, supported by the relevant theoretical convergence analysis. The approach includes non-neutral DDEs and, potentially, REs and coupled systems, but has never been specifically considered for periodic solutions, where ω is an unknown parameter. Thus, starting from DDEs, we show that periodic BVPs can be expressed in the form required in [1], and then verify that, using the right BVP formulation, all theoretical assumptions needed in [1] can be satisfied. We comment on the role of the period as parameter in determining the amount of work required for the proofs. We discuss the issues that arise when trying to use the other BVP formulation mentioned above. We conclude with a brief remark on the numerical significance of the method for DDEs and its relationship with the numerical method described in [2].

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Hartman–Wintner theorems for nonlinear differential systems

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In this talk we consider nonlinear versions of the classic Hartman–Wintner theorem, applied to different types of differential equations. In this context, "nonlinear" means that the space terms are not linear to leading order. The goal is to obtain necessary and sufficient conditions on the problem data (often perturbation or forcing terms) under which the solution x of the perturbed equation obeys $x(t)/y(t) \to 1$ as $t \to \infty$, where y is the solution of an unperturbed equation. The results apply to ODEs, FDEs and stochastic differential equations (SDEs), and also throw light on the performance of numerical methods used to recover the rates of growth or decay in solutions of ODEs.

Consensus problems in coupled delay differential equations on directed networks

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We consider a class of agreement models on networks, where the information transmission between the nodes is subject to time delays. We present some (rather unexpected) novel effects of the delay on the convergence speed and behavior. Interestingly, most of these effects manifest themselves only in directed networks and indicate a complex interaction of the temporal and spatial aspects of the system. We discuss some of the challenges in the analysis posed by directed networks. By contrast, undirected networks constitute a much better-understood case where exact results can be obtained.

A differential equation with a state-dependent queueing delay

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We consider a differential equation with a state-dependent delay motivated by a queueing process. The time delay is determined by an algebraic equation involving the length of the queue for which a discontinuous differential equation holds. The new type of state-dependent delay raises some problems that are studied in this talk. We formulate an appropriate framework to handle the problem, and show that the solutions define a Lipschitz continuous semiflow in the phase space. The second main result guarantees the existence of slowly oscillating periodic solutions.

On the existence and stabilization of an upper unstable limit cycle of the damped forced pendulum

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We consider the forced damped pendulum. By the use of intervalmethodsit is proventhat there exists an unstable periodic solutionto the dampedand periodically forced pendulum, and discuss some stabilization techniques for the upper, unstable equilibrium solution. One of these methods uses the current solution parameters (the angle and speed of the pendulum), thus this is a feedback technique. The other method does not use this kind of information, it is then a no feedback technique. We also analyzed by reliable computational methods whether these methods really stabilize the unstable solution. We were able to prove the necessary conditions for the stabilization of the unstable solution.

On the stability of a system of delayed logistic equations with patch structure

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The delayed logistic equation is a classical simple looking nonlinear delay differential equation whose global behavior is notoriously difficult to describe. It was proposed for modeling growth of a single species explaining observed oscillatory behavior. We explore extending existing results known for the scalar equation to a system with patch structure. We investigate what migration rates justify considering patch structure and, also, the stability of the system in terms of local growth and migration parameters. In particular, we will discuss the corner case of one-way migration.

Positive solutions of a class of linear discrete equations with delays

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A class of linear discrete delayed equations with delays

$$\Delta y(n) = -\sum_{s=1}^{k} \left(\frac{k}{k+1} \right)^s \frac{\alpha_s}{k+1} y(n-s)$$

is studied where $n \to \infty$, k is a fixed positive integer and α_s, $s = 1, \ldots, k$ are non-negative numbers. It is proved that if coefficients α_s, $s = 1, \ldots, k$ satisfy several conditions, then the equation has two positive solutions $y_1(n)$, $y_2(n)$ such that $\lim_{n\to\infty} y_1(n)/y_2(n) = 0$. Perturbed equations are also studied and the boundaries for perturbations guaranteeing the existence of positive solutions are given.

In the proof, the discrete variant of Wazewski's topological method is used.

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Stability analysis of an equation with two time delays and delay-dependant coefficients

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A stage-structured population model motivated by the regulation of platelet production is derived and analysed for its stability. The mature population is described by a delay-differential equation with two time delays and the linear stability of its equilibria leads to an equation with delay-dependant coefficients. The roles of both the death rate and the survival time are investigated, and a nonlinear analysis leads to the existence of an invariant torus.

Periodic and connecting orbits for Mackey–Glass type differential-delay equations

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We are looking for stable periodic solutions of

$$x'(t) = -ax(t) + b\frac{x^k(t-1)}{1+x^n(t-1)}$$

Mackey–Glass type equations, where $a > 0$, $b > 0$ are real parameters, $k > 1$ and $n > 1$ are integers. The $k = 1$ case gives the classic Mackey–Glass equation. The $k > 1$ case is important for modeling the so-called Allee effect in population dynamics. We know little about the dynamics generated by the equations both in case $k = 1$ and $k > 1$.

The solutions show a variety of behaviors for different values of the parameters: there are periodic and complicated solutions, the description of the dynamics is an intensely researched area.

By fixing $a > 0$, $b > 0$ and $k > 1$ as $n \to \infty$, we achieve a non-continuous equation

$$x'(t) = -ax(t) + bf(x(t-1))$$

where $f(x) = x^k$ if $x \in (0,1)$, $f(1) = 1/2$, and $f(x) = 0$ for $x > 1$.

First, for some $b > a > 0$ and $k > 1$ parameter values, we construct an orbitally asymptotically stable periodic solution for the non-continuous equation. Then we prove that for every large n, and for the above $b > a > 0$ and $k > 1$ parameter values, the original equation also has an orbitally asymptotically stable periodic solution close to the corresponding orbit of the non-continuous equation.

In addition to the stable periodic solution obtained, we prove the existence of certain heteroclinic orbits.

Asymptotic inference for linear stochastic differential equations with time delay

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Assume that we observe a stochastic process $(X(t))_{t\in[-r,T]}$, which satisfies the linear stochastic delay differential equation

$$\mathrm{d}X(t) = \vartheta \int_{[-r,0]} X(t+u)\, a(\mathrm{d}u)\, \mathrm{d}t + \mathrm{d}W(t), \qquad t \geq 0,$$

where a is a finite signed measure on $[-r, 0]$. The local asymptotic properties of the likelihood function are studied.

Firstly, we study a special case, when the delay is uniform. Namely, when a is the Lebesgue-measure and $r = 1$. In this special model local asymptotic normality is proved in case of $\vartheta \in \left(-\frac{\pi^2}{2}, 0\right)$, local asymptotic mixed normality is shown if $\vartheta \in (0, \infty)$, periodic local asymptotic mixed normality is valid if $\vartheta \in \left(-\infty, -\frac{\pi^2}{2}\right)$, and only local asymptotic quadraticity holds at the points $-\frac{\pi^2}{2}$ and 0.

In the general model we have not chance to determinate the exact values of the parameter, where the appropriate property is valid. However we can add a sufficient condition for this. Local asymptotic normality is proved in case of $v_\vartheta^* < 0$, local asymptotic quadraticity is shown if $v_\vartheta^* = 0$, and, under some additional conditions, local asymptotic mixed normality or periodic local asymptotic mixed normality is valid if $v_\vartheta^* > 0$, where v_ϑ^* is an appropriately defined quantity. As an application, the asymptotic behaviour of the maximum likelihood estimator $\widehat{\vartheta}_T$ of ϑ based on $(X(t))_{t \in [-r, T]}$ can be derived as $T \to \infty$.

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Stable periodic solutions for Nazarenko's equation

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We consider the scalar delay differential equation

$$\dot{x}(t) + px(t) - \frac{qx(t)}{r + x^n(t-\tau) = 0}, \quad t \in \mathbb{R}, \tag{1}$$

under the assumption that

$$p, q, r, \tau \in (0, \infty), n \in \mathbb{N} = \{1, 2, \dots\} \quad \text{and} \quad \frac{q}{p} > r.$$

Under this assumption, there exists a unique positive equilibrium: $[-\tau, 0] \ni t \mapsto K = (q/p - r)^{1/n} \in \mathbb{R}^+$.

This equation was proposed by Nazarenko in 1976 to study the control of a single population of cells [1].

Song, Wei and Han showed that a series of Hopf bifurcations takes place as τ passes through the critical values

$$\tau_k = \frac{q}{np(q-pr)}\left(2k\pi + \frac{\pi}{2}\right), \quad k \geq 0,$$

see [2]. Then they verified the global existence of the bifurcating periodic solutions by applying the global Hopf bifurcation theory of Wu. Song and his coauthors could not determine the stability of the periodic orbits for τ far away from the local Hopf bifurcation values. Uniqueness of the slowly oscillatory periodic solutions was not studied either.

In this talk we show that if $\tau > 0$ is large enough, then equation (1) has a unique positive periodic solution $p : \mathbb{R} \to \mathbb{R}$ oscillating slowly about K (up to translation of time), and p is asymptotically stable.

We also discuss the asymptotic shape of these periodic solutions as $n \to \infty$.

This is a joint work with Gabriella Vas.

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Instabilities in MBE growth equation

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The coarsening of growing interfaces, in the presence of Ehrlich–Schwoebel–Villain barrier that induces a pyramidal or mound-type structure without slope selection, is investigated. Our aim is to obtain results on the coarsening process by inspecting the behavior of branch of the steady state periodic solutions. The steady state solutions of the generalized phenomenological equation are analytically analyzed. It is found that the equation has periodic and not periodic solutions as well. For large slope of the particular solution we gave the connection between the slope and the amplitude or the wave length. For the periodic solutions the dispersion relation will be given, we show how the height amplitude varies indicating that the model exhibits the coarsening phenomena.

Mean-field models of go-or-grow type with realistic cell cycle length distributions

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We consider the mean-field approximation of individual-based models describing cell motility and proliferation, which incorporates the volume exclusion principle, the go-or-grow hypothesis and an explicit cell cycle delay. To utilise the framework of on-lattice agent-based models, we make a range of biological assumptions that allow us to incorporate cell cycle length into agent based models. The mean-field models are expressed by systems of delay differential equations and include variables such as the number of motile cells, proliferating cells, waiting cells, reserved sites and empty sites. For the case when cells enter mitosis only if they can secure an additional site for the daughter cell, and in which case they occupy two lattice sites until the completion of mitosis, we prove the convergence of biologically feasible solutions: eventually all available space will be filled by mobile cells, after an initial phase when the proliferating cell population is increasing then diminishing. By comparing the behaviour of the mean-field models for different parameter values and initial cell distributions, we illustrate that the total cell population may follow a logistic-type growth curve, or may grow in a step-function-like fashion. By explicit stochastic simulation, generalizing the Gillespie algorithm to a non-Markovian case, we investigate dependence of the speed of travelling wave solutions on the key biological parameters.

