

Goal (Lemma 2.11): to verify that $Z_7(z_2, t) = -1$

It is obvious (verified) that $z_2 = -\sqrt{z_2 s} \in (-1, 1)$

General remarks.

We will verify the equality in an equivalent form, namely that
 $(1 + b_0 + b_2 z_2^2 + b_4 z_2^4 + b_6 z_2^6) = -(b_1 z_2 + b_3 z_2^3 + b_5 z_2^5 + b_7 z_2^7)$.

We do not claim that this is the simplest way.

We give a particular example first (assuming that $z_2 s, b_0, \dots, \omega$, etc are defined in terms of t)

$$\text{In[*]:= } \omega = \sqrt{\frac{4 - 11t + 8t^2}{t}};$$

$$\text{In[*]:= } z_2 s =$$

$$\begin{aligned} & (-6 + 25t + 14t^2 - 266t^3 + 648t^4 - 752t^5 + 464t^6 - 144t^7 + 16t^8 + (-1 + 2t)\omega\omega(2 - 3t - 8t^2 + 20 \\ & t^3 - 14t^4 + 4t^5 + \sqrt{2}\sqrt{(t(10 - 41t - 48t^2 + 590t^3 - 1482t^4 + 1870t^5 - 1328t^6 + \\ & 526t^7 - 104t^8 + 8t^9 + t(19 - 124t + 322t^2 - 422t^3 + 292t^4 - 100t^5 + 12t^6) \\ & \omega\omega))) / (16(-1 + t)^3 t^2 (-1 + 5t - 6t^2 + t^3)) \end{aligned}$$

$$\begin{aligned} \text{Out[*]:= } & (-6 + 25t + 14t^2 - 266t^3 + 648t^4 - 752t^5 + \\ & 464t^6 - 144t^7 + 16t^8 + (-1 + 2t)\omega\omega(2 - 3t - 8t^2 + 20t^3 - 14t^4 + 4t^5 + \\ & \sqrt{2}\sqrt{(t(10 - 41t - 48t^2 + 590t^3 - 1482t^4 + 1870t^5 - 1328t^6 + 526t^7 - \\ & 104t^8 + 8t^9 + t(19 - 124t + 322t^2 - 422t^3 + 292t^4 - 100t^5 + 12t^6) \\ & \omega\omega))) / (16(-1 + t)^3 t^2 (-1 + 5t - 6t^2 + t^3)) \end{aligned}$$

$$\text{In[*]:= } q = (2t - 1)^{10};$$

$$p61 = 16t^5(-10 + 27t + 288t^2 - 2022t^3 +$$

$$5825t^4 - 9477t^5 + 9336t^6 - 5494t^7 + 1712t^8 - 122t^9 - 80t^{10} + 16t^{11});$$

$$p62 = (-16t^6)(-25 + 206t - 726t^2 + 1459t^3 - 1927t^4 + 1870t^5 - 1394t^6 + 724t^7 - 210t^8 + 24t^9);$$

$$p41 = (-4t^3)(50 - 687t + 4203t^2 - 15695t^3 + 41282t^4 - 82204t^5 + 125852t^6 -$$

$$144150t^7 + 117216t^8 - 62520t^9 + 18496t^{10} - 1080t^{11} - 960t^{12} + 192t^{13});$$

$$p42 = 4t^3(10 - 121t + 677t^2 - 2521t^3 + 7132t^4 - 15412t^5 + 24800t^6 -$$

$$29742t^7 + 27184t^8 - 18840t^9 + 9072t^{10} - 2520t^{11} + 288t^{12});$$

$$p21 = (-2 + 40t - 360t^2 + 2245t^3 - 11098t^4 + 42924t^5 - 126314t^6 + 282304t^7 - 483968t^8 +$$

$$638896t^9 - 640864t^{10} + 470400t^{11} - 234144t^{12} + 65792t^{13} - 2784t^{14} - 3840t^{15} + 768t^{16});$$

$$p22 = -t^3(65 - 774t + 4344t^2 - 15694t^3 + 41224t^4 - 82096t^5 + 124784t^6 -$$

$$144768t^7 + 127840t^8 - 83808t^9 + 37824t^{10} - 10080t^{11} + 1152t^{12});$$

$$p01 = (1 - t)(1 - 19t + 161t^2 - 924t^3 + 4066t^4 - 13822t^5 + 35840t^6 - 70584t^7 + 105400t^8 -$$

