Computer-Assisted Assessment of Mathematical Knowledge

Márta K. Németh, Szeged (Hungary) and Róbert Vajda, Szeged (Hungary)

Abstract: In recent years, arguments for the use of computers (especially computer algebra systems -- CAS) as an effective tool in supporting teaching and learning of mathematics, are well established. Accepting these arguments it is also clear that new, computer-based assessment models should be worked out in mathematical education. Due to our investigation we discuss two cases separately:

1. CAS extended with an (external) intelligent learning environment assessing basic mathematical knowledge contained in introductory courses.
2. CAS environment to assess complex problem solving skills and creativity.

After a short overview of the challenges and principles that had to be taken into consideration we give several examples how we have used these effective tools in both cases.

Key Words and Phrases: computer algebra systems, computer aided assessment, web-based learning environment

ZDM-Classification: D65, N85, U55, U75

1. Introduction

Assessment is an important part of the learning-teaching process, its purposes are [8]:

- informing learners about their own learning;
- informing teachers about their own teaching strategies;
- characterizing quantitatively the degree of achievement ;
- informing colleagues and researchers about the role of the given course in the educational system.

Traditional assessment methods, like writing tests and oral exams, are well-known. But why should we use computers in the assessment of mathematical knowledge?

1. At universities, instructors have to face the heterogeneity and the increasing number of students. The instructors find it difficult to use traditional assessment methods: It is hard to manage making a large amount of exercises, to assure uniformity in the level of difficulty of exercises for different groups and to guarantee fairness of the evaluation process at the same time. New, computer-based assessment models could help to handle this new situation in mathematical education.
2. The other problem is the small number of practical courses where the instructor is also present personally. The interaction between students and teacher could help to the students to check their level of knowledge. If there are fewer contact seminars, homework and self-assessment become a lot more significant, and computer algebra systems are effective tools in this situation, too.
3. The CAS, which help us to tackle the previous problems, are also qualitatively new tools in assessment. In a CAS environment new kinds of questions and exercises can appear: With these systems it is possible to measure such mathematical knowledge (e.g. complex problem-solving skills, mathematical model-building abilities etc.) that would be impossible to test without computers. The new questions are much more realistic, they are nearer to 'real-world' problems and the activity that should be done in this new assessment environment bears much more relationship to the job that mathematicians do at a company or a research institute than the activity in the traditional "paper-pencil" and "time-constrained" written examinations. This fact can give legitimation to computer assisted assessment from the direction of society as well.

Recent research papers emphasize that during the academic integration of a new technology into the curriculum we cannot neglect the fact to what extent they are at students' disposal and to what extent the students are familiar with them. Trouche in [16] makes a distinction between the notion of artifact and instrument and describes the instrumental genesis, i.e. the long process in which the pure artifacts become an instrument. In the sequel we try to take this theoretical framework into consideration insofar we notice that the possibilities provided by computer algebra systems appear in different ways at the different levels of the whole teaching-learning cycle.

1. Hearing the phrase “assessment with CAS” one usually thinks of a situation where a student sits in front of a locally installed CAS typing appropriate commands or programs in order to solve the posed mathematical exercise. But at freshmen level the students do not have any programming and syntactical abilities. We handle this problem by using interactive systems based on services of computer algebra systems. These systems provide a general easy-to-use interface to the students. So the students need not have any special syntactical knowledge, but they can enjoy the advantages of using computers. Communicating via graphical user interface elements the presupposed syntactical knowledge on the student side is reduced to the question of how to type in some numbers, polynomials or a list of elements. By self-assessment they can practice the given types of mathematical exercises as much as they want, their interactive mathematical activities can be evaluated and if needed, with hints and explanations can be extended automatically. Continuous and real-time feedback from one’s knowledge level is supported. Computers with appropriate software environments help the instructor in assessing large classes since they support automatic test paper generation with auto-generated solutions. In this described situation, we prefer free web-based solutions that enable (self)assessment of mathematics without presupposing any rigid syntactical and programming knowledge on the student’s side.
2. At advanced level, we have other assessment objectives. We assume that our students already have some basic mathematical and programming skills. We focus on gaining information about the students’ problem-solving skills. Built-in knowledge and high level programming language, expandable with new algorithms and data structures, make it possible to achieve our purposes in this level.

In the following chapters we discuss separately the two different computer-assisted assessment situations mentioned above. First we focus on intelligent interactive systems which exploit the services of a computer algebra system when exercises are generated and evaluated but require no prior programming knowledge on the students’ side. Secondly we investigate the challenges if all CAS-services are allowed during exams.