Joint work with Ruth Baker (Oxford) and Gergely Röst (Szeged).

Dynamics of planar S-systems

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S-systems are simple examples of power-law dynamical systems (polynomial systems with real exponents). For planar S-systems, we study global stability of the unique positive equilibrium, solve the center problem, and characterize permanence. Further, we construct a planar S-system with three limit cycles.

Stability of periodic orbits of renewal equations

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The Floquet theory and the principle of linearized stability, which allow to study the local stability of periodic solutions, are proved in [3] for abstract integral equations in the framework of sun-star calculus, assuming the validity of two hypotheses. In particular, in [3] the sun-star calculus is applied to retarded functional differential equations, and it is proved that those two hypothesis hold true in this case. The sun-star calculus is extended to renewal equations in [2]. In this talk I present the work [1], where we prove that the two hypotheses are satisfied in the case of renewal equations under suitable and reasonable conditions, thus ensuring the validity of the Floquet theory and of the principle of linearized stability. In [1] we also provide a detailed proof of [3, Theorem XIV.3.3] (principle of linearized stability for abstract integral equations), whose novel idea I present as well.

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Spirals in a delayed reaction-diffusion equation: from a thin annulus to a circle and back again

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Consider an optical system with an annular aperture, a refractive optical modulator, and a delayed feedback loop. The state of the system evolves according to the Debye relaxation equation for the phase difference induced by the modulator:

$$u_t = D\Delta u - u + K|e^{\mathrm{i}z_0\Delta}e^{\mathrm{i}u(t-T)}|^2, \quad r < \rho < R, \quad 0 \le \theta < 2\pi.$$

Here, $u(\rho, \theta, t)$ is a periodic real-valued function, which—with the help of special equipment—can be forced to satisfy oblique derivative boundary conditions

$$\rho u_\rho - \tan(\alpha) u_\theta = 0, \quad \rho = r, R.$$

To study periodic solutions such as spirals, one could attempt to construct a Hopf bifurcation normal form along the lines of [1] but the coefficients of this normal form cannot be computed explicitly because of the orthogonal projections and boundary value problems involved in the process. As a consequence, the connection between the physical parameters of the optical system and the stability properties of spiral waves is concealed, which complicates their modeling.

Helpful ideas can be drawn from the fact that in practice the annular domain is usually thin, $r \lesssim R$. As was shown in the seminal paper [2], the dynamics of reaction-diffusion equations on thin domains is closely related to the dynamics of their infinitely thin limit equations. We used these ideas to model rotating and standing waves in a system with Neumann boundary conditions [3], and by analyzing the Hopf bifurcation normal form of the one-dimensional model on a circle—which can be constructed explicitly [4]—managed to excite two-dimensional waves in an annulus.

For the oblique derivative system, the limit equation reads

$$\tilde{u}_t = \tfrac{1+\tan(\alpha)^2}{r^2} D\tilde{u}_{\theta\theta} - \tilde{u} + K|e^{\mathrm{i}\frac{1+\tan(\alpha)^2}{r^2} z_0 \partial_\theta^2} e^{\mathrm{i}\tilde{u}(t-T)}|^2, \quad 0 \leq \theta < 2\pi,$$

with periodic boundary conditions, and the system acquired an additional reflection symmetry. On analyzing the more complicated yet tractable O(2) Hopf bifurcation normal form of the limit equation instead of the simpler yet intractable SO(2) Hopf bifurcation normal form of the initial system, we observed stable (on the center manifold) rotating and pulsating spirals in our numerical experiments.

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Global attraction in systems of DDEs via the study of difference equations

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One tool to obtain information about global attraction in delay differential equations (DDEs) is to study certain related difference equations whose asymptotic behaviour is inherited by DDEs. The earlier works of Mallet-Paret and Nussbaum [3] and Ivanov and Sharkovsky [1] provided interesting results in this line for a DDE which is written in terms of a linear decay and a delayed feedback. Therefore, via this link, one can use results coming from discrete dynamical systems to obtain information about the qualitative properties of DDEs.

An extension of this type of results to some particular cases of systems of DDEs was considered in [2], where the concept of strong attractor plays a crucial role. Such concept recovers the essential features of convergence that are needed in the results of the scalar case. In this talk we will show several ideas to extend some theoretical results in [2] to more systems of DDEs by using a weaker concept.

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Dynamics of impulsive fractional stochastic evolution equations with unbounded delay

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This talk is concerned with the well-posedness and dynamics of delay impulsive fractional stochastic evolution equations with time fractional differential operator $\alpha \in (0, 1)$. After establishing the well-posedness of the problem, and a result ensuring the existence and uniqueness of mild solutions globally defined in future, the existence of a minimal global attracting set is investigated in the mean-square topology, under general assumptions not ensuing the uniqueness of solutions. Furthermore, in the case of uniqueness, it is possible to provide more information about the geometrical structure of such global attracting set. In particular, it is proved that the minimal compact globally attracting set for the solutions of the problem becomes a singleton. It is remarkable that the attraction property is proved in the usual forward sense, unlike the pullback concept used in the context of random dynamical systems, but the main point is that the model under study has not been proved to generate a random dynamical system.

Delay-dependent stability switches in fractional differential equations

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We discuss some qualitative properties of a linear fractional delay differential system

$$D^{\alpha}x(t) = Ax(t) + Bx(t-\tau)\,,$$

where $0 < \alpha < 1$ is a real number, D^{α} means the Caputo derivative, A, B are commutative real matrices and τ is a positive real time delay. As a main result, the explicit stability dependence on a changing time delay is described, including conditions for the appearance and calculations of stability switches when the stability property turns into instability and vice versa in view of a monotonically increasing lag. Some supporting asymptotic results are stated as well. Also, we compare the presented results with existing ones for the integer-order case.

On the mild solutions of a second-order integro-differential inclusion

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We study the following problem

$$x''(t) \in A(t)x(t) + \int_0^t K(t,s)F(s,x(s))ds, \quad x(0) = x_0, x'(0) = y_0 \tag{1}$$

where $F : [0,T] \times X \to \mathcal{P}(X)$ is a set-valued map, X is a separable Banach space, $\{A(t)\}_{t\geq 0}$ is a family of linear closed operators from X into X that genearates an evolution system of operators $\{S(t,s)\}_{t,s\in[0,T]}$, $\Delta = \{(t,s) \in [0,T] \times [0,T]; t \geq s\}$, $K(.,.) : \Delta \to \mathbf{R}$ is continuous and $x_0, y_0 \in X$. The general framework of operators $\{A(t)\}_{t\geq 0}$ that define problem (1) has been introduced by Kozak [2] and, afterwards, improved by Henriquez [1].

We consider this problem in the case when the set-valued map is not convex valued, but is Lipschitz in the second variable. We obtain several existence results for mild solutions of this problem using fixed point techniques and classical selection results as: Kuratowsky and Ryll-Nardzewski, Bressan and Colombo, De Blasi and Pianigiani. At the same time, we provide the arcwise connectedness of the solution set of problem (1).

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On the computer aided part of the proof for the Wright conjecture on a delay differential equation

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In 1955, E. M. Wright proved that all solutions of the delay differential equation $\dot{x} = -\alpha(e^{x(t-1)} - 1)$ converge to zero as $t \to \infty$ for $\alpha \in (0, 3/2]$ [5], and conjectured that this is even true for $\alpha \in [1.5, 1.5706]$ (compare with $\pi/2 = 1.570796\ldots$). The proof is based on the proven fact that it is sufficient to guarantee the nonexistence of slowly oscillating periodic solutions with small amplitudes cannot exist [1,2,3]. The talk will give details on the computer assisted proof part that exclude slowly oscillating periodic solutions with large amplitudes. We discuss also the relation of our work to the final proof [4].

Joint work with B. Bánhelyi, T. Krisztin and A. Neumaier.

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Periodicity, stability and instability in elementary way

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We consider the motions of an excited pendulum about their equilibria: two second order ODEs with step-function coefficient as excitation. We are looking for the values of parameters in the excitation for wich the upper equilibrium is stable and the lower one is unstable. Simple geometric ideas are applied for the construction of periodic solutions.

Modelling varicella vaccination in Hungary

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In our poster, we consider simple models of varicella infections including vaccination strategies. After theoretical study of equilibria, we present experiments on the effect of different vaccination strategies. The model parameters fit the Hungarian specialties. We also take into account the strong seasonality of varicella. Finally, we give some insight from the age-structured models.

Research is based on a joint research with János Karsai, János Dudás, Ágnes Dánielisz, Zsuzsanna Molnár and Gergely Röst.

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Two biological applications of a simple linear chain

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We consider a simple linear chain, which represents how particles (or, as in our particular case, cells) pass through a number of intermediate stages before reaching a final compartment. We determine the final state of this linear system as a function of initial data and transition rates, then we provide two biological applications. First, we model a laboratory experiment of chlamydia bacteria infecting human cells. We show that our simple model can replicate an important empirical finding of the chlamydia literature, and we can infer biological information such as the number of cell differentiation cycles and the rates of cell division. In a second application, we consider a stem cell population that differentiates into progenitor and eventually mature cells. In the absence of regulatory feedback, this is again can be modelled by a linear chain, where we can determine the eventual size of a developing organ from the initial stem cell population and the rates of self-renewal.

Joint work with Gergely Röst (Szeged).

Pseudospectral approximation for bifurcation problems in delay equations

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Pseudospectral approximation of delay equations was introduced in 2005 by Breda et al. as a numerical method to approximate eigenvalues of delay equations by eigenvalues of ordinary differential equations.

The pseudospectral approximation has a natural interpretation as a discretisation of the delay equation in the sun-star-framework. Therefore, the approximating ordinary differential equations 'inherit' the structure from the delay equation. Because of this, it has been conjectured that the pseudospectral method can be used as a numerical method for bifurcation problems in delay equations.

In this talk, this topic will be adressed in the context of the Hopf bifurcation. In particular, we will discuss the way in which 'direction of bifurcation' (i.e. the sign of the Lyapunov coefficient) is preserved in the pseudospectral approximation.

This is joint work with Odo Diekmann, Francesca Scarabel and Sjoerd Verduyn Lunel.

Global dynamics of an SIS model with multiple strains, superinfection and patch structure

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We study the global stability of a multistrain SIS model with superinfection and patch structure. We establish an iterative procedure to calculate a sequence of threshold parameters, and show that these completely determine the global dynamics of the system: for any number of patches and strains with different virulences, the stable coexistence of any subset of the strains is possible, and we completely characterize all scenarios.

Joint work with Y. Muroya and G. Röst.