$$\begin{aligned}
& 118\,168\,t^9 + 96\,704\,t^{10} - 54\,208\,t^{11} + 17\,760\,t^{12} - 1\,440\,t^{13} - 1\,024\,t^{14} + 256\,t^{15}); \\
p02 = & t^3 (25 - 290\,t + 1636\,t^2 - 6010\,t^3 + 15\,992\,t^4 - 32\,064\,t^5 + 48\,928\,t^6 - \\
& 56\,632\,t^7 + 49\,024\,t^8 - 30\,752\,t^9 + 13\,120\,t^{10} - 3360\,t^{11} + 384\,t^{12}); \\
p11 = & 8\,t^3 (t - 1) (-125 + 4005\,t - 60\,169\,t^2 + 570\,158\,t^3 - 3\,862\,166\,t^4 + 20\,045\,945\,t^5 - \\
& 83\,223\,823\,t^6 + 284\,217\,439\,t^7 - 813\,613\,881\,t^8 + 1\,978\,202\,656\,t^9 - 4\,124\,892\,528\,t^{10} + \\
& 7\,432\,364\,016\,t^{11} - 11\,645\,707\,296\,t^{12} + 15\,960\,414\,880\,t^{13} - 19\,242\,695\,952\,t^{14} + \\
& 20\,530\,254\,512\,t^{15} - 19\,487\,643\,408\,t^{16} + 16\,504\,985\,216\,t^{17} - 12\,441\,863\,808\,t^{18} + \\
& 8\,263\,236\,224\,t^{19} - 4\,749\,495\,424\,t^{20} + 2\,309\,017\,088\,t^{21} - 926\,839\,040\,t^{22} + \\
& 300\,215\,040\,t^{23} - 76\,503\,296\,t^{24} + 14\,585\,856\,t^{25} - 1\,794\,048\,t^{26} + 73\,728\,t^{27} + 8192\,t^{28}); \\
p12 = & 8\,t^3 (1 - t) (-25 + 765\,t - 11\,171\,t^2 + 105\,634\,t^3 - 736\,638\,t^4 + 4\,055\,851\,t^5 - 18\,297\,419\,t^6 + \\
& 69\,029\,409\,t^7 - 220\,386\,161\,t^8 + 599\,953\,820\,t^9 - 1\,399\,432\,680\,t^{10} + 2\,804\,818\,368\,t^{11} - \\
& 4\,835\,265\,904\,t^{12} + 7\,165\,985\,152\,t^{13} - 9\,113\,381\,872\,t^{14} + 9\,916\,520\,688\,t^{15} - 9\,197\,207\,856\,t^{16} + \\
& 7\,238\,388\,416\,t^{17} - 4\,811\,313\,152\,t^{18} + 2\,688\,147\,584\,t^{19} - 1\,256\,283\,520\,t^{20} + 488\,377\,856\,t^{21} - \\
& 156\,990\,208\,t^{22} + 41\,689\,856\,t^{23} - 9\,309\,952\,t^{24} + 1\,799\,168\,t^{25} - 280\,576\,t^{26} + 24\,576\,t^{27}); \\
p31 = & 8\,t^3 (t - 1) (-125 + 4005\,t - 59\,369\,t^2 + 553\,998\,t^3 - 3\,730\,470\,t^4 + \\
& 19\,565\,545\,t^5 - 83\,537\,119\,t^6 + 297\,885\,767\,t^7 - 901\,975\,729\,t^8 + \\
& 2\,350\,136\,384\,t^9 - 5\,333\,296\,784\,t^{10} + 10\,652\,922\,784\,t^{11} - 18\,880\,321\,184\,t^{12} + \\
& 29\,852\,879\,392\,t^{13} - 42\,255\,474\,560\,t^{14} + 53\,630\,116\,864\,t^{15} - \\
& 60\,971\,031\,168\,t^{16} + 61\,759\,492\,352\,t^{17} - 55\,128\,623\,872\,t^{18} + 42\,657\,332\,224\,t^{19} - \\
& 28\,037\,451\,008\,t^{20} + 15\,314\,942\,464\,t^{21} - 6\,803\,466\,496\,t^{22} + 2\,405\,327\,616\,t^{23} - \\
& 657\,957\,120\,t^{24} + 131\,469\,312\,t^{25} - 16\,441\,344\,t^{26} + 663\,552\,t^{27} + 73\,728\,t^{28}); \\
p32 = & 8\,t^3 (1 - t) (-25 + 765\,t - 11\,171\,t^2 + 107\,634\,t^3 - 787\,518\,t^4 + 4\,665\,067\,t^5 - \\
& 22\,973\,819\,t^6 + 95\,297\,577\,t^7 - 336\,259\,225\,t^8 + 1\,016\,942\,748\,t^9 - 2\,647\,457\,192\,t^{10} + \\
& 5\,940\,364\,352\,t^{11} - 11\,479\,425\,312\,t^{12} + 19\,070\,848\,736\,t^{13} - 27\,171\,006\,848\,t^{14} + \\
& 33\,097\,766\,976\,t^{15} - 34\,341\,234\,112\,t^{16} + 30\,212\,895\,232\,t^{17} - 22\,415\,990\,528\,t^{18} + \\
& 13\,930\,544\,128\,t^{19} - 7\,190\,272\,256\,t^{20} + 3\,053\,491\,200\,t^{21} - 1\,060\,255\,488\,t^{22} + \\
& 303\,176\,448\,t^{23} - 73\,578\,240\,t^{24} + 15\,455\,232\,t^{25} - 2\,525\,184\,t^{26} + 221\,184\,t^{27}); \\
p51 = & 128\,t^8 (t - 1) (-1500 + 28\,960\,t - 257\,939\,t^2 + 1\,404\,604\,t^3 - 5\,211\,955\,t^4 + \\
& 13\,795\,890\,t^5 - 25\,855\,137\,t^6 + 29\,614\,248\,t^7 + 3\,063\,112\,t^8 - 113\,012\,293\,t^9 + \\
& 333\,031\,055\,t^{10} - 652\,968\,101\,t^{11} + 993\,241\,608\,t^{12} - 1\,221\,269\,400\,t^{13} + \\
& 1\,224\,164\,696\,t^{14} - 994\,694\,744\,t^{15} + 647\,101\,024\,t^{16} - 331\,790\,224\,t^{17} + \\
& 131\,533\,872\,t^{18} - 39\,147\,024\,t^{19} + 8\,229\,120\,t^{20} - 1\,046\,016\,t^{21} + 41\,472\,t^{22} + 4608\,t^{23}); \\
p52 = & 128\,t^8 (1 - t) (-100 + 80\,t + 20\,039\,t^2 - 260\,728\,t^3 + 1\,786\,957\,t^4 - 8\,269\,540\,t^5 + 28\,531\,279\,t^6 - \\
& 77\,414\,974\,t^7 + 170\,435\,054\,t^8 - 309\,998\,835\,t^9 + 470\,441\,223\,t^{10} - 598\,385\,907\,t^{11} + \\
& 638\,090\,092\,t^{12} - 567\,729\,488\,t^{13} + 417\,123\,064\,t^{14} - 249\,429\,768\,t^{15} + 119\,783\,328\,t^{16} - \\
& 46\,124\,976\,t^{17} + 14\,587\,824\,t^{18} - 3\,960\,432\,t^{19} + 919\,872\,t^{20} - 157\,824\,t^{21} + 13\,824\,t^{22}); \\
p71 = & 2048\,t^{13} (t - 1) (-25 + 56\,t + 2520\,t^2 - 25\,565\,t^3 + 120\,032\,t^4 - 330\,197\,t^5 + \\
& 533\,239\,t^6 - 339\,915\,t^7 - 575\,358\,t^8 + 1\,899\,107\,t^9 - 2\,748\,394\,t^{10} + 2\,566\,194\,t^{11} - \\
& 1\,675\,233\,t^{12} + 781\,067\,t^{13} - 257\,689\,t^{14} + 57\,232\,t^{15} - 7392\,t^{16} + 288\,t^{17} + 32\,t^{18}); \\
p72 = & 2048\,t^{13} (1 - t) (-1 - 50\,t + 626\,t^2 - 2319\,t^3 - 1948\,t^4 + 47\,711\,t^5 - \\
& 201\,331\,t^6 + 485\,563\,t^7 - 779\,658\,t^8 + 878\,711\,t^9 - 712\,736\,t^{10} + 425\,072\,t^{11} - \\
& 193\,639\,t^{12} + 72\,003\,t^{13} - 23\,071\,t^{14} + 6068\,t^{15} - 1096\,t^{16} + 96\,t^{17});
\end{aligned}$$

```
In[ ]:= RootReduce [
  1 / q {Sqrt[p71 + p72 ωω], p61 + p62 ωω, -Sqrt[p51 + p52 ωω], p41 + p42 ωω, Sqrt[p31 + p32 ωω],
    p21 + p22 ωω, -Sqrt[p11 + p12 ωω], p01 + p02 ωω}.
  Table[(-Sqrt[z2s])^j, {j, 7, 0, -1}] /. ωω → ω /. {t → 4/5}]
```