In the sequel the contribution of the new technology for improving our assessment systems is emphasized. However, we are aware of the drawbacks and difficulties related to the new tools (e.g. the vanishing of the original concept or the confusion of the notion and its representation - just to mention some), and we do not assert that for evaluating each kind of mathematical knowledge the computer assissted assessment is the most suitable. A more balanced combined assessment system will be outlined in the fourth chapter.

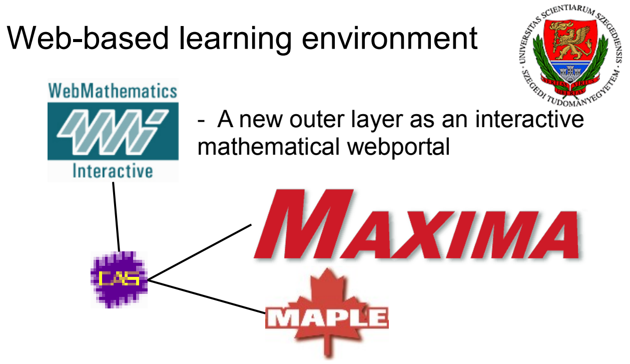
2. Assessing of Basic Skills with Interactive Systems

In this chapter we focus on intelligent interactive systems which exploit the services of a computer algebra system when exercises are generated and evaluated but require no prior programming knowledge on the students’ side. The assessment of a classical algebra topic (computing in polynomial rings) is demonstrated with our system WebMathematics Interactive.

2.1. Intelligent Interactive Systems

At a freshmen level, the explicit introduction of CAS opens up new possibilities but new difficulties [7] as well. CAS were originally designed for research purposes, they are command oriented with rigorous syntax. If we would like to integrate CAS thoroughly into education at the beginning of the curriculum, the new students should also acquire the basic syntactical conventions and programming knowledge (i.e. data and controlling structures) in parallel with the core mathematical concepts (e. g. limit, continuity, derivatives etc.). This usually causes great problems to the average students in our courses. It should be mentioned that the management of such courses in computer labs often cause difficulties on the instructor’s side as well. The least we can say that the didactic capabilities of computer algebra systems must be improved.

At the University of Szeged, just like in many other places, most of the teaching that is done by the Department of Mathematics is a service to other disciplines (computer science, physics, chemistry etc.). Due to the large number of students in these faculties and the limited lab and instructional capabilities of our department, we decided to improve by an external interface the didactic capabilities of certain computer algebra systems that were available for us. We developed WebMathematics Interactive (WMI) - a new web learning environment based on the services of CAS [17]. To assess the services of WMI the user needs only an internet connection and arbitrary web-browser.



Analogous systems on the internet (e.g. AIM, Cow-culus, Mathe-Online, MathServ, WIMS etc.) show that these systems are used very often.

During developing the thematic modules within WMI we mostly focused on the topics covered by introductory mathematical courses such as Calculus, Discrete Mathematics, Linear Algebra and Analytic Geometry.

Interacting with computer algebra systems and exploiting the mathematical services of such systems allowed the automated question generation and evaluation.

There are several types of exercises that could be used in (self)assessing mathematical knowledge with the help of these intelligent interactive systems:

- static open- and closed-ended tests,
- dynamically generated open- and closed-ended tests,
- (more) complex exercises with hints and basic services,
- multistep - exercises,
- matching mathematical objects with drag and drop technology,
- real-time evaluated multiple choice tests,
- filling out missing data in an incomplete text using mathematical rules and relations given in a wider context,
- transformation of information (e.g. reading out coordinates of a point from a graph and clicking on points of local extrema).

2.2. Didactical Principles

Since we cannot demonstrate all these features, we give explanations and remarks only for the first three types. Static open- and closed-ended tests are also well-known in the traditional "paper-pencil" exams. From static questions stored in database tests can be constructed with respect to certain principles (i.e. selected topic, the difficulty of questions etc.) and can be automatically evaluated by machine.

The didactic novelty comes from using dynamically generated tests. The mathematical objects in these questions (from simple numerical parameters such as matrix entries or coefficients of a polynomial until complex mathematical structures such as regions of integration or functions of a given type) are being dynamically generated. If the webpage is reloaded, the user gets a new question of the same type, since the mathematical objects are changed in the question. This technology has advantages both in assessment and self-assessment. In self-assessing, different questions are available for practicing basic types of exercises (i.e. the user can practice till the mathematical knowledge becomes a routine and trivialized for himself / herself). In assessing, these randomly generated tests with auto-solutions could be a base to give different test-exams for students or for group of students in order to avoid forbidden copy (plagiarism) on the students' side and extra work on the instructor's side.

Corresponding to the elaboration of more complex questions within the WMI system we wanted to implement the well known White-Box – Black-Box principle [5]. We consider the invention of mathematical knowledge as a spiral process, where one takes several steps (problem specification, experimentation, conjecture, proof, application, etc.) to gain the appropriate knowledge. If we complete one circle of the whole spiral, the gained knowledge can be added into our knowledge base [9].