Exponential stability tests for linear delayed differential systems depending on all delays

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Linear delayed differential systems

$$\dot{x}_i(t) = \sum_{j=1}^{m} \sum_{k=1}^{r_{ij}} a_{ij}^k(t) x_j(h_{ij}^k(t)), \ \ i = 1, \dots, m$$

are considered on a half-infinity interval $t \geq 0$. It is assumed that m and r_{ij}, $i, j = 1, \dots, m$ are natural numbers and the coefficients $a_{ij}^k \colon [0, \infty) \to \mathbb{R}$ and delays $h_{ij}^k \colon [0, \infty) \to \mathbb{R}$ are Lebesgue measurable functions. New explicit results on uniform exponential stability, depending on all delays, are derived. The conditions obtained do not require the dominance of diagonal terms over the off-diagonal terms as most of the existing stability tests for non-autonomous delay differential systems do.

The existence of solitary wave solutions of perturbed delayed Camassa–Holm equations

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In this talk, we discuss the Camassa–Holm equation, which is a model for shallow water waves. We first establish the existence of solitary wave solutions for the equation without delay. And then we prove the existence of solitary wave solutions for the equation with a special local delay convolution kernel and a special nonlocal delay convolution kernel by using the method of dynamical system, especially the geometric singular perturbation theory and invariant manifold theory. According to the relationship between solitary wave and homoclinic orbit, the Camassa–Holm equation is transformed into the ordinary differential equations with fast variables by using the variable substitution. It is proved that the equation with disturbance also possesses homoclinic orbit, and there exists solitary wave solution of the delayed Camassa–Holm equation.

An approach to blow-up times detection for delay-differential equations

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A problem of singular solutions of ordinary differential equations, when the solutions "blows up" to infinity in a finite time has been intensively studied both analytically and numerically. Often it is important to distinguish between blow-ups and an unbounded growth in infinite time interval. However almost no studies of blow-up solutions detection for delay differential equations exist. In a recent paper A. Takayasu and his co-authors suggested an approach to determine whether the solution of an ODE with a polynomial right-hand side reaches the infinity and whether there is a singularity. It uses the space and time compactification into a bounded domain with its boundary representing the infinity, thus making numerical solution easier, and construction of a Lyapunov function for the solution which helps to determine whether the solution is a blow-up. In the talk we try to apply the same technique to DDEs with polynomial right-hand sides. Indeed, the compactification (resulting in a system of state-dependent DDEs) helps obtaining the blow-up solutions and find blow-up time moments. However we could not yet find a way to construct suitable Lyapunov functions for DDEs so by now we do not have a reliable way to say if there is a blow-up or not. We present several test examples to demonstrate how the mentioned approach works for DDEs.

Reliable mathematical modeling of extended malaria model

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In order to have an adequate model, the continuous and the corresponding numerical models on some fixed mesh should preserve the basic qualitative properties of the original phenomenon. In the talk we focus our attention on some mathematical models of epidemic models and we will investigate their qualitative properties. First we investigate the SIR model. We give those conditions for the continuous and finite difference discrete models, under which the nonnegativity and some other basic qualitative properties are valid. Special attentions will be paid to the propagation of malaria with including vital dynamics. We formulate and investigate different mathematical models and we give sufficient conditions for the preservation of the basic qualitative properties. Numerical examples demonstrate the sharpness of the results.

Periodic solutions for differential equations with infinite delay and nonlinear impulses

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For a family of scalar periodic differential equations with infinite delay and nonlinear impulses, sufficient conditions for the existence of at least one positive periodic solution are established. The main technique used here is the Krasnoselskii–Guo fixed point theorem: this scenario has been used extensively, however the novelty of our results derives from the consideration of an original operator.

Our criteria are applied to some classes of Volterra integro-differential equations with unbounded or periodic delay, and with nonlinear impulses whose signs may vary.

This is a joint work with Sebastián Buedo-Fernández (Universidade de Santiago de Compostela, Spain).

New directions of generalized ODEs

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We present new results involving generalized ODEs and its applications.

Infinitely many stable *and* rapid Duffing oscillations, by delay

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For many decades it has been know that stable periodic solutions in scalar first order delay equations with monotone feedback are necessarily *slowly* oscillating. Occasionally, and in contrast to the first order case, it has been remarked how scalar second order delay equations may well exhibit stable *rapid* oscillations.

Based on some numerics and rather formal calculations, Richard Rand and others have in fact suggested, recently, that the delayed Duffing oscillator might exhibit an infinity of stable rapidly oscillating solutions, with amplitudes tending to infinity and periods tending to zero. We pursue the mathematical basis for this phenomenon, which also arises in the desirable context of linear delayed noninvasive feedback control of the standard Duffing oscillator.

Results are based on joint work with Alejandro Lopez, Richard Rand, Simohamed Sah, Isabelle Schneider, and Babette de Wolff. See also the talks by Alejandro, Babette, and Isabelle at this conference, as well as

http://dynamics.mi.fu-berlin.de/

Asymptotic problems for nonlinear ordinary differential equations with φ-Laplacian

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In this talk, we consider the asymptotic problems for the nonlinear differential equation

$$\big(a(t)\varphi(x')\big)' + b(t)|x|^{\gamma} \operatorname{sgn} x = 0,$$

where $\varphi : \mathbb{R} \to (-\sigma, \sigma)$ with $0 < \sigma \leq \infty$ is odd, continuous, strictly increasing, bijective, the real-valued functions $a(t)$ and $b(t)$ are positive and continuous on (t_0, ∞), and $\gamma \neq 1$ is a positive constant.

We give necessary and sufficient conditions for the oscillation of solutions of this equation. Moreover, we discuss the classification of unbounded solutions with different asymptotic behavior, such as asymptotically linear solutions, weakly increasing solutions, and extremal solutions. Our proofs are based on the Tychonov fixed point theorem.

This is a joint work with Professor Zuzana Došlá (Masaryk University, Czech Republic).

Absence of superexponential solutions for a cyclic system of delay differential equations

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The question of existence of superexponential solutions – i.e. nonzero solutions that converge to 0 faster than any exponential function – is an important and often challenging problem peculiar to infinite dimensional systems.

In this talk we consider the unidirectional cyclic system of delay differential equations

$$\dot{x}^i(t) = g^i(x^i(t), x^{i+1}(t - \tau^i)), \qquad 0 \leq i \leq N, \tag{1}$$

where the indices are taken modulo $N+1$, with $N \in \mathbb{N}_0$, $\tau^i > 0$, and the feedback functions $g^i : \mathbb{R}^2 \to \mathbb{R}$ are C^1-smooth, and each of them satisfy either a positive or a negative feedback condition in the second variable.

We show that all components of a superexponential solutions of (1) must have infinitely many sign-changes on any interval of length $\sum_{i=0}^{N} \tau^i$. As a consequence one gets that provided (1) possesses a global attractor, then it does not contain any superexponential solution.

Finally, we show how the result can be applied to construct a nontrivial Morse decomposition of the global attractor of certain equations of type (1).

On the dynamics of a Chua–Yang ring network in 8D

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A circle of eight identical Chua–Yang oscillators with two–parameter nearest–neighbor coupling is investigated. For the standard piecewise linear and saturated activation function, a complete description of equilibrium bifurcations and of all orbit connections between nonzero equilibria is given. Our second activation function discussed is the tangent hyperbolic function. We observe numerically that the essential dynamics is the same in both cases.

A system of differential equations with state-dependent delay from cell biology – differentiability, linearized stability and research in progress

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The talk will be on a system of differential equations with state-dependent time delay from cell biology, where the delay is defined implicitly as the time when the solutions of an ordinary differential equation parametrized by the history of a state variable meet a threshold. Recently it has been proven in [1], based on an approach of Walther and collaborators [3], that the solutions of the system define a differentiable semiflow. The differentiability property allowed in turn to prove a linearized stability theorem [1,2] based on approaches of [3] and a result of E. Stumpf. In [2] there is established analytical and numerical evidence that the interior equilibrium is stable upon emergence in a transcritical bifurcation and may destabilise in a Hopf bifurcation. Research topics in progress, on some of which I may go into during the talk, include convex and compact sets that are invariant under the time-t-map, oscillations, global asymptotic stability and dissipativity.

The research is based on joint work with István Balázs, Mats Gyllenberg, Gergely Röst, Francesca Scarabel and others.

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Computer-assisted proofs of chaos in low-dimensional dynamical systems

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By chaos in a continuous dynamical system we understand the existence of *symbolic dynamics* for the Poincaré map P on some Poincaré section. More precisely, we find a semi-conjugacy of P with the shift map on the space of bi-infinite sequences of two symbols.

Such a semi-conjugacy appears naturally if one finds a sequence of *covering relations* between some sets on the Poincaré section. If those sets are homeomorphic to cubes, then we can use interval arithmetic, which is easy to work on with use of computer, especially with Computer–Assisted Proofs in Dynamics (CAPD) C++ library (http://capd.ii.uj.edu.pl).

An illustrative example of the method is our recent work on the model of the rotation of Saturn's moon Hyperion. The angle of inner rotation θ fulfills a three-dimensional ordinary differential equation

$$\begin{cases} \theta' = \phi \\ \phi' = -\frac{\omega^2}{2r^3}\sin 2(\theta - f) \\ f' = \frac{(1+e\cos f)^2}{(1-e^2)^{3/2}} \end{cases} .$$

The model is expected to be chaotic for large range of parameters e, ω. We have rigorously proven the existence of symbolic dynamics for the Poincaré map on the 2-dim section $\{f = 0\}$ for 'Hyperion's' values of parameters $\omega = 0.89$, $e = 0.1$.

I will also present the application of covering relations for 3-dimensional sets for the 3D Henon map

$$H(x, y, z) = (a - y^2 - bz, x, y), \quad a = 1.76, \quad b = 0.1,$$

where one can prove the symbolic dynamics in the 'folded towel' attractor.

On models of physiologically structured populations and their reduction to ordinary differential equations

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Considering the environmental condition as a given function of time, we formulate a physiologically structured population model as a linear non-autonomous integral equation for the, in general distributed, population level birth rate. We take this renewal equation as the starting point for addressing the following question: When does a physiologically structured population model allow reduction to an ODE without loss of relevant information? We formulate a precise condition for models in which the state of individuals changes deterministically, that is, according to an ODE. Specialising to a one-dimensional individual state, like size, we present various sufficient conditions in terms of individual growth-, death-, and reproduction rates, giving special attention to cell fission into two equal parts and to the catalogue derived in an other paper of ours (submitted). We also show how to derive an ODE system describing the asymptotic large time behaviour of the population when growth, death and reproduction all depend on the environmental condition through a common factor (so for a very strict form of physiological age).