Out[]:= -1

```
In[ ]:= {RootReduce [
  1 / q (q + {p61 + p62 ωω, p41 + p42 ωω, p21 + p22 ωω, p01 + p02 ωω}.Table[(-Sqrt[z2s])^j,
    {j, 6, 0, -2}]) /. ωω → ω /. {t → 4/5}], RootReduce [
  1 / q {-Sqrt[p71 + p72 ωω], Sqrt[p51 + p52 ωω], -Sqrt[p31 + p32 ωω], Sqrt[p11 + p12 ωω]}.
  Table[(-Sqrt[z2s])^j, {j, 7, 1, -2}] /. ωω → ω /. {t → 4/5}]}
```

Out[]:= {0.720 ..., 0.720 ...}

Now we will show two canonical forms of the LHS and RHS

(a) with radicals

(b) as bivariate integral polynomial $p(t,y)$ (labeled by qq45) whose zero set is a superset of the curve $(t,LHS(t))=(t,RHS(t))$ ($t \in (t_0,1)$)

```
In[ ]:= qq45 = -4096 + 148 480 t - 2 750 720 t^2 + 34 749 824 t^3 - 336 526 192 t^4 + 2 659 653 476 t^5 -
  17 816 570 327 t^6 + 103 669 805 970 t^7 - 532 673 407 533 t^8 + 2 444 555 432 428 t^9 -
  10 101 516 992 278 t^10 + 37 808 103 046 200 t^11 - 128 736 341 031 482 t^12 +
  400 097 778 377 888 t^13 - 1 137 744 898 086 151 t^14 + 2 965 536 850 326 350 t^15 -
  7 093 301 577 214 869 t^16 + 15 579 473 043 083 968 t^17 - 31 424 558 991 009 676 t^18 +
  58 188 795 780 620 280 t^19 - 98 832 142 780 309 764 t^20 + 153 771 297 026 767 552 t^21 -
  218 772 834 488 230 896 t^22 + 283 964 518 819 782 336 t^23 - 335 334 923 634 828 768 t^24 +
  359 082 661 480 751 680 t^25 - 347 305 120 815 149 760 t^26 + 302 022 806 671 910 528 t^27 -
  234 884 594 325 708 288 t^28 + 162 340 823 171 777 024 t^29 - 98 977 513 449 947 392 t^30 +
  52 763 299 284 114 944 t^31 - 24 330 463 229 814 016 t^32 + 9 577 133 976 019 968 t^33 -
  3 164 554 413 297 664 t^34 + 858 818 914 918 400 t^35 - 185 838 351 155 200 t^36 +
  30 727 222 591 488 t^37 - 3 631 831 056 384 t^38 + 272 092 889 088 t^39 - 9 663 676 416 t^40 +
  (-512 t^3 + 36 480 t^4 - 862 368 t^5 + 10 578 472 t^6 - 71 258 236 t^7 + 129 341 869 t^8 +
  2 786 256 320 t^9 - 38 165 474 712 t^10 + 295 155 126 592 t^11 - 1 688 825 202 801 t^12 +
  7 745 491 543 804 t^13 - 29 539 529 842 228 t^14 + 95 567 084 428 168 t^15 -
  265 211 205 189 724 t^16 + 634 568 767 617 900 t^17 - 1 308 675 104 905 300 t^18 +
  2 309 673 007 928 456 t^19 - 3 422 964 145 264 132 t^20 + 4 063 930 244 975 440 t^21 -
  3 328 525 537 082 832 t^22 + 362 287 730 145 472 t^23 + 5 031 272 957 570 880 t^24 -
  11 916 590 513 322 944 t^25 + 18 346 637 582 050 816 t^26 - 22 202 048 974 222 720 t^27 +
  22 291 811 606 586 368 t^28 - 18 958 202 256 457 728 t^29 + 13 762 753 646 345 216 t^30 -
  8 537 042 285 750 272 t^31 + 4 508 930 756 182 016 t^32 - 2 012 599 688 642 560 t^33 +
  750 296 622 743 552 t^34 - 229 579 662 491 648 t^35 + 56 196 285 202 432 t^36 -
  10 581 868 544 000 t^37 + 1 438 010 834 944 t^38 - 125 275 471 872 t^39 + 5 234 491 392 t^40) y +
  (128 t^5 + 944 t^6 - 121 024 t^7 + 2 435 295 t^8 - 25 309 618 t^9 + 150 214 572 t^10 - 338 717 930 t^11 -
```