Once a basic mathematical knowledge is trivialized it becomes a black-box. "The black-box phase is exactly the moment for applying 'technology', i.e. the current math systems". WMI can offer such basic services for the user in a composite problem situation to enable the concentration to problem solving on a higher level. It should be also mentioned that with sound control of basic services available for the user in a problem situation, an appropriate answer can be given to the highly debated question, that asks: "How can we use CAS, when the built-in services are too powerful ?", since such services are made hidden in the given context. The didactical principles and the types of exercises are illustrated by showing several concrete examples in the next chapter.

2.3. Example: Algebra — Polynomial Rings

Polynomial rings are one of the basic topics of classical algebra. They are taught for freshmen who learn mathematics. This topic is especially good for being assessed with CAS, because it includes algorithms and questions that often have explicit computable symbolic or numerical answers.

There are more levels of questions. On the lowest level we are interested in how students can learn the most basic algorithms like polynomial division. We can assess it by open-ended tests. Using WMI we can take the advantages of automatically generated arguments and the evaluation by the computer.

In Figure 1, a dynamic open-ended question can be seen. If the student clicks on the „Reload" button of the browser, the polynomials in the question change. Their coefficients are automatically generated. If the user faces some problems in typing the answer, the help of the system provides some syntactical information. Typing the answer and clicking on to this button, the system evaluates the answer by using the service of a CAS.

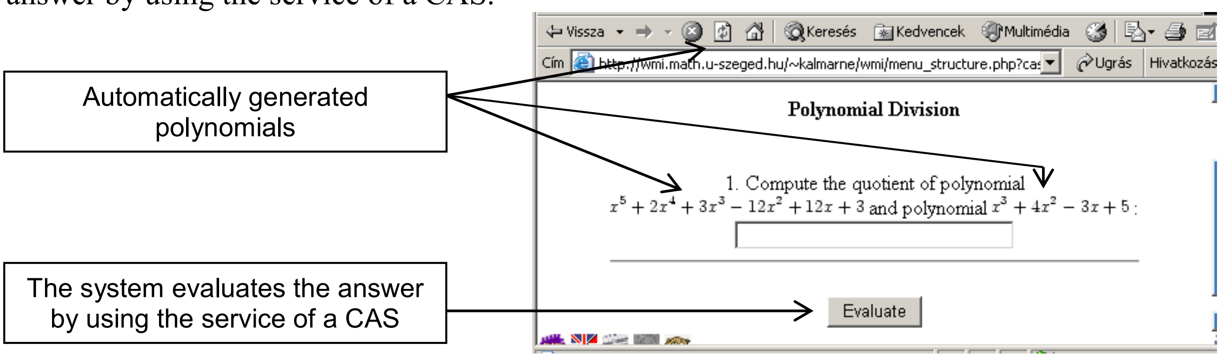


Figure 1: An open-ended test

On the higher level we can use complex exercises with hints and basic services. In Figure 2, a dynamic complex exercise can be seen (GCD: greatest common divisor).

The polynomials in the question are automatically generated. We can carry out this automatic generation in two ways. The system can generate the coefficients of the polynomials, or can choose from a database. The advantage of the latter way is that it protects the students from difficult calculations.

Clicking on the „Hint" button on the webpage, the student gets some help from the system. It can be very useful during self-assessment.

On this level the student can use CAS services like polynomial division. Clicking on the

„Polynomial Division” button, an interactive window opens, where the student can give two polynomials and gets their quotient and remainder. So the Euclidean Algorithm can be carried out without any mistakes in calculation.