The talk is based on joint work with Odo Diekmann and Hans Metz

Qualitative properties of a class of retarded dynamical systems

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In this work we study the qualitative behaviour of a linearized system of differential equation with one discrete delay. The characteristic function of the mentioned system has the form

$$\Delta(z,\tau) := p(z) + q(z)e^{-z\tau} + r(z)e^{-2z\tau}$$

where p, q and r are polynomials with real coefficients and $deg(r) > deg(q)$. Using the Mikhailov criterion we give for special p, q and r an estimation on the length of delay τ for which no stability switching occurs. In the proof we use higher order Taylor polynomials, and compare our results with other methods. Then we give a condition under which stability is independent on the delay. Finally, a formula for Hopf bifurcation is calculated in terms of p, q and r giving an explicit formula for the Hopf bifurcation parameter. We apply these results for the so called "Love affair dynamics" and for the full Brusselator model.

Unbounded and blow-up solutions for a delay logistic equation with positive feedback

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We study bounded, unbounded and blow-up solutions of a delay logistic equation without assuming the dominance of the instantaneous feedback. It is shown that there can exist an exponential (thus unbounded) solution for the nonlinear problem, and in this case the positive equilibrium is always unstable. We obtain a necessary and sufficient condition for the existence of blow-up solutions, and characterize a wide class of such solutions. There is a parameter set such that the non-trivial equilibrium is locally stable but not globally stable due to the co-existence with blow-up solutions.

This is a joint work with Yukihiko Nakata and Gergely Röst.

Sigmoidal approximation of Heaviside functions in neural lattice models

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A sigmoidal neural field lattice system developed from Amari–Hopfield neural field lattice models will be discussed, in which the Heaviside function is replaced by a simplifying sigmoidal function characterized by a small parameter ε. The solutions of the resulting sigmoidal lattice system are studied and shown to converge to the solution of the Heaviside lattice system as ε approaches 0.

On the quasilinearization method for a parameter estimation problem in neutral functional differential equations with state-dependent delays

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In this talk we consider several classes of functional differential equations with state-dependent delays including explicit and implicit neutral equations. We discuss the problem of smooth dependence of the solutions on parameters. We formulate the quasilinearization method for the parameter estimation problem, and we present numerical studies to illustrate the convergence of the method.

On the problem of damping

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The equation

$$x'' + h(t, x, x')x' + f(x) = 0 \qquad (x \in \mathbb{R},\ xf(x) \geq 0,\ t \in [0, \infty))$$

is considered, where the damping coefficient h allows an estimate

$$a(t)|x'|^{\alpha} w(x, x') \leq h(t, x, x') \leq b(t)W(x, x').$$

Sufficient conditions on the lower and upper control functions a, b are given guaranteeing that along every motion the total mechanical energy tends to zero as $t \to \infty$. The key condition in the main theorem is of the form

$$\int_0^{\infty} a(t)\psi(t; a, b)\mathrm{d}t = \infty,$$

which is required for every member ψ of a properly defined family of test functions. Corollaries are deduced from this general result formulated by explicit analytic conditions on a, b containing certain integral means.

A sharpening of the Duhamel's principle for the forced vibration of an infinite string

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Cauchy problem is considered in the whole plane for the infinite string with continuous force under homogeneous initial conditions. We shall prove the existence of the classical solution under the condition: the force is smooth (only) in some direction.

Validated numerics for the unstable manifold of delay differential equations

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We briefly review a method to obtain the unstable manifold of equilibria for DDEs; the resulting parameterization is not a graph, which is particularly appreciated to investigate its potential folds. Moreover, we will discuss how to obtain rigorous error bounds for this parameterization. In particular, the technique is applied for the cubic Ikeda equation, the Wright's equation and the Mackey–Glass equation.

Dynamics of mass action systems

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With mass action kinetics, every chemical reaction network can be translated into a polynomial system of ODEs. This results in a rich class of dynamical systems, including gradient systems, oscillating systems, etc. In this survey talk I will present old conjectures and recent results on the global attractor conjecture, the persistence and permanence conjecture.

Dynamics and bifurcations of DDE cell-cycle models

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The Burns–Tannock model of the cell cycle can be expressed mathematically as a nonlinear scalar delay differential equation (DDE) with a single delay. This DDE can be used to model the dynamics of a population of hematopoietic stem cells (HSCs), and coupled with other equations to model the production of circulating blood cells. In dynamical diseases the concentrations of circulating blood cells oscillate, which could be due to an inherent instability of the HSC production, or from feedback from one of the circulating cell lines. To investigate the first possibility we study the dynamics and bifurcations of the stand-alone scalar Burns–Tannock DDE. We demonstrate that very long period orbits, quasi-periodic orbits and chaotic solutions may arise. We focus in particular on an apparent canard explosion that arises. Previous explorations of canards in DDEs only considered equations with at least two spatial dimensions.

Joint work with Daniel Câmara De Souza (U. Edinburgh)

Threshold dynamics in a model for Zika virus disease with seasonality

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In this talk, we present a compartmental model to study the transmission of Zika virus disease including spread through sexual contacts and asymptomatic carriers. To incorporate the impact of the periodicity of weather on the spread of Zika, we apply a non-autonomous model with time-dependent mosquito birth, death and biting rates, which shown that the global dynamics is determined by the basic reproduction number R_0 which is defined through the spectral radius of a linear integral operator. If $R_0 < 1$, then the disease free periodic solution is globally asymptotically stable and if $R_0 > 1$, then the disease persists.

Although a regular periodic recurrence of Zika has not been observed yet, it is expected that this might be altered by the climate change. We show numerical examples to study what kind of parameter changes might lead to a periodic recurrence of Zika.

A computer assisted proof of Jones' conjecture: counting and discounting slowly oscillating periodic solutions to Wright's equation

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A classical example of a nonlinear delay differential equations is Wright's equation: $y'(t) = -\alpha y(t-1)[1+y(t)]$, considering $\alpha > 0$ and $y(t) > -1$. This talk discusses two conjectures associated with this equation: Wright's conjecture, which states that the origin is the global attractor for all $\alpha \in (0, \pi/2]$; and Jones' conjecture, which states that there is a unique slowly oscillating periodic solution for $\alpha > \pi/2$.

To prove Wright's conjecture our approach relies on a careful investigation of the neighborhood of the Hopf bifurcation occurring at $\alpha = \pi/2$. Using a rigorous numerical integrator we characterize slowly oscillating periodic solutions and calculate their stability, proving Jones' conjecture for $\alpha \in [1.9, 6.0]$ and thereby all $\alpha \geq 1.9$. We complete the proof of Jones conjecture using global optimization methods, extended to treat infinite dimensional problems.

Non-oscillation criteria for half-linear difference and differential equations with asymptotically periodic coefficients

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The aim of this talk is to compare and present new non-oscillation criteria for half-linear difference and differential equations with asymptotically periodic coefficients. The used technique of proofs is based on the adapted generalized Riccati equation.

We consider the difference equation

$$\Delta \left[k^{\alpha} r_k^{1-p} \Phi(\Delta x_k)\right] + k^{\alpha-p} c_k \Phi(x_{k+1}) = 0,$$

where $\Phi(x) = |x|^{p-1} \operatorname{sgn} x$, $p > 1$, $\alpha \in (0, p-1)$, $\{r_k\}_{k\in\mathbb{N}}$, $\{c_k\}_{k\in\mathbb{N}}$ are positive asymptotically periodic sequences. At the same time, we consider the corresponding differential equation

$$\left[t^{\alpha} r^{1-p}(t) \Phi(x')\right]' + t^{\alpha-p} c(t) \Phi(x) = 0,$$

where r, c are asymptotically periodic continuous functions, r is positive.

This talk is mainly based on papers [1] and [2].

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Modelling the spread of varicella in Hungary

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Talk is based on a joint research with Rita Csuma-Kovács, Teodóra Borsos, János Dudás, Dánielisz Ágnes, Molnár Zsuzsanna and Gergely Röst.

Varicella is one of the most common infectious diseases of children, which can be prevented by vaccine. The epidemiology of the varicella-zoster virus (VZV) is quite complicated. It causes not only the chickenpox of children, but also the herpes zoster at an older age. Hence, modeling studies of VZV are extremely important, especially from the point of view of the impact of vaccination. In Hungary, obligatory vaccination will be introduced in September 2019, which gives the actuality of our research.

In our talk, first we consider a simple autonomous model of the spread of VZV without age-structures. We present results on the properties of disease-free and endemic equilibria and investigate the sensitivity on the parameters. Since chickenpox shows strong seasonality due to the school year, we modify the model with periodic infection, and perform parameter estimations to the Hungarian data of the Varicella incidence. Then, incorporating vaccination we experimentally study the effect of different vaccination strategies. Finally, we try to present rather realistic predictions in Hungary using detailed age-structured models.

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On a randomized backward Euler method for nonlinear evolution equations with time-irregular coefficients

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In this talk we introduce a randomized version of the backward Euler method that is applicable to stiff ordinary differential equations and nonlinear evolution equations with time-irregular coefficients. I will focus on the finite-dimensional case and consider Carathéodory-type functions satisfying a one-sided Lipschitz condition. After investigating the well-posedness and the stability properties of the randomized scheme, we prove the convergence to the exact solution with a rate of 0.5 in the root-mean-square norm assuming only that the coefficient function is square integrable with respect to the temporal parameter.

Qualitative properties of a class of retarded dynamical systems

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In this work we study the qualitative behaviour of a linearized system of differential equation with one discrete delay. The characteristic function of the mentioned system has the form

$$\Delta(z,\tau) := p(z) + q(z)e^{-z\tau} + r(z)e^{-2z\tau}$$

where p, q and r are polynomials with real coefficients and $deg(r) > deg(q)$. Using the Mikhailov criterion we give for special p, q and r an estimation on the length of delay τ for which no stability switching occurs. In the proof we use higher order Taylor polynomials, and compare our results with other methods. Then we give a condition under which stability is independent on the delay. Finally, a formula for Hopf bifurcation is calculated in terms of p, q and r giving an explicit formula for the Hopf bifurcation parameter. We apply these results for the so called "Love affair dynamics" and for the full Brusselator model.

Waveform relaxation method for nonlinear problems

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The method of waveform relaxation (WR) is an iterative method which can be applied for a large class of problems. The first description of the method can be found in [1], where it was used to approximate the solution of a system of ordinary differential equations (ODEs) describing large scale circuits. They considered the general implicit case, applied WR to approximate the solution of the equation $F(\dot{u}(t), u(t), t) = 0$. Since then many works have been devoted to investigate the convergence of the method for different types of problems, for example functional-differential equations [2], delay equations [3], general results for semi-linear partial differential equations can be found in [4].

Now the waveform relaxation method is investigated for nonlinear differential equations of the form

$$\dot{u}(t) = f(u(t)), \quad u(0) = u_0,$$

where f is a Lipschitz continuous operator. In WR the right hand side is split into two parts $f = f_1 + f_2$ and an iteration is define for $m = 1, 2, \ldots$ the following way:

$$\dot{v}_m(t) = f_1(v_m(t)) + f_2(v_{m-1}(t)), \quad v_m(0) = u_0, \quad v_0(t) = u_0.$$

The convergence of this method for these kind of problems is discussed in [5]. Here the results of [5] are refined. By carrying out a more subtle analysis a better estimation is given for the iteration error $\|u(t) - v_m(t)\|$, consequently the effect of *windowing* can better be described. Compared to that similar study [6], here we give an explicit error estimate, that is, an estimate depending only on the Lipschitz constants, step size and the number of steps.