$$\begin{aligned}
& 2\,716\,061\,235\,t^{12} + 35\,586\,205\,432\,t^{13} - 228\,758\,901\,298\,t^{14} + 1\,038\,778\,158\,408\,t^{15} - \\
& 3\,641\,811\,626\,338\,t^{16} + 10\,209\,673\,867\,420\,t^{17} - 23\,229\,240\,646\,864\,t^{18} + 43\,067\,989\,979\,784 \\
& t^{19} - 64\,999\,863\,846\,764\,t^{20} + 80\,441\,318\,627\,968\,t^{21} - 87\,334\,674\,313\,120\,t^{22} + \\
& 105\,688\,325\,071\,488\,t^{23} - 180\,692\,976\,848\,352\,t^{24} + 356\,158\,390\,490\,240\,t^{25} - \\
& 627\,743\,179\,812\,288\,t^{26} + 917\,571\,987\,183\,616\,t^{27} - 1\,106\,630\,331\,519\,232\,t^{28} + \\
& 1\,109\,792\,927\,015\,936\,t^{29} - 931\,896\,661\,092\,352\,t^{30} + 657\,693\,440\,925\,696\,t^{31} - \\
& 390\,390\,706\,241\,536\,t^{32} + 194\,478\,675\,722\,240\,t^{33} - 80\,920\,576\,049\,152\,t^{34} + \\
& 27\,904\,990\,773\,248\,t^{35} - 7\,880\,314\,781\,696\,t^{36} + 1\,787\,806\,613\,504\,t^{37} - \\
& 315\,265\,908\,736\,t^{38} + 40\,653\,291\,520\,t^{39} - 3\,388\,997\,632\,t^{40} + 134\,217\,728\,t^{41})y^2 + \\
(8\,t^8 - 202\,t^9 - 273\,t^{10} + 87\,928\,t^{11} - 1\,942\,927\,t^{12} + 24\,965\,964\,t^{13} - 227\,100\,424\,t^{14} + \\
1\,579\,091\,256\,t^{15} - 8\,743\,747\,008\,t^{16} + 39\,530\,964\,420\,t^{17} - 148\,289\,139\,060\,t^{18} + \\
466\,280\,733\,784\,t^{19} - 1\,235\,973\,648\,972\,t^{20} + 2\,765\,663\,780\,976\,t^{21} - 5\,205\,285\,494\,432\,t^{22} + \\
8\,148\,562\,651\,392\,t^{23} - 10\,336\,766\,132\,480\,t^{24} + 9\,948\,814\,128\,128\,t^{25} - \\
5\,692\,194\,596\,864\,t^{26} - 1\,905\,393\,430\,528\,t^{27} + 10\,212\,250\,810\,368\,t^{28} - \\
15\,862\,040\,682\,496\,t^{29} + 16\,889\,421\,414\,400\,t^{30} - 13\,848\,250\,941\,440\,t^{31} + \\
9\,035\,515\,887\,616\,t^{32} - 4\,720\,780\,378\,112\,t^{33} + 1\,959\,608\,975\,360\,t^{34} - 633\,610\,960\,896\,t^{35} + \\
154\,131\,496\,960\,t^{36} - 26\,576\,158\,720\,t^{37} + 2\,900\,361\,216\,t^{38} - 150\,994\,944\,t^{39})y^3 + \\
(-t^{10} + 50\,t^{11} - 1203\,t^{12} + 18\,544\,t^{13} - 205\,715\,t^{14} + 1\,748\,874\,t^{15} - 11\,847\,485\,t^{16} + \\
65\,652\,532\,t^{17} - 303\,151\,596\,t^{18} + 1\,182\,149\,760\,t^{19} - 3\,931\,550\,832\,t^{20} + 11\,232\,064\,320\,t^{21} - \\
27\,706\,743\,616\,t^{22} + 59\,214\,199\,808\,t^{23} - 109\,855\,142\,400\,t^{24} + 177\,017\,808\,896\,t^{25} - \\