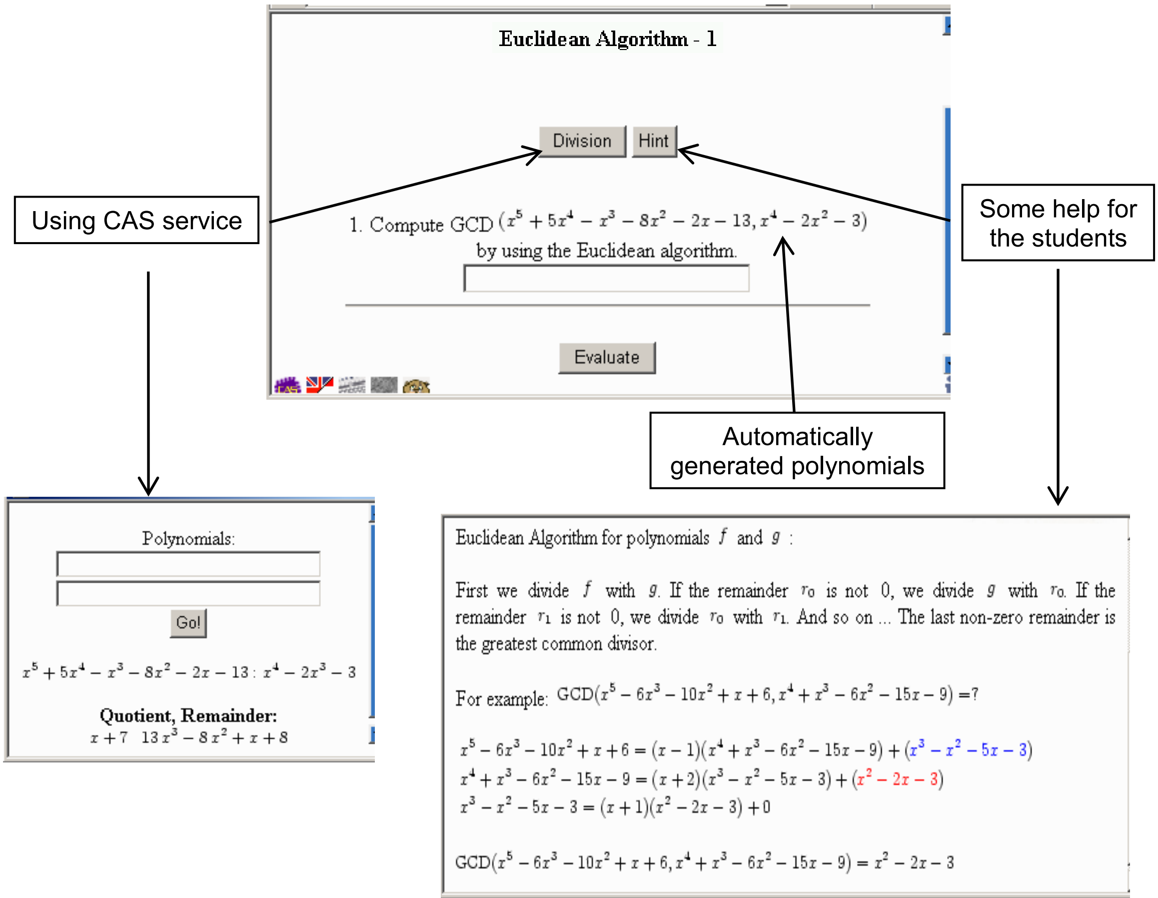


Figure 2: An open-ended test with hint and using CAS service

The „Evaluate” button has been mentioned before, but at this point we can find an explanation in the evaluation. Clicking to the „?” button in the evaluation, an automatically generated detailed explanation will be seen, which shows the steps of the Euclidean Algorithm referring to the polynomial in the current question and gives the right answer. So the student can check where she/he has made the mistake. In Figure 3 an evaluated open-ended test with explanation can be seen.

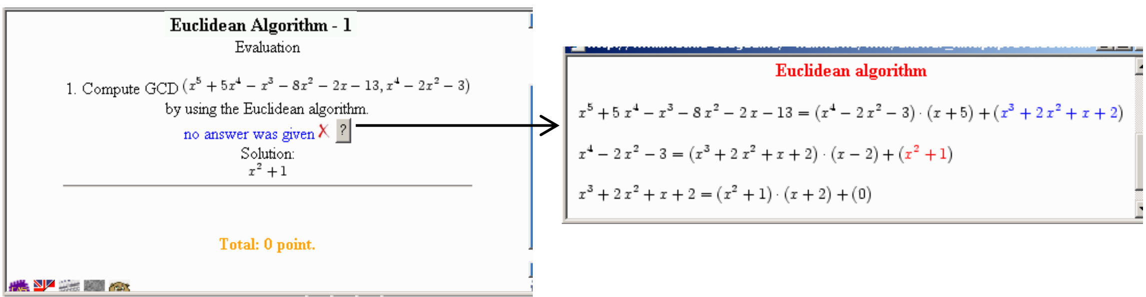


Figure 3: Automatically generated evaluation

If we want to assess the interconnections of the subject, we can use closed-ended tests. The advantages of the computer based closed-ended tests against the traditional ones are the following:
- they can have automatically generated arguments in the question,
- the selection and the sorting of the questions from which they are built up can be automated,

- their evaluations can be also automated,
- they can be expanded with additional explanations in the evaluation.

In Figure 4 we can see an evaluated closed-ended test with explanations.

Polynomial Equations, Lagrange Interpolation

Evaluation

1. Is the pair $u_0 = -x,\ v_0 = x^2$ a solution of polynomial equation $(x^2-1)u-(x-1)v = 2x^3-x^2-x$?

◉ Yes. ✗ ?

○ No.