In practice the subproblems are solved numerically. Error analysis is presented for combinations of WR with convergent numerical methods. With the concept of *cumulative numerical error*, an overall error estimate can be formulated. The convergence of such combined methods is proven. Finally some numerical experiments are presented to confirm the theoretical results.

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Bifurcation analysis of a harmonically forced delay equation

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We consider the classical one-degree-of-freedom tool vibration model, a delay-differential equation with quadratic and cubic nonlinearity, and periodic forcing. The perturbation method called the method of multiple scales is used to derive the slow flow equations. We investigate the stability and bifurcations of equilibria of the slow flow equations. Analytical expressions are obtained for the Hopf and saddle-node bifurcation points. Bifurcation analysis is also carried out numerically. Sub- and supercritical Hopf, cusp, fold, generalized Hopf (Bautin), Bogdanov–Takens bifurcations are found. The analysis demonstrates the rich dynamics of the system. This is joint work with my supervisor Tamás Kalmár-Nagy.

Recent developments of computer-assisted proofs in infinite dimensional dynamical systems

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In the study of infinite dimensional dynamical systems exploring the dynamics in the entire phase space is impossible. One strategy to tackle this problem is to focus on a set of special solutions that act as organizing centers. To single out these solutions computer-assisted proofs are being developed to find, for example, fixed points, periodic orbits and connecting orbits between those. Computer-assisted proofs in dynamics combines the strength of scientific computing, nonlinear analysis, numerical analysis, applied topology, functional analysis and approximation theory. While in the past decade, these techniques have primarily been applied to ODEs, we are starting to witness their applicability for infinite dimensional nonlinear dynamics generated by partial differential equations (PDEs), integral equations, delay differential equations (DDEs), and infinite dimensional maps. In this talk I will present recent advances in this direction, with a special emphasize on the dynamics of DDEs and PDEs.

How harvesting affects population stability: insights from discrete-time models

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An important challenge in harvesting and management theory is predicting population responses to the removal of individuals, and it is commonly assumed that increasing harvesting tends to stabilize the dynamics of exploited populations. These effects have been studied using discrete-time models of population dynamics at least since the influential Ricker's paper from 1954, but many questions remain as open problems even for simple models.

In this talk we present some new results for typical maps used in population dynamics (quadratic, Ricker, Bellows), providing precise conditions under which increasing harvesting can stabilize or destabilize the equilibrium. We review the role that some issues play in the population responses to harvesting: harvest policies, stock-recruitment relationship, age–structure, and seasonality.

Some stability properties of Runge–Kutta and multistep methods for ODEs

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When computing time discretizations of systems of ODEs (e. g., by Runge–Kutta or linear multistep methods) arising, for example, from semi-discretizations of certain PDEs (e.g., hyperbolic conservation laws), it is often desirable to preserve certain qualitative properties of the original PDE—such as positivity, dissipativity, monotonicity, the TVD property, or various maximum or minimum principles. Depending on the form of the right-hand side of the system of ODEs, various step-size coefficients have been introduced in the literature, including the strong-stability preserving coefficient (also known as the SSP coefficient), or the step-size coefficient for boundedness. These coefficients govern the maximum allowable step size of the time discretization that guarantees some of the above preservation properties. After giving an overview of these step-size coefficients, we present some results about their exact determination.

Global dynamics of Lasota's discrete-time model for blood cell production

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In an attempt to explain experimental evidence of chaotic behaviour in blood cell populations, A. Lasota proposed in [1] the following discrete-time model:

$$x_{n+1} = (1-\sigma)x_n + p_n, \tag{1}$$

where x_n denotes the number of cells at time $n \in \mathbb{N} \cup \{0\}$, σx_n ($0 < \sigma < 1$) is the amount of them destroyed in the time interval $[n, n+1)$ and p_n is the quantity of cells produced at the bone marrow during the same period, which based on experimental results was proposed to have the form

$$p_n = p(x_n) = (cx_n)^\gamma e^{-x_n}, \quad (c, \gamma > 0).$$

With the aim of explaining the influence of σ on the dynamics of equation (1), Lasota fixed $c = 0.47$ and $\gamma = 8$ and chose some values of σ. These values allowed him to describe the dynamics in some relevant clinical cases: normal conditions ($\sigma = 0.1$: depending on the initial condition, solutions either go to extinction or converge to a positive equilibrium), non-severe disease ($\sigma = 0.4$: the positive equilibria are unstable and there is a 2-periodic attractor), and severe disease ($\sigma = 0.8$, where Lasota observed the presence of a 3-periodic orbit and therefore chaotic behaviour).

In a recent joint work with E. Liz [2], we studied the model in detail by means of an analytical approach combined with numerical simulations. In particular, we revisit the results which appear in the original paper, but we also discover new interesting phenomena. In the analytical part, we find sufficient conditions for extinction and stability (including a sharp global stability condition for $\gamma \leq 1$), depending on the involved parameters. In a second part, we show some numerical bifurcation diagrams using either γ or σ as bifurcation parameters, while keeping c=0.47, as in [1]. These diagrams allow us to discover the rich dynamics of the model, which exhibits such phenomena as stability switches (bubbles), extinction windows, hydra effects and sudden collapses, among others.

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Characterization of periodic solutions for scalar delay equations with monotone feedback and even-odd symmetry

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A standard technique in the study of periodic solutions to a delay differential equation (DDE) is the reduction to an ordinary differential equation (ODE) whose periodic solutions happen to solve the original DDE via a symmetry assumption. In such a way one can at least prove the existence of some periodic solutions, but not necessarily all of them.

In the talk I will present sufficient conditions that allow to fully characterize the periodic solutions of a DDE in terms of an underlying planar ODE system.

Intricate structure of the analyticity set for a class of integral equations

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We consider the integral operator $L : C_{2\pi} \to C_{2\pi}$ given by

$$(Lx)(t) = \int_{\eta(t)}^{t} x(s)\, ds,$$

where $C_{2\pi}$ is the space of 2π-periodic functions with the usual sup norm. Here $\eta : \mathbb{R} \to \mathbb{R}$ is continuous, satisfying $\eta(t) \leq t$ and $\eta(t + 2\pi) = \eta(t) + 2\pi$ for all $t \in \mathbb{R}$. We first give necessary and sufficient conditions for the spectral radius $r(L)$ of L to be positive; in this case, by the Krein–Rutman Theorem, there exists a nonnegative eigenfunction $x \in C_{2\pi}$ to the eigenproblem $Lx = \kappa x$, where $\kappa = r(L)$. If further the function η is analytic, and under certain dynamical conditions on the map η, we show that the eigenfunction x is analytic for $t \in \mathcal{A} \subseteq \mathbb{R}$ where $\mathcal{A}$ is an open dense subset of $\mathbb{R}$, and where its complement $\mathcal{N} = \mathbb{R} \setminus \mathcal{A}$ is a generalized Cantor set.

This is joint work with Roger Nussbaum.

Asymptotic behavior of solutions of linear integral equations with two delays

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In this talk we consider a linear integral equation with two delays. We present some necessary and sufficient conditions for the zero solution of the equation to be exponentially stable by using analysis of characteristic roots. We also investigate the limit of solutions in the critical case where the equation loses its exponential stability. More precisely, if the solution tends to an equilibrium point or a periodic orbit, we establish the explicit expressions depending on given initial functions by applying the formal adjoint theory.

Generalized ODEs and measure FDEs with infinite time-dependent delays

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In this work, we introduce measure functional differential equations (MFDEs) with infinite time-dependent delay, and we study the correspondence between the solutions of these equations andthe solutions of the generalized ordinary differential equations (GODEs) in Banach spaces. Using the theory of GODEs, we obtain results concerning the existence and uniqueness of solutions and continuous dependence on parameters of measure functional differential equations with infinite time-dependent delay. We also establish a result of existence of solutions for a MFDE with infinite time-dependent delay in the presence of a perturbation independent of the state. We develop the theory in the context of phase spaces defined axiomatically. Our results in this paper generalize several previousworks on MFDEs with infinite time-independent delay. This is a joint work with Claudio Gallegos and Hernan Henríquez.

How to teach one step and linear multistep methods

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One step and linear multistep methods serve as a popular choice to solve initial value problems. When we teach these topics at a course, usually we try to omit the techniqual and time consuming parts as the stability proofs and the "consistency + stability = convergence" type theorem is presented in an oversimplified way. It could be also troublesome that later the student will also learn how to approximate the solution of BVPs and elliptic problems by the FDM and may draw a conclusion that these areas of numerical mathematics are completely different especially when we compare the tools we apply there. In this talk we try to resolve these problems and show that these areas can be presented in the same framework. Moreover, the usual tools we apply at the latter mentioned topic are useful for one step and linear multistep methods as well.

Neumann fractional p-Laplacian: eigenvalues and problems with source

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We develop some properties of the p-Neumann derivative for the fractional p-Laplacian in bounded domains with general $p > 1$. In particular, we prove the existence of a diverging sequence of eigenvalues and we introduce the associated evolution problem, studying the basic properties of solutions. Finally, we study a nonlinear problem with source in absence of the Ambrosetti–Rabinowitz condition.

Optimal temporary non-pharmaceutical intervention strategies for epidemic outbreaks

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We propose temporary non-pharmaceutical intervention (NPI) strategies in the SIR disease outbreak model, where NPIs start when the density of infected individuals reaches a threshold level, and continues until the density of susceptibles drops below a critical level such that the infection can not spread anymore even without further intervention. Costs are assigned to NPIs and disease burden, and we investigate which one of the two-parameter family of ITHIR (intervene till herd immunity reached) strategies gives the minimal cost. We compare a range of possible cost functions. We found that the whole parameter domain is divided into two regions where the total cost is infinite in one, while it is finite in the other. Restricting the domain to the feasible region, when the cost of NPIs is very small compared to the cost of disease burden, the infimum of total costs is at the boundary between the two regions. We determine the optimal strategies by setting a maximum of acceptable length of the intervention. When the NPI is very expensive, the minimal cost is attained without any intervention. However, when these costs are of similar magnitudes, we uncover some counter-intuitive phenomena, namely the total cost can be a non-monotone function of the control intensity and the threshold value, and then we can determine wich strategy gives the minimal total cost.

Joint work with Gergely Röst (Szeged).

On the stability of periodic solution under variable delay

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We are interested in the impact of state-dependent delay on the behaviour of solutions to a differential equation of the form

$$x'(t) = g(x(t-d)) \tag{1}$$

and obtain results about destabilization of a periodic solution and bifurcation of Floquet multipliers.