247\,564\,564\,480\,t^{26} + 299\,858\,362\,368\,t^{27} - 313\,427\,288\,064\,t^{28} + 281\,219\,727\,360\,t^{29} - \\
215\,000\,383\,488\,t^{30} + 138\,664\,345\,600\,t^{31} - 74\,425\,892\,864\,t^{32} + 32\,629\,850\,112\,t^{33} - \\
11\,380\,981\,760\,t^{34} + 3\,036\,676\,096\,t^{35} - 581\,959\,680\,t^{36} + 71\,303\,168\,t^{37} - 4\,194\,304\,t^{38})y^4;
\end{aligned}$$

```

In[ ]:= rp1 = 8 + 54 t - 2385 t^2 + 21528 t^3 - 100271 t^4 + 284780 t^5 - 512936 t^6 + 546616 t^7 - 185728 t^8 -
          398908 t^9 + 761100 t^10 - 684968 t^11 + 377396 t^12 - 129808 t^13 + 25824 t^14 - 2304 t^15;
rp2 = (16 - 8 t - 1275 t^2 + 9302 t^3 - 31095 t^4 + 53980 t^5 - 28974 t^6 - 80664 t^7 + 217764 t^8 -
       264956 t^9 + 189108 t^10 - 76880 t^11 + 11764 t^12 + 3552 t^13 - 1888 t^14 + 256 t^15);
rq = 4 (-1 + t)^6 t^2 (-1 + 2 t)^6;
rp3 = 2 (12960 - 292016 t + 2854698 t^2 - 14792140 t^3 + 32588991 t^4 + 85143228 t^5 -
        925203078 t^6 + 3299368184 t^7 - 4942483971 t^8 - 7668657248 t^9 + 62474910910 t^10 -
        170963084656 t^11 + 246192050150 t^12 - 34672143568 t^13 - 789233922456 t^14 +
        2311627705560 t^15 - 4090792828656 t^16 + 5282309403760 t^17 - 5223088300728 t^18 +
        3967261586528 t^19 - 2228596974256 t^20 + 799166250432 t^21 - 35198764320 t^22 -
        180928743808 t^23 + 144029257712 t^24 - 66339007104 t^25 + 20749249664 t^26 -
        4414670336 t^27 + 584887296 t^28 - 34091008 t^29 - 1572864 t^30 + 262144 t^31);
rp4 = 2 (6624 t - 161376 t^2 + 1695878 t^3 - 9509785 t^4 + 24259520 t^5 + 42626172 t^6 -
        628269302 t^7 + 2609963815 t^8 - 5418910112 t^9 + 384019718 t^10 +
        38614387936 t^11 - 152399515228 t^12 + 362607645744 t^13 - 617651612344 t^14 +
        779178270248 t^15 - 703631968400 t^16 + 373394610624 t^17 +
        57362354376 t^18 - 372214821664 t^19 + 456997541776 t^20 - 358199885632 t^21 +
        201881079680 t^22 - 82461667616 t^23 + 22845169104 t^24 - 3148220352 t^25 -
        456967680 t^26 + 354630144 t^27 - 87122944 t^28 + 10960896 t^29 - 589824 t^30);
rqq = 4 (-1 + t)^6 t^2 (-1 + 2 t)^6;

```

```

In[ ]:= RootReduce [
  {Root[qq45 /. t -> 4/5, 3], (rp1 + rp2 ωω - Sqrt[(rp3 + rp4 ωω])) / rqq /. ωω -> ω /. t -> 4/5}]

```

```

Out[ ]:= {0.720 ..., 0.720 ...}

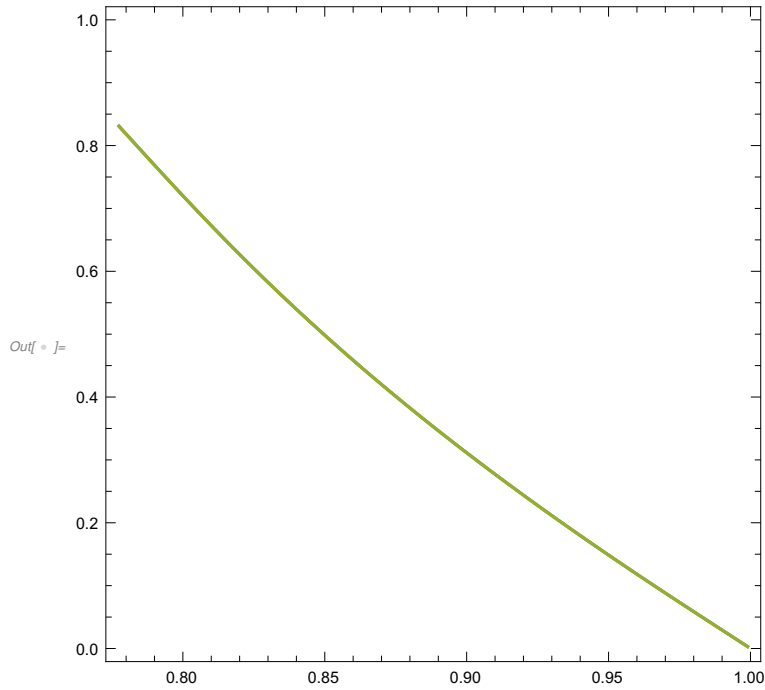
```