2. Is it true that all solutions of the polynomial equation $(x^3+2x^2+2x+1)u+(x^3+3x^2+3x+2)v = x^3-1$ are of the form $u = 1-x-(x+2)t(x)$, $v = x-1-(x+1)t(x)$, $(t(x) \in \mathbf{R}[x])$?

◉ Yes. ✓

○ No.

3. Is there a quadratic polynomial that takes the values 3,4,5 at places 1,2,3 ?

◉ Yes. ✗ ?

○ No.

4. Is it true that there exists a quartic polynomial that takes the values 3,4,5 at places 1,2,3 ?

◉ Yes. ✓

○ No.

Total: 2 points.

Figure 4: A closed-ended test

There are some topics like Lagrange interpolation, in which we can find difficulties during assessment with CAS. If we are interested in interpolation polynomials, and we try to use closed-ended tests, the student will not calculate the correct polynomial, but only substitute some values into the given ones. If we ask an open-ended question, and the student makes a little mistake in the calculation, the computer will ignore the answer. Applying multi-step exercises to assess knowledge about Lagrange-interpolation, we can not be sure that the student has understood the whole algorithm or just followed the previously defined steps of the exercise. We can partially handle this problem if we enumerate the most frequent mistakes in the menu, from which the appropriate answer must be chosen, or if we combine the different assessment elements mentioned above. For a more detailed description of the problem of partial credits we refer to [14].

We could see that interactive systems like WMI can take over some duties of the teacher and give effective help in the assessment of basic skills.

3. Assessing Problem-Solving Skills with Open-Structured CAS

3.1. Didactical Principles

In this chapter we focus on assessing situations where students can use all services of CAS. So they can solve difficult, complex problems that develop their analyzing and synthesizing skills.

Within the same environment we have the opportunity to assess the state of development of their problem-solving strategies and creativity. This is due to the fact that a well-posed question do not determine the way to its solution uniquely and in this environment the solution way (the method) could be also assessed in contrast to the environment in Ch. 2. The method of solution chosen by the student will depend at least on the theoretical and technical background knowledge and several times also on the student's style/way of thinking. The open-structured CAS environment enables to measure ‘creativity’, since the subproblems are solved perhaps with the aid of self-defined functions and procedures or with an unexpected combination of the built-in elements.

It is true that the same advantages could be said about the traditional paper-pencil environment for assessing, but using CAS we can ask new kinds of questions. There are some interesting exercises which are time-consuming or contain difficult computations without CAS. We ignore them in traditional tests, but we can pose these problems, if the students are allowed to use CAS during their solutions. One can think of real problems of the field of applied mathematics as well, where first a mathematical model of the problem must be created, and after the elaboration of the exercise the solution gained within the model needs additional interpretation and a validity check.

The disadvantage of this CAS environment is that we can hardly use it to assess basic skills, because computer algebra systems are too powerful and there is no possibility to hide their services.

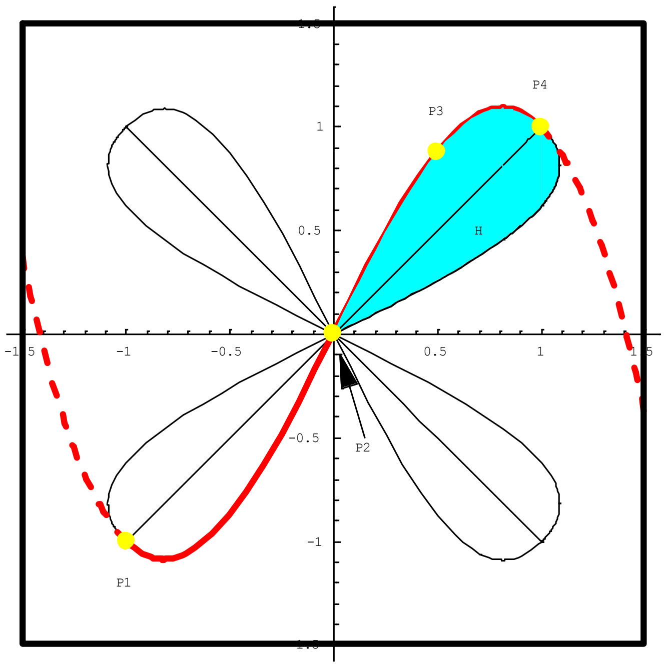
Without any intelligent interactive environment (i.e. where services can be reached using standard GUI elements) students need to know the syntax of a CAS. At the University of Szeged, we first give the students basic mathematical knowledge and teach syntactical and programming skills later, on a higher level. When students learn to work with CAS and solve mathematical problems supported by these systems, they already have basic mathematical skills.