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Period two solutions of distributed delay differential equations

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We would like to present a class of distributed delay differential equations that has periodic solutions of period 2, where the maximum delay is 1. The existence of the periodic solutions can be proven, following the idea by Kaplan and Yorke (1974): an ansatz deduces a second order ordinary differential equation. We show that, for some special equations, there exist periodic solutions that can be expressed in terms of the Jacobi elliptic functions explicitly.

Dynamics of recurrent neural networks with piece-wise constant activation functions

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Recently, increased attention surrounds the dynamics of continuous-time recurrent neural networks where both excitatory and inhibitory neurons are present. A neuron's activation function is usually a continuous sigmoid function. The dynamics can be complex, and available results are mostly about small networks and stability of steady states. In this paper, we examine excitatory-inhibitory networks with step functions as activation functions. This can be thought as a rough approximation of the continuous case. The obvious advantage of this approach is that the understanding of the dynamics is easier for large networks as well. In the case of the two and three valued step functions, we give a detailed description of the dynamics. The connection between continuous and step activation functions is also considered.

Spectral theory of functions and asymptotic behavior of solutions of some functional differential equations

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We propose a spectral theory of unbounded functions and its applications to study the asymptotic behavior of solutions of some classes of functional differential equations. The results are stated in terms of the countability of spectrum of the equations and asymptotic stability of individual solutions of the equations.

C^1-smooth dependence with respect to history in Sobolev space $W^{1,1}$ and constant delay: beyond Lipschitz continuous histories

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The objective of this talk is to clarify the connection between the regularity of initial histories and the smooth dependence of solutions with respect to history and delay. By using the fixed point argument, we obtain the C^1-smooth dependence with respect to history in Sobolev space $W^{1,1}$ and constant delay as an application of the uniform contraction theorem for Banach spaces. In previous results, the history space is restricted to the Lipschitz continuous histories. The result of this talk extends the regularity of histories to $W^{1,1}$-functions.

Nonautonomous Hopf bifurcation and Li–Yorke chaos

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We analyze the characteristics of the global attractor of a type of dissipative nonautonomous dynamical systems in terms of the Sacker and Sell spectrum of its linear part. The model gives rise to a pattern of nonautonomous Hopf bifurcation which can be understood as a generalization of a classical autonomous one. We pay special attention to the dynamics at the bifurcation point, showing the possibility of occurrence of Li–Yorke chaos in the corresponding attractor and hence of a high degree of unpredictability.

Global attractivity of the periodic solution for a periodic model of hematopoiesis

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In this presentation, we show sufficient conditions for the global attractivity of the positive periodic solution of the generalized periodic model of the hematopoiesis

$$x'(t) = -a(t)x(t) + \sum_{i=1}^{m} \frac{b_i(t)}{1 + x(t - \tau_i(t))^n},$$

with $0 < n \leq 1$. We emphasize that the result presented improves previous ones in the literature.

At the end of this presentation, a numerical example is given, illustrating the effectiveness of the new criterion.

Teresa Faria is a co-author of this research.

Hyers–Ulam stability for first-order linear differential equations with periodic coefficient

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This talk is concerned with Hyers–Ulam stability(HUS) of the first-order homogeneous linear differential equation $x' - a(t)x = 0$ on $\mathbb{R}$, where $a : \mathbb{R} \to \mathbb{R}$ is a continuous periodic function. It is well known that if $a(t)$ is a constant a, then $x' - ax = 0$ has HUS on $\mathbb{R}$ if and only if $a \neq 0$. The purpose of this study is to find a necessary and sufficient condition for HUS of $x' - a(t)x = 0$ on $\mathbb{R}$. Furthermore, the best constant in HUS is clarified.

On positive solutions of boundary value problems with Riemann–Stieltjes integrals

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In this talk, we prove some new results on the positive solutions of second-order boundary value problems with and without ϕ-Laplacians and nonlocal boundary conditions.

Asymptotic behavior of supercritical multi-type CBI processes

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Under a first order moment condition on the immigration mechanism, we show that an appropriately scaled supercritical and irreducible multi-type continuous state and continuous time branching process with immigration (CBI process) converges almost surely. If an $xlog(x)$ moment condition on the branching mechanism does not hold, then the limit is zero. If this $xlog(x)$ moment condition holds, then we prove L_1 convergence as well. The projection of the limit on any left non-Perron eigenvector of the branching mean matrix is vanishing. If, in addition, a suitable extra power moment condition on the branching mechanism holds, then we provide the correct scaling for the projection of a CBI process on certain left non-Perron eigenvectors of the branching mean matrix in order to have almost sure and L_1 limit. A representation of the limits is also provided under the same moment conditions. Our results extend some results of Kyprianou, Palau and Ren [1] to certain left non-Perron eigenvectors of the branching mean matrix of the multi-type CBI process in question.

Under a second order moment condition on the branching and immigration mechanisms, we show that an appropriately scaled projection of a supercritical and irreducible multi-type CBI process on the other left non-Perron eigenvectors of the branching mean matrix is asymptotically mixed normal. In case of a non-vanishing immigration, with an appropriate random scaling, we prove asymptotic normality as well. Note that these results can be considered as counterparts of Theorems 3 and 4 and Corollaries 1 and 2 in Section 8 in Chapter V in Athreya and Ney [2] (which are derived for multidimensional discrete state continuous time Markov branching processes without immigration). In case of a non-vanishing immigration, we prove the almost sure convergence of the relative frequencies of distinct types of individuals as well.

The full presentation of our results can be found in [3] and [4].

Joint work with Mátyás Barczy (MTA-SZTE Analysis and Stochastics Research Group, Bolyai Institute, University of Szeged) and Sandra Palau (Department of Statistics and Probability, Instituto de Investigaciones en Matemáticas Aplicadas y en Sistemas, Universidad Nacional Autónoma de México, México)

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Conditionally oscillatory half-linear differential equations and Hille–Nehari type criteria

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Considering a nonoscillatory half-linear second order differential equation

$$(r(t)\Phi(x'))' + c(t)\Phi(x) = 0, \quad \Phi(x) = |x|^{p-1}\operatorname{sgn} x, \quad p > 1,$$

with the use of its solution h, it is possible to construct its perturbation so that the resulting equation is conditionally oscillatory. We present Hille–Nehari type integral oscillation and nonoscillation criteria for the perturbed conditionally oscillatory equation with the critical coefficient

$$(r(t)\Phi(x'))' + \left[c(t) + \frac{1}{2qh^p(t)R(t)(\int^t R^{-1}(s)\,\mathrm{d}s)^2} + \tilde{c}(t)\right]\Phi(x) = 0$$

and discuss its further generalization with more perturbation terms which preserve the property of conditionally oscillation.

Global bifurcation for nonlinear dynamic equations on time scales

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In this work we characterize the existence of a continuous of solutions (x, λ) for the nonlinear parametrized problem on time scales

$$x^\Delta(t) + \lambda\phi(t, x(t), x^\Delta(t)) + \lambda\psi(t, x(t)) = 0, \qquad \lambda \geq 0, \tag{1}$$

where $x : [0, T]_{\mathbb{T}} \to \mathbb{R}^n$, along with appropriate boundary conditions. We extend the main result in [Benevieri et al., 2005] to the context of time scales. For this, we use the concepts of orientation of nonlinear Fredholm maps of index zero, and topological degree of a triple $(v, W, 0)$, where v is a suitable function and $W \subset \mathbb{R}^n \times [0, \infty)$ is an open subset, to apply these results to the problem (1).

Semistability of complex balanced kinetic systems with time delays

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In this talk we introduce a class of delayed kinetic systems derived from mass action type reaction network models. We define the time delayed positive stoichiometric compatibility classes and the notion of complex balanced equilibria. We prove the existence and uniqueness of equilibria within the time delayed positive stoichiometric compatibility classes. We prove the semistability of equilibria for time delayed complex balanced sytems using an appropriate Lyapunov–Krasovskii functional and LaSalle's invariance principle. As a consequence, we show that every positive complex balanced equilibrium is locally asymptotically stable relative to its positive stoichiometric compatibility class. The talk is based on a joint work with Katalin M. Hangos, György Lipták and Gábor Szederkényi (Institute for Computer Science and Control, Hungarian Academy of Sciences, Budapest, Hungary).

Delayed neural field equations in two-dimensional spatial domains

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Neural field models describe the activity of neuronal populations at a mesoscopic level. We consider a single population of neurons, distributed over a two-dimensional bounded, connected, open region, whose state is described by their membrane potential. These potentials are assumed to evolve according to an integro-differential equation with space dependent delay

$$\frac{\partial}{\partial t}u(t,x) = -\alpha u(t,x) + \int_\Omega J(x,r)S(u(t-\tau(x,r),r))\,dr, \quad \alpha > 0.$$

In this lecture we discuss our analysis on the spectrum of the linearized equation in case of a rectangular spatial domain.

Bifurcations in periodic integrodifference equations

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Integrodifference equations (IDEs for short) are infinite-dimensional discrete dynamical systems popular in theoretical ecology to describe the temporal evolution and spatial dispersal of populations with nonoverlapping generations. We provide several bifurcation criteria for time-periodic IDEs. These theoretical results are illustrated both analytically and numerically. In the latter case, we work with Nyström-type discretizations and apply them to various models from theoretical ecology. This is based on joint work with Christian Aarset.

Stability and stabilization of a model occurring in hydraulics

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There is considered a system of two sets of hyperbolic partial differential equations describing the waterhammer in a hydroelectric power plant containing the dynamics of the tunnel, of the turbine penstock, of the surge tank and hydraulic turbine as follows

$$
\begin{aligned}
&\partial_t q_i + \frac{a_i}{\delta_i}\partial_{x_i}\left(h_i + \frac{1}{2g}\left(\frac{q_i}{f_i}\right)^2\right) + \frac{a_i}{\delta_i}\cdot\frac{\lambda_i}{2D_i}\left(\frac{q_i}{f_i}\right)\left|\frac{q_i}{f_i}\right| = 0\\
&\partial_t h_i + a_i\delta_i\partial_{x_i} q_i = 0\ ;\ i = 1,2\ ,\ 0 \le x_i \le L_i \quad t > 0\\
&h_1(0,t) = 1\ ,\ h_1(L_1,t) - \frac{\lambda_s'}{T_s}q_1(L_1,t) = z(t) - \frac{\lambda_s'}{T_s}q_2(0,t)\\
&h_2(0,t) + \frac{\lambda_s'}{T_s}q_2(0,t) = z(t) + \frac{\lambda_s'}{T_s}q_1(L_1,t)\ ,\ T_s\frac{\mathrm{d}z}{\mathrm{d}t} = q_1(L_1,t) - q_2(0,t)\ ,\\
&q_2(L_2,t) = f_\theta(t)\sqrt{h_2(L_2,t)}\ ,\ T_a\frac{\mathrm{d}\varphi}{\mathrm{d}t} = f_\theta(t)q_2(L_2,t)^3 - \nu_g\ ;\\
&0 \le f_\theta \le 1\ ,\ 0 \le \nu_g \le 1
\end{aligned}
\tag{1}
$$

Under standard simplifying assumptions (neglect of Darcy Weisbach losses $(\lambda_i/(2D_i))(q_i/f_i)|q_i/f_i|$ and of the dynamic head $(1/2g)(q_i/f_i)^2$ space variations [1]) a system of functional differential equations of neutral type can be associated and existence, uniqueness and continuous data dependence can be established. Inherent stability of the equilibria (Stability Postulate of Četaev) and their asymptotic feedback stabilization are then discussed using a Lyapunov functional deduced from the energy identity.