the three expressions are the same (lhs, rhs, radical expression)

```

In[ ]:= ContourPlot [
  Evaluate [{y == 1/q (q + {p61 + p62 ωω, p41 + p42 ωω, p21 + p22 ωω, p01 + p02 ωω}.Table[
    (-Sqrt[z2s])^j, {j, 6, 0, -2}]} /. ωω → ω,
  y == 1/q {-Sqrt[p71 + p72 ωω], Sqrt[p51 + p52 ωω], -Sqrt[p31 + p32 ωω],
    Sqrt[p11 + p12 ωω]}.Table[(-Sqrt[z2s])^j, {j, 7, 1, -2}]} /. ωω → ω,
  y == (rp1 + rp2 ω - Sqrt[(rp3 + rp4 ω)]/rq), {t, 0.777..., .999},
  {y, 0, 1}, WorkingPrecision → 80,
  PlotPoints → 80]

```

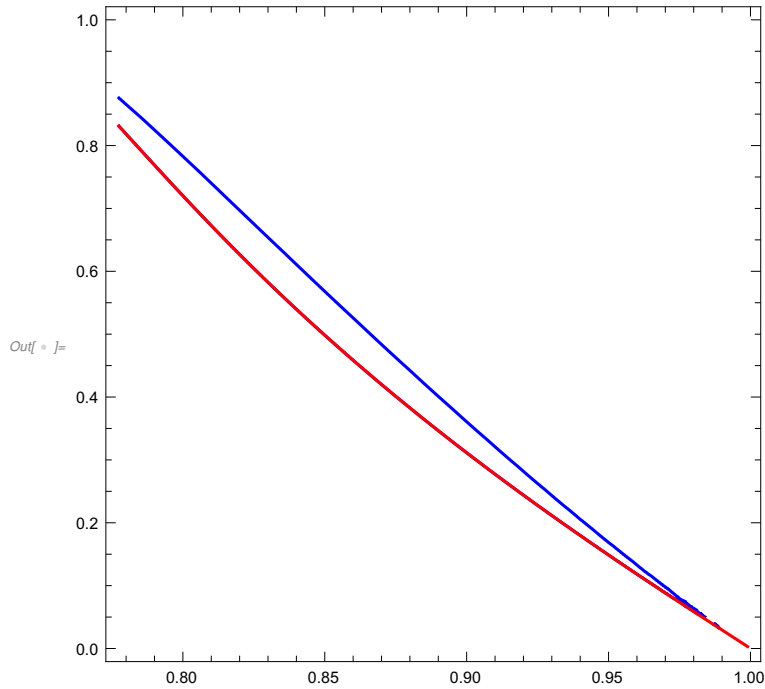


radical expression and the superset as a zeroset of a bivariate polynomial

```

In[ ]:= ContourPlot[Evaluate[{{qq45 == 0, y == (rp1 + rp2 ω - Sqrt[(rp3 + rp4 ω])/rq}}],
  {t, {0.777..., .999}}, {y, 0, 1}, WorkingPrecision -> 80,
  PlotPoints -> 80, ContourStyle -> {Blue, Red}]

```



Now the formal verification consist of showing that starting from the LHS/RHS we can derive the same radical expression/bivariate polynomial.

It seems that showing everything “on the polynomial” level turns out to be the standard way. If the LHS-RHS polynomials coincide, then one can show that zero set of qq45 consist of 4 separate continuous curves above (t0,1) and therefore testing one point (t=4/5) for identifying the radical expression (the 3rd curve part) is sufficient, QED.

To derive from LHS the radical expression, we take $z2s = (z2)^2$ as a sum.

$z2s$ as $z2s = z2ms + z2mms$ for the derivation (binomial theorem)

```

In[ ]:= z2ms = (-6 + 25 t + 14 t^2 - 266 t^3 + 648 t^4 - 752 t^5 + 464 t^6 - 144 t^7 + 16 t^8 +
  (-1 + 2 t) ω ω (2 - 3 t - 8 t^2 + 20 t^3 - 14 t^4 + 4 t^5 + 0)) / ((16 (-1 + t)^3 t^2 (-1 + 5 t - 6 t^2 + t^3)))
Out[ ]:= (-6 + 25 t + 14 t^2 - 266 t^3 + 648 t^4 - 752 t^5 + 464 t^6 - 144 t^7 + 16 t^8 +
  (-1 + 2 t) (2 - 3 t - 8 t^2 + 20 t^3 - 14 t^4 + 4 t^5) ω ω) / ((16 (-1 + t)^3 t^2 (-1 + 5 t - 6 t^2 + t^3)))

```

In[]:= z2mms =

$$\left((-1 + 2t) \omega \left(\sqrt{2} \sqrt{(t(10 - 41t - 48t^2 + 590t^3 - 1482t^4 + 1870t^5 - 1328t^6 + 526t^7 - 104t^8 + 8t^9 + t(19 - 124t + 322t^2 - 422t^3 + 292t^4 - 100t^5 + 12t^6) \omega)} \right) \right) / (16(-1+t)^3 t^2 (-1+5t-6t^2+t^3))$$