The students can realize that the CAS is a good tool for their work. On the one hand if they face a mathematical problem at their workplace, they will have the skill of using computers during problem-solving. On the other hand for some students the CAS is not a great help but a new problem because they can find difficulty in syntax and programming. The other disadvantage of using CAS without any interactive environment is the danger that we measure programming skills instead of mathematical knowledge [11]. The latest version of computer algebra systems support such interfaces (Maplets and Mathematica Palettes and recently the GUIKit package), that minimize the occurring technical problems.

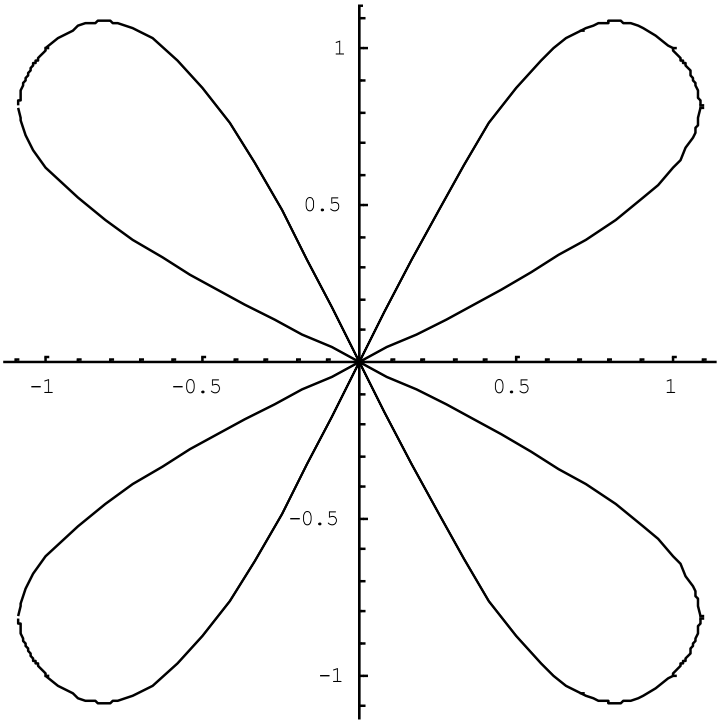
If we have decided in favour of using CAS in assessment, we have to reevaluate our traditional exercises. The questions written for a CAS-free environment are classified according to the following four categories: trivial with CAS, easy with CAS, difficult with CAS and CAS-proof (the CAS provides little help in solution of CAS-proof problems). To measure how suitable our tests are for a CAS-supported environment, we can use the CAS index [12]. The index ranges from 0 to 10, 0 means that the question is unsuitable in the CAS environment, 10 means that the question is the most suitable.

3.2. Example

We pose a problem whose elaboration requires a computer algebraic environment (in this special case: Mathematica). Let us consider the picture below which represents a planar region constructed by the help of a cubic polynomial. All the following questions are related to this plot.



a. GIVE a cubic polynomial p that fits to the points P1 (-1,-1), P2 (0,0), P3 (1/2,7/8), P4 (1,1).
b. INVESTIGATE the parity of the function p.
c. GIVE the local extrema of the function p (exact values).
d. Using symmetry CONSTRUCT AND DRAW the closed curve shown in the diagram by the aid of CAS without coloring the region H and without the points P1-P4 (see picture below). How can the symmetrical properties of the closed curve be exploited during the construction?



e. CALCULATE the (exact) area of the filled region H.
***f.** CALCULATE the arc length s of a curve $y=p(x)$ contained between the two points P1 and P4 with accuracy 10^{-3} (plotted with red in the first diagram).

The theoretical prerequisites needed for the solution is not beyond the average material of the basic mathematical courses. For this reason it can be also part of a final matura exam in a system where CAS is fully integrated into mathematical education at the high-school level [2, 3, 11].

We suppose that the basic knowledge is needed as part of the composite solution (e.g. derivation, integration etc. can already be handled as black-boxes) and we can focus on problem solving strategies and applications.

First we emphasize that both constructing a suitable exercise of this type and the evaluation of it requires extra expenditure on the instructor's side. As part of the problem (question d) we demand an explicit construction (plotting) of a closed curve with the help of Mathematica. Posing such kinds of questions neither makes sense in the traditional assessment environment nor is it possible using a similar environment which was presented in the second chapter. It is not surprising that finding an appropriate representation (i.e. functional description) of the curve remarkably simplifies the construction of the plot. Finally we list and overview some proposed solution schemes for the given questions. These are shown to demonstrate that the questions in the exercise are not only open-ended but they also leave enough degree of freedom for students to mobilize their background knowledge and to deliver diverging solutions.