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Decoupling of quasilinear systems on time scales

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We consider the dynamic system in a Banach space on unbounded above and unbounded below time scales [1]

$$\begin{cases} x^{\Delta} = A(t)x + f(t,x,y), \\ y^{\Delta} = B(t)y + g(t,x,y). \end{cases} \tag{1}$$

Based on the integral version of the idea developed in the article [2] we generalize [3] and find sufficient condition under which the system (1) is dynamic equivalent to

$$\begin{cases} x^{\Delta} = A(t)x + f(t,x,u(t,x)), \\ y^{\Delta} = B(t)y + g(t,v(t,y),y). \end{cases} \tag{2}$$

or

$$\begin{cases} x^{\Delta} = A(t)x + f(t,x,u(t,x)), \\ y^{\Delta} = B(t)y + g(t,k(t,x,y),y). \end{cases} \tag{3}$$

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Asymptotic constancy of solutions of linear Volterra and other functional differential equations

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This talk examines the asymptotic behaviour of continuous solutions $x : [a, \infty) \to \mathbb{R}^n$ of

$$x(t) = f(t) + \int_a^t K(t,s)x(s)\,ds, \quad t \geq a. \tag{1}$$

Sharp conditions on the kernel and forcing function which force the solution $x(t)$ to converge to a limit $x(\infty)$ as $t \to \infty$ are presented. The conditions on the kernel are that it be of continuous convergent type, and that the absolute value of the kernel obey

$$\rho\left(\lim_{T\to\infty} \sup_{t\geq T} \int_T^t |K^{\star p}(t,s)|\,ds\right) < 1, \tag{2}$$

where $K^{\star p}$ is the pth iterated kernel of K. Though there is no explicit formula for $x(\infty)$, a limit formula involving it is found. The solutions of (1) can also be represented in terms of the resolvent kernel, which inherits properties from the kernel K provided (2) is satisfied.

One of the motivations for this research is the study of the growth and decay of solutions of

$$y(t) = h(t) + \int_a^t H(t,s)y(s)\,ds, \quad t \geq a, \tag{3}$$

The idea is to weight the solution by a function $\Gamma : [a, \infty) \to \mathbb{R}^{n\times n}$. For then if $x(t) = \Gamma(t)y(t)$, then x obeys (1) with

$$K(t,s) = \Gamma(t)H(t,s)\Gamma(s)^{-1}, \quad f(t) = \Gamma(t)g(t),$$

assuming for simplicity that $\Gamma(t)$ is always invertible. If Γ can be constructed so that all the entries of $x(\infty)$ are non-trivial, the asymptotic behaviour of y has been exactly described.

Time permitting, the method will be applied to functional differential equations other than classical linear Volterra ones.

Our work builds on well-known theory which has been expounded for instance in books by Corduneanu, and Gripenberg, Londen & Staffens, as well as an extensive survey article by Tsalyuk. As well as extending research by us on Volterra integral and summation equations, we also use ideas from the work of Horváth & Győri on Volterra difference equations.

This is joint work with J. A. D. Appleby and Győri, I.

Global dynamics of a new delay logistic equation arisen in cell biology

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The delayed logistic equation (also known as Hutchinson's equation or Wright's equation) was originally introduced to explain oscillatory phenomena in ecological dynamics. While it motivated the development of a large number of mathematical tools in the study of nonlinear delay differential equations, it also received criticism from modellers because of the lack of a mechanistic biological derivation and interpretation. Here we propose a new delayed logistic equation, which has clear biological underpinning coming from cell population modelling. This nonlinear differential equation includes terms with discrete and distributed delays. The global dynamics is completely described, and it is proven that all feasible nontrivial solutions converge to the positive equilibrium. The main tools of the proof rely on persistence theory, comparison principles and an L^2-perturbation technique. Using local invariant manifolds, a unique heteroclinic orbit is constructed that connects the unstable zero and the stable positive equilibrium, and we show that these three complete orbits constitute the global attractor of the system. Despite global attractivity, the dynamics is not trivial as we can observe long-lasting transient oscillatory patterns of various shapes. We also discuss the biological implications of these findings and their relations to other logistic type models of growth with delays.

Joint work with Ruth Baker (Oxford) and Péter Boldog (Szeged).

Criteria of global attraction in systems of delay differential equations with mixed monotonicity

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In this talk we derive some criteria of global attraction to a non-trivial equilibrium for systems of delay differential equations with mixed monotonicity. The method of proof is reminiscent to the classical approach of "decomposing +embedding" developed by H. L. Smith. Our results have two strengths: (i) We derive delay-dependent conditions of global attraction. (ii) We drop some common monotonicity conditions of the classical approach. We also show several applications in classical models of Mathematical Biology. This is a joint work with Hassan A. El-Morshedy.

Equivariant Pyragas control: theoretical mechanism versus experimental verification

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Following an idea of Kestutis Pyragas, we want to stabilize periodic orbits by noninvasive time-delayed feedback control. We consider an unstable periodic orbit near Hopf bifurcation in an equivariant system, e.g., a network of symmetrically coupled Stuart–Landau oscillators. The spatio-temporal symmetry of the periodic orbit is given by a chosen isotropy subgroup.

The main question of the talk is: How should one adapt the idea of time-delayed feedback control to selectively stabilize periodic orbits of prescribed symmetry type?

As a solution, we propose an extended Pyragas control scheme which includes the spatio-temporal symmetry of the periodic orbit. Necessary and sufficient conditions for the stabilization can be proved in the context of coupled Stuart–Landau oscillators.

The control mechanism suggests a wider range of applications, therefore we test this hypothesis experimentally on neuromorphic chemical oscillators. While the experiments confirm stabilization via equivariant Pyragas control, they also reveal a number of interesting features which demand further mathematical investigation. The experimental part was done in cooperation with Jan Totz and David Hering (TU Berlin).

On qualitative behavior of multiple solutions of quasilinear parabolic functional equations

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We shall consider weak solutions of initial-boundary value problems for quasilinear parabolic functional differential equations (containing nonlocal terms). Some particular cases will be considered when the problem has several solutions. Further, qualitative properties of the solutions will be studied as the time tends to infinity.

Lyapunov functions to neural network models

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The ODE model of a network of n identical units (e.g. neurons) is studied. The units, having their own differential equations, are joined by the adjacency matrix of the network, leading to a system of n non-linear ODEs. The Hopfield model of a neural network, the Lotka–Volterra model of a population or epidemic spread on a social network are examples for this complex system. The structure of the network has a strong effect on the dynamical behaviour of the system that has been widely studied. We show three classes of networks when suitable Lyapunov functions guarantee the global stability of a unique steady state. There will be emphasis on networks with negative weights on the edges that are important in modeling the effect of inhibitory neurons.

Sturmian separation theorems on unbounded intervals

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This is a joint work with Peter Šepitka (Masaryk University, Brno). We present Sturmian separation theorems on unbounded intervals for linear Hamiltonian systems under no controllability assumption. Our results extend the corresponding theory on compact intervals by J. Elyseeva (2016) and the authors (2017). Our results are also new for completely controllable systems, in particular for the even order Sturm–Liouville differential equations.

Non-linear and spectral analysis of neural field models with transmission delays and diffusion

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Neural Field Equations model the large scale behaviour of large groups of neurons. We extend results of Van Gils et al. (2013) and Dijkstra et al. (2015) by including diffusion in

the Neural Field, which models direct, electrical connections. We extend known sun-star calculus results for delay equations to be able to include diffusion and explicitly characterize the essential spectrum. When the connectivity of the Neural Field has a specific form, we are able to compute the spectral properties and normal form coefficients on the center manifold. By examining a numerical example of a Hopf-bifurcation, we conclude that the addition of diffusion suppresses spatial modes, while leaving temporal modes unaffected.

Multichromatic travelling waves for lattice Nagumo reaction diffusion equation

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Nagumo reaction diffusion equations serve as a prototype system in which two competing states coexist in a delicate balance. As such it served as a simplified model in mathematical ecology as well as in the modelling of propagation of action potentials through nerve fibers. Discrete-space models (on lattices or graphs) provide a rich structrure of solutions. In this talk we discuss multichromatic front solutions to the bistable Nagumo lattice differential equation. Such fronts connect the stable spatially homogeneous equilibria with spatially heterogeneous n-periodic equilibria and hence are not monotonic like the standard monochromatic fronts. These multichromatic fronts can disappear and reappear as the diffusion coefficient is increased. In addition, these multichromatic waves can travel in parameter regimes where the monochromatic fronts are also free to travel. This leads to intricate collision processes where an incoming multichromatic wave can reverse its direction and turn into a monochromatic wave.

Counting and ordering periodic stationary solutions of lattice Nagumo equations

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We study the rich structure of periodic stationary solutions of Nagumo reaction diffusion equation on lattices. By exploring the relationship with Nagumo equations on cyclic graphs we are able to divide these periodic solutions into equivalence classes that can be partially ordered and counted. In order to accomplish this, we use combinatorial concepts such as necklaces, bracelets and Lyndon words.

Positive monotone solutions of a neutral-type equation system

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The existence of positive monotonous solutions of the system of neutral equations is studied by using an equivalent criterion in the form of the existence of positive functions that meet the appropriate inequalities. The result is a generalization of known criteria for a system of delayed differential equations. The results are derived by the retraction method. An important role is played by the design of a system of suitable initial functions that suits sewing conditions.

Some improvements in a rigorous integration of delay differential equations

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An algorithm for the forward in time integration of scalar delay differential equations (DDEs) with a constant delay was recently presented in [1]. The algorithm can be used to obtain rigorous bounds of the solutions in the phase-space for a rather general form of the initial value problem for DDEs:

$$\begin{cases} \dot{x}(t) = f(x(t-\tau), x(t)), & t \geq 0, \\ x(t) = \psi(t), & t \in [-\tau, 0], \end{cases} \tag{1}$$

with f being a function *smooth enough* that can be represented as a computer program. Wright and Mackey–Glass equations are good examples that fall into this category.

In the talk, some developments to the mentioned algorithm will be presented. The new implementation is much more optimized in terms of computation time and it uses smoothing properties of the DDE to produce enclosures of the solutions of a much better quality than the previous one. This in turn enables to prove existence results for a wider class of solutions, including unstable periodic solutions to Mackey–Glass equation.