Out[]:= $(-1 + 2t) \omega$

$$\sqrt{(t(10 - 41t - 48t^2 + 590t^3 - 1482t^4 + 1870t^5 - 1328t^6 + 526t^7 - 104t^8 + 8t^9 + t(19 - 124t + 322t^2 - 422t^3 + 292t^4 - 100t^5 + 12t^6) \omega)} / (8 \sqrt{2} (-1+t)^3 t^2 (-1+5t-6t^2+t^3))$$

In[]:= Together [z2s - (z2ms + z2mms)]

Out[]:= 0

In[]:= Together [(rp1 + rp2 ω) / rq - (1 + 1/q {p61 + p62 ω , p41 + p42 ω , p21 + p22 ω , p01 + p02 ω }. {z2ms ^ 3 + 3 z2ms z2mms ^ 2, z2ms ^ 2 + z2mms ^ 2, z2ms, 1}) / . $\omega \rightarrow \omega$]

Out[]:= 0

In[]:= Together [(rp3 + rp4 ω) / rq ^ 2 - (1/q {p61 + p62 ω , p41 + p42 ω , p21 + p22 ω , p01 + p02 ω }. {z2mms ^ 2 z2mms + 3 z2ms ^ 2 z2mms, 2 z2ms z2mms, z2mms, 0}) ^ 2 / . $\omega \rightarrow \omega$]

Out[]:= 0

The above computation verifies that LHS and the canonical radical expression are the same. One can also derive from the canonical radical expression the bivariate polynomial.

Now e.g. to derive from RHS the bivariate integral polynomial is a lengthy, tedious, resultant computation (see below), but it works!

Auxiliary goal: rhs has the same bivariate polynomial as lhs above (qq45)

In[]:= qq45 = -4096 + 148480 t - 2750720 t² + 34749824 t³ - 336526192 t⁴ + 2659653476 t⁵ - 17816570327 t⁶ + 103669805970 t⁷ - 532673407533 t⁸ + 2444555432428 t⁹ - 10101516992278 t¹⁰ + 37808103046200 t¹¹ - 128736341031482 t¹² + 400097778377888 t¹³ - 1137744898086151 t¹⁴ + 2965536850326350 t¹⁵ - 7093301577214869 t¹⁶ + 15579473043083968 t¹⁷ - 31424558991009676 t¹⁸ + 58188795780620280 t¹⁹ - 98832142780309764 t²⁰ + 153771297026767552 t²¹ - 218772834488230896 t²² + 283964518819782336 t²³ - 335334923634828768 t²⁴ + 359082661480751680 t²⁵ - 347305120815149760 t²⁶ + 302022806671910528 t²⁷ - 234884594325708288 t²⁸ + 162340823171777024 t²⁹ - 98977513449947392 t³⁰ +