•**a. proposed solution schemes**

- using the built-in command

```
pl = {{-1, -1}, {0, 0}, {1/2, 7/8}, {1, 1}};
TableForm[Transpose[%]]
```

$$\begin{array}{cccc} -1 & 0 & \frac{1}{2} & 1 \\ -1 & 0 & \frac{7}{8} & 1 \end{array}$$

```
P = Expand[InterpolatingPolynomial[pl, x]]
```

$2x - x^3$

- using another built-in function that can cause trouble.
!! Dangerous, but still valid after a tiny explanation.

```
Fit[pl, {1, x, x^2, x^3}, x]
P = Chop[%]
Table[ P /. x -> pl[[i, 1]] , {i, Length[pl]}]
```

$2.13691\times 10^{-16} + 2.\ x - 6.15935\times 10^{-16}\ x^2 - 1.\ x^3$

$2.\ x - 1.\ x^3$

{-1., 0, 0.875, 1.}

- solving a linear equation system that is equivalent to the original problem

```
P = a x^3 + b x^2 + c x + d;
rul = Flatten[Solve[Table[(P /. (x → pl[[i, 1]])) == pl[[i, 2]],
    {i, Length[pl]}]]
      ]
```

{d → 0, a → -1, b → 0, c → 2}

```
P /. rul
```

$2x - x^3$

- using another construction with fundamental interpolating polynomials

$$\text{Expand}\left[\sum_{j=1}^{\text{Length[pl]}} \text{pl[[j, 2]]} \prod_{i=1}^{\text{Length[pl]}} \frac{\text{If[i == j, 1, x - pl[[i, 1]]]}}{\text{If[i == j, 1, pl[[j, 1]] - pl[[i, 1]]]}}\right]$$

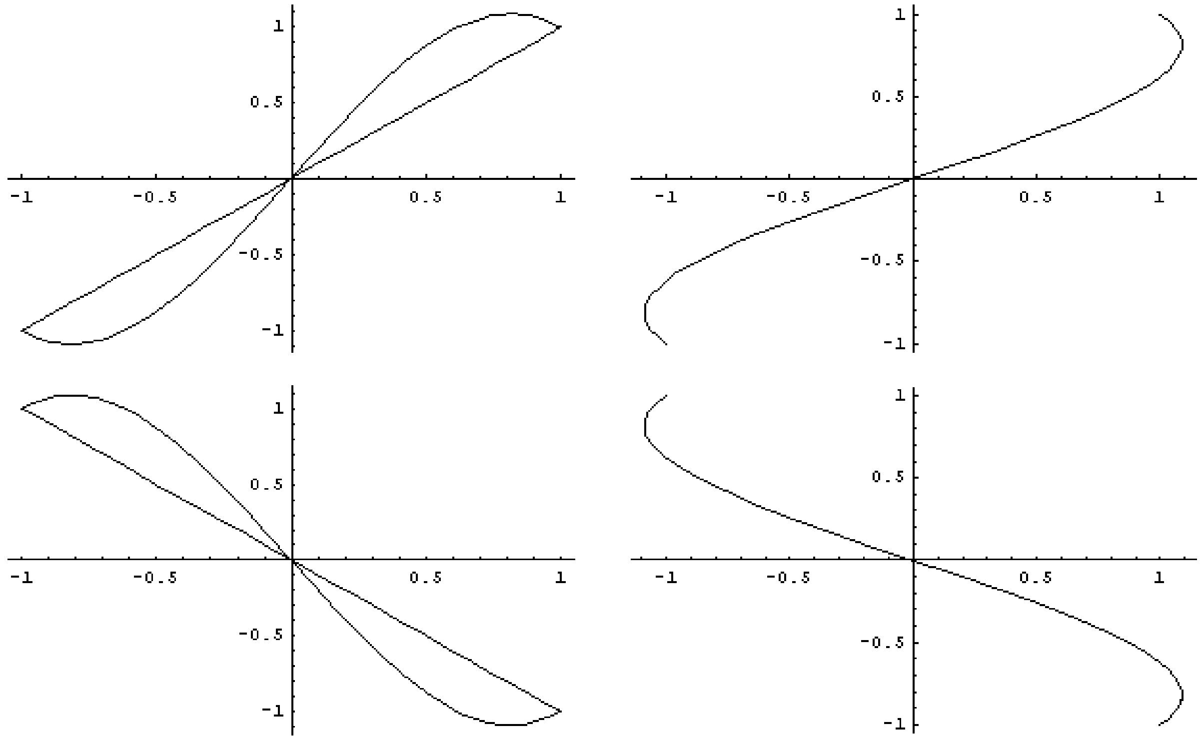
$2x - x^3$

■ d. proposed solution schemes

parametric representation, implicit and explicit functional relations, contour lines of a function with two variables