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Stability and oscillations in multistage SIS models depend on the number of stages

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We consider multistage SIS models of infectious diseases, where infected individuals are passing through infectious stages $I_1, I_2, \ldots, I_n$ and then return to the susceptible compartment. First we calculate the basic reproduction number $\mathcal{R}_0$, and prove that the disease dies out for $\mathcal{R}_0 \leq 1$, while a unique endemic equilibrium exists for $\mathcal{R}_0 > 1$. Our main result is that the stability of the endemic equilibrium depends on the number of stages: the endemic equilibrium is always stable when $n \leq 3$, while for $n > 3$ it can be either stable or unstable, depending on the particular choice of the parameters. We point out an error in the literature for multistage SEIRS models as well. Our results have important implications on the discretization of infectious periods with varying infectivity.

Joint work with Gergely Röst (Szeged).

On the form of kinetic differential equations

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The induced kinetic differential equation of the reaction

$$\sum_{m=1}^{M} \alpha(m,r)\mathrm{X}(m) \to \sum_{m=1}^{M} \beta(m,r)\mathrm{X}(m) \quad (r = 1, 2, \ldots, R)$$

assuming mass action type kinetics is

$$\dot{c_m} = \sum_{r=1}^{R} (\beta(m,r) - \alpha(m,r)) k_r \prod_{p=1}^{M} c_p^{\alpha(p,r)} \quad (m = 1, 2, \ldots, M)$$

or shortly $\dot{\boldsymbol{c}} = (\boldsymbol{\beta} - \boldsymbol{\alpha}) \cdot \boldsymbol{k} \odot \boldsymbol{c}^{\boldsymbol{\alpha}}$ [6,9]. What kind of an equation is this? It is a polynomial system. Is it a Volterra system, a Kolmogorov type system, a monotone one, an S-system? Is the first orthant its invariant set? The answers to these questions may help use results from the qualitative theory of differential equations.

In the last 50 years a large theory has emerged called **Formal Reaction Kinetics** (or the **Chemical Reaction Network Theory**) which treats the behaviour of solutions of kinetic differential equations, and provides results applicable in the field of the qualitative theory of differential equations. Let us mention a simple example. We are interested in proving the existence of a positive stationary point of the equation

$$\begin{gathered}\dot{x}_1 = X(x_1, x_2, x_5), \quad \dot{x}_2 = X(x_2, x_2, x_5), \quad \dot{x}_3 = X(x_3, x_4, x_1), \\ \dot{x}_4 = X(x_4, x_4, x_1), \quad \dot{x}_5 = X(x_5, x_6, x_3), \quad \dot{x}_6 = X(x_6, x_6, x_3), \\ \text{with } X(u, v, w) := s - gu + \frac{bv}{1 + v + w}, \quad \text{where } s, g, b > 0\end{gathered} \tag{1}$$

coming from a biological application (it is a repressilator model [5]). A naive approach does not help. The standard mathematical approach using recent results in algebra gives the answer but not cheaply. Our approach is to find a reversible reaction having a kinetic differential equation dynamically equivalent to Eq. (1), thus allowing the needed consequence to be deduced.

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Large-amplitude periodic solutions for delay equations

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This talk aims to give an overview on the so-called large-amplitude periodic (LAP) solutions of the delay equation

$$\dot{x}(t) = -\mu x(t) + f(x(t-1)$$

with monotone positive feedback.

As it is well-known, $\hat{\chi} \in C([-1,0],\mathbb{R})$ is an unstable equilibrium if $\hat{\chi}(s) = \chi$ for all $s \in [-1,0]$, where χ is an unstable fixed point of $\mathbb{R} \ni x \mapsto f(x)/\mu \in \mathbb{R}$ (that is, $f(\chi)/\mu = \chi$ and $f'(\chi)/\mu > 1$). A periodic solution is said to have large amplitude if it oscillates about at least two unstable fixed points.

We discuss the bifurcation and the existence of the LAP solutions when the dynamical system has two unstable equilibria. We describe the geometric properties of the unstable set of a specific LAP orbit in detail. Complicated configurations of LAP solutions appear when the dynamical system has several unstable equilibria – we also consider this case.

Delay equations and twin semigroups

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A delay equation is a rule for extending a function of time towards the future on the basis of the (assumed to be) known past. By translation along the extended function (i.e., by updating the history), one defines a dynamical system. If one chooses as state-space the continuous initial functions, the translation semigroup is continuous, but the initial data corresponding to the fundamental solution is not contained in the state space.

In ongoing joint work with Odo Diekmann, we choose as state space the space of bounded Borel functions and thus sacrifice strong continuity in order to gain a simple description of the variation-of-constants formula.

The aim of the lecture is to introduce the perturbation theory framework of twin semigroups on a norming dual pair of spaces, to show how renewal equations fit in this framework and to sketch how neutral equations can be covered. The growth of an age-structured population serves as a pedagogical example.

Multiplicity of critical points for minimax functionals and applications to differential equations

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We discuss the multiplicity of critical points for functionals of a general minimax geometry on Banach spaces. Firstly, we show the existence of at least two critical points under a key additional assumption on max-stability of a base set w.r.t. descending homotopies. Secondly, we show that the functional possesses at least three critical points provided it is globally bounded from below. Both of these general results can be applied to derive specific extensions of Mountain Pass Theorem, Saddle Point Theorem, and Linking Theorem. Finally, we introduce two applica tions to boundary value problems for differential equations. A resonant problem based on a Landesman–Lazer-type condition and a nonlinear equation describing, e.g., a type of semi-positone problems are studied.

Validated numerics for period-tupling and touch-and-go bifurcations of symmetric periodic orbits in reversible system

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We propose a framework for computer-assisted verification of the presence of symmetry breaking, period-tupling and touch-and-go bifurcations of symmetric periodic orbits for reversible maps. As the application of the developed framework will be presented numerical results for the Michelson system, Falkner–Skan equation and Three Body Problem. The paper is prepared based on the publication "Validated numerics for period-tupling and touch-and-go bifurcations of symmetric periodic orbits in reversible systems", *Commun. Nonlinear Sci. Numer. Simul.* **74**(2019), 30–54.

Solutions with dense short segments from regular delays

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Simple-looking autonomous delay differential equations

$$x'(t) = f(x(t-r))$$

with a real function f and single time lag $r > 0$ can generate complicated (chaotic) solution behaviour, depending on the shape of f. The same could be shown for equations with a variable, state-dependent delay $r = d(x_t)$, even for the case $f(\xi) = -\alpha\,\xi$ linear, with $\alpha > 0$ [1]. Here the argument x_t of the *delay functional* d is the segment, or history, of the solution x between $t-r$ and t defined as the function $x_t : [-r, 0] \to \mathbb{R})$ given by $x_t(s) = x(t+s)$.

So the delay alone may be responsible for complicated solution behaviour. In both cases the complicated behaviour which could be established occurs in a thin dust-like invariant subset of the infinite-dimensional Banach space or *solution manifold* of functions $[-r, 0] \to \mathbb{R}$ on which the delay equation defines a semiflow of differentiable solution operators [2,3].

The lecture presents results which grew out of an attempt to obtain complicated motion on a larger set with non-empty interior, as certain numerical experiments seem to suggest. In [4] a delay functional $d : Y \to (0, r)$ was constructed on an infinite-dimensional subset Y of the space $C^1([-r, 0], \mathbb{R})$, with $r > 1$, so that the equation

$$x'(t) = -\alpha\, x(t - d(x_t)) \tag{1}$$

has a solution whose *short segments* $x_t|_{[-1,0]}$, $t \geq 0$, are dense in the space $C^1([-1, 0], \mathbb{R})$. This implies a new kind of complicated behaviour of the flowline $[0, \infty) \ni t \mapsto x_t \in C^1_r$.

The set Y in [4] is small in the sense that it has infinite codimension, and it is not smooth like the said solution manifolds of finite codimension. Recent work [5] concerns the construction of a delay functional on an *open* subset of the space $C^1([-r, 0], \mathbb{R})$ so that Eq. (1) defines a nice semiflow on the solution manifold, and has a solution whose short segments are dense in an open subset of the space $C^1([-1, 0], \mathbb{R})$.

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Temporal dissipative solitons in systems with time delay

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Localized states are an universal phenomenon in spatially extended dissipative nonlinear systems. Recently, also temporally localized states have gained some attention, mainly driven by applications in various optical systems, where the dynamics of short optical pulses, i.e. a temporal localization of optical fields, can be described by DDE models. We present a theory for such states, which appear as periodic solutions with a period close to the large delay of the system. We study such solutions by using the limit of large delay, derive a desingularized equation for the solution profile, and provide a classification of the Floquet spectrum into point and pseudo-continuous spectrum. In particular, we point out some analogies and differences to the classical theory for spatially localized states in partial differential equations.

On absolute stability of delay differential equations with one discrete delay

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The absolute stability of a linear DDE is a stability for an arbitrary positive delay. We present a simple criterium for the absolute stability of systems of DDEs with one discrete delay. In particular, the conditions for the absolute stability are the same as for the stability for sufficiently large delay.

Effect of impulsive controls in a model system for age-structured population over a patchy environment

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In this talk, I will present a very general model of impulsive delay differential equations in $n - patches$ that describes the impulsive eradication of population of a single species over $n-$patches. The model allows an age structure consisting of immatures and matures, and also includes mobility and culling of both matures and immatures. Conditions are obtained for extinction and persistence of the model system under three special scenarios: (i) without impulsive control; (ii) with impulsive culling of the immatures only; and (iii) with impulsive culling of the matures only, respectively. In the case of persistence, the persistence level is also estimated for the systems in the case of identical n patches, by relating the issue to the dynamics of multi-dimensional maps. Two illustrative examples and their numerical simulations are given to show the feasibility and effectiveness of the results. Based on the theoretical results, some strategies of impulsive culling are provided for eradicating the population of a pest species.

Tuning the parameters of a mechanistic chain oscillator model to match the Kolmogorov spectrum of turbulence

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Turbulence can be phenomenologically modelled with a mechanistic binary tree, which represents Richardson's energy cascade model: the largest eddies are unstable and break up into smaller vortices and the smallest whirls dissipate the kinetic energy of the flow through viscosity. In this model, the biggest mass represents the biggest whirl and it is connected to smaller masses with springs. At the smallest level, the masses are connected with springs and dampers to the ground in order to dissipate energy. The energy of the mechanistic "turbulence" is quantified by the kinetic energy of the masses, which is transferred through the potential energy of the springs and finally dissipated by the dampers. The aim of the study is to tune the mechanical parameters of the model to approach as precisely as possible the Kolmogorov-spectrum considering the energy dissipation of the system. This is a joint work with Tamás Kalmár-Nagy.

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