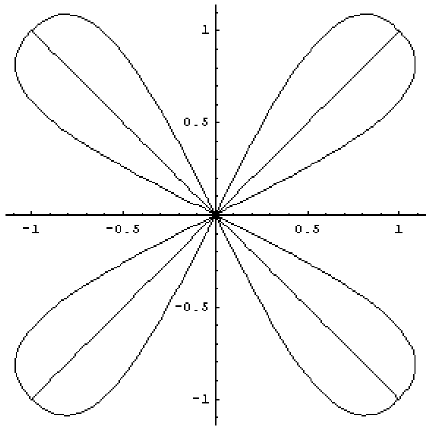
Symmetry: changing the coordinates + t→(-t) substitution

```
t1 = ParametricPlot[{{t, -t ^3 + 2 t}, {t, t}}, {t, -1, 1}];
t2 = ParametricPlot[{-t ^3 + 2 t, t}, {t, -1, 1}];
t3 = ParametricPlot[{{t, t ^3 - 2 t}, {t, -t}}, {t, -1, 1}];
t4 = ParametricPlot[{t ^3 - 2 t, t}, {t, -1, 1}];
```



Symmetry: squaring the original equation, changing the variables

```
eq2 = y^2 == (-x^3 + 2 x)^2;
<< Graphics`ImplicitPlot`
pp1 = ImplicitPlot[{x == y, eq2}, {x, -1, 1}, DisplayFunction → Identity];
pp2 = ImplicitPlot[{y == -x, eq2 /. {x → y, y → x}}, {x, -4 / 3 Sqrt[2 / 3], 4 / 3 Sqrt[2 / 3]},
   {y, -1, 1}, DisplayFunction → Identity];
Show[pp1, pp2, DisplayFunction → $DisplayFunction];
```



We cannot list all the solution schemes but already from this little part (a. and d.) one can recognize that technical difficulties (and these are not necessarily the same using another CAS!) can influence the problem solving effectiveness. In part a. we see the effect caused by the fact that we cannot hide the most powerful services during the exam in this environment. If the purpose of the assessment is the quickest application of the theoretical knowledge then the first proposed solution (i.e the built-in InterpolatingPolynomial, see attached notebook) has priority. But assessing the ability to construct somehow or effectively the interpolating polynomial, the last solution is more valuable [cf. Ch. 2.3] but in this case the use of the high-level built-in commands should be excluded during the assessment. A lot of authors underline that if CAS used in mathematical education, at least some of the built-in high-level commands (such as InterpolatingPolynomial in Mathematica) must go through the white-box phase (i.e. a possible or current implementation should be explored algorithmically in low-level CAS terms before we use them permanently). The second proposed solution is also based on a built-in function, but its (numerical) output should be handled with more care because an exact polynomial expression is demanded.

In part d. two relatively nice solutions are given to the plotting problem assuming the knowledge of parametrical and implicit functions and their graphical representation in Mathematica. The reader is asked to think about the origin of constants (e.g. -4/3Sqrt[2/3]) seen in the second proposed solution and about the difficulties in the construction if one uses only explicit (one-valued) functional relations and symmetry can be exploited in the plot by using inverse functions.

4. Integrating Technology Support into Multi-layered Assessment

However none of the above mentioned arguments imply that all kinds of mathematical knowledge and skills should be assessed with the aid of computers.

Due to the recent international literature, a multi-layered assessment strategy with respect to a given course or curriculum proved to be suitable. Kutzler proposes a two-tier examinational system: in the first part, no technology is allowed and basic mental fitness is assessed, while in the second (in time longer) part, problem-solving skill is tested and every technology is allowed (i.e. the whole symbolic, numerical and graphical services of a CAS could be used during the exam) [10]. We could extend this system according to Shirmer-Saneff with a third component: we can also include “self-elaborated focus papers” or project works to the final evaluation of a course or module [15]. The ability to find relevant sources and references to the given project problem independently, to make own ideas accessible to others and to structure and present the elaborated material in a high professional level is becoming more and more important since society demands a lot of cooperative teamwork.

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URLs of Some Known Web-based Systems:

[AIM]	http://www.mat.bham.ac.uk/aim/
[CowCulus]	http://cow.math.temple.edu/~cow/
[Mathe Online]	http://www.mathe-online.at
[Mathserv]	http://mss.math.vanderbilt.edu/~pscrooke/toolkit.shtml
[WIMS]	http://wims.unice.fr
[WMI]	http://wmi.math.u-szeged.hu

Authors

K. Németh, Márta, University of Szeged, Bolyai Institute, Aradi vértanúk tere 1, H-6720 Szeged, Hungary, e-mail: kalmarne@math.u-szeged.hu

Vajda, Róbert, University of Szeged, Bolyai Institute, Aradi vértanúk tere 1, H-6720 Szeged, Hungary, e-mail: vajda@server.math.u-szeged.hu