$$\begin{aligned}
& 52\,763\,299\,284\,114\,944\,t^{31} - 24\,330\,463\,229\,814\,016\,t^{32} + 9\,577\,133\,976\,019\,968\,t^{33} - \\
& 3\,164\,554\,413\,297\,664\,t^{34} + 858\,818\,914\,918\,400\,t^{35} - 185\,838\,351\,155\,200\,t^{36} + \\
& 30\,727\,222\,591\,488\,t^{37} - 3\,631\,831\,056\,384\,t^{38} + 272\,092\,889\,088\,t^{39} - 9\,663\,676\,416\,t^{40} + \\
& (-512\,t^3 + 36\,480\,t^4 - 862\,368\,t^5 + 10\,578\,472\,t^6 - 71\,258\,236\,t^7 + 129\,341\,869\,t^8 + \\
& \quad 2\,786\,256\,320\,t^9 - 38\,165\,474\,712\,t^{10} + 295\,155\,126\,592\,t^{11} - 1\,688\,825\,202\,801\,t^{12} + \\
& \quad 7\,745\,491\,543\,804\,t^{13} - 29\,539\,529\,842\,228\,t^{14} + 95\,567\,084\,428\,168\,t^{15} - \\
& \quad 265\,211\,205\,189\,724\,t^{16} + 634\,568\,767\,617\,900\,t^{17} - 1\,308\,675\,104\,905\,300\,t^{18} + \\
& \quad 2\,309\,673\,007\,928\,456\,t^{19} - 3\,422\,964\,145\,264\,132\,t^{20} + 4\,063\,930\,244\,975\,440\,t^{21} - \\
& \quad 3\,328\,525\,537\,082\,832\,t^{22} + 362\,287\,730\,145\,472\,t^{23} + 5\,031\,272\,957\,570\,880\,t^{24} - \\
& \quad 11\,916\,590\,513\,322\,944\,t^{25} + 18\,346\,637\,582\,050\,816\,t^{26} - 22\,202\,048\,974\,222\,720\,t^{27} + \\
& \quad 22\,291\,811\,606\,586\,368\,t^{28} - 18\,958\,202\,256\,457\,728\,t^{29} + 13\,762\,753\,646\,345\,216\,t^{30} - \\
& \quad 8\,537\,042\,285\,750\,272\,t^{31} + 4\,508\,930\,756\,182\,016\,t^{32} - 2\,012\,599\,688\,642\,560\,t^{33} + \\
& \quad 750\,296\,622\,743\,552\,t^{34} - 229\,579\,662\,491\,648\,t^{35} + 56\,196\,285\,202\,432\,t^{36} - \\
& \quad 10\,581\,868\,544\,000\,t^{37} + 1\,438\,010\,834\,944\,t^{38} - 125\,275\,471\,872\,t^{39} + 5\,234\,491\,392\,t^{40})y + \\
& (128\,t^5 + 944\,t^6 - 121\,024\,t^7 + 2\,435\,295\,t^8 - 25\,309\,618\,t^9 + 150\,214\,572\,t^{10} - 338\,717\,930\,t^{11} - \\
& \quad 2\,716\,061\,235\,t^{12} + 35\,586\,205\,432\,t^{13} - 228\,758\,901\,298\,t^{14} + 1\,038\,778\,158\,408\,t^{15} - \\
& \quad 3\,641\,811\,626\,338\,t^{16} + 10\,209\,673\,867\,420\,t^{17} - 23\,229\,240\,646\,864\,t^{18} + 43\,067\,989\,979\,784 \\
& \quad t^{19} - 64\,999\,863\,846\,764\,t^{20} + 80\,441\,318\,627\,968\,t^{21} - 87\,334\,674\,313\,120\,t^{22} + \\
& \quad 105\,688\,325\,071\,488\,t^{23} - 180\,692\,976\,848\,352\,t^{24} + 356\,158\,390\,490\,240\,t^{25} - \\
& \quad 627\,743\,179\,812\,288\,t^{26} + 917\,571\,987\,183\,616\,t^{27} - 1\,106\,630\,331\,519\,232\,t^{28} + \\
& \quad 1\,109\,792\,927\,015\,936\,t^{29} - 931\,896\,661\,092\,352\,t^{30} + 657\,693\,440\,925\,696\,t^{31} - \\
& \quad 390\,390\,706\,241\,536\,t^{32} + 194\,478\,675\,722\,240\,t^{33} - 80\,920\,576\,049\,152\,t^{34} + \\
& \quad 27\,904\,990\,773\,248\,t^{35} - 7\,880\,314\,781\,696\,t^{36} + 1\,787\,806\,613\,504\,t^{37} - \\
& \quad 315\,265\,908\,736\,t^{38} + 40\,653\,291\,520\,t^{39} - 3\,388\,997\,632\,t^{40} + 134\,217\,728\,t^{41})y^2 + \\
& (8\,t^8 - 202\,t^9 - 273\,t^{10} + 87\,928\,t^{11} - 1\,942\,927\,t^{12} + 24\,965\,964\,t^{13} - 227\,100\,424\,t^{14} + \\
& \quad 1\,579\,091\,256\,t^{15} - 8\,743\,747\,008\,t^{16} + 39\,530\,964\,420\,t^{17} - 148\,289\,139\,060\,t^{18} + \\
& \quad 466\,280\,733\,784\,t^{19} - 1\,235\,973\,648\,972\,t^{20} + 2\,765\,663\,780\,976\,t^{21} - 5\,205\,285\,494\,432\,t^{22} + \\
& \quad 8\,148\,562\,651\,392\,t^{23} - 10\,336\,766\,132\,480\,t^{24} + 9\,948\,814\,128\,128\,t^{25} - \\
& \quad 5\,692\,194\,596\,864\,t^{26} - 1\,905\,393\,430\,528\,t^{27} + 10\,212\,250\,810\,368\,t^{28} - \\
& \quad 15\,862\,040\,682\,496\,t^{29} + 16\,889\,421\,414\,400\,t^{30} - 13\,848\,250\,941\,440\,t^{31} + \\
& \quad 9\,035\,515\,887\,616\,t^{32} - 4\,720\,780\,378\,112\,t^{33} + 1\,959\,608\,975\,360\,t^{34} - 633\,610\,960\,896\,t^{35} + \\
& \quad 154\,131\,496\,960\,t^{36} - 26\,576\,158\,720\,t^{37} + 2\,900\,361\,216\,t^{38} - 150\,994\,944\,t^{39})y^3 + \\
& (-t^{10} + 50\,t^{11} - 1203\,t^{12} + 18\,544\,t^{13} - 205\,715\,t^{14} + 1\,748\,874\,t^{15} - 11\,847\,485\,t^{16} + \\
& \quad 65\,652\,532\,t^{17} - 303\,151\,596\,t^{18} + 1\,182\,149\,760\,t^{19} - 3\,931\,550\,832\,t^{20} + 11\,232\,064\,320\,t^{21} - \\
& \quad 27\,706\,743\,616\,t^{22} + 59\,214\,199\,808\,t^{23} - 109\,855\,142\,400\,t^{24} + 177\,017\,808\,896\,t^{25} - \\
& \quad 247\,564\,564\,480\,t^{26} + 299\,858\,362\,368\,t^{27} - 313\,427\,288\,064\,t^{28} + 281\,219\,727\,360\,t^{29} - \\
& \quad 215\,000\,383\,488\,t^{30} + 138\,664\,345\,600\,t^{31} - 74\,425\,892\,864\,t^{32} + 32\,629\,850\,112\,t^{33} - \\
& \quad 11\,380\,981\,760\,t^{34} + 3\,036\,676\,096\,t^{35} - 581\,959\,680\,t^{36} + 71\,303\,168\,t^{37} - 4\,194\,304\,t^{38})y^4;
\end{aligned}$$

particular check, then building up, step by step, the “radical/root“-tower

In[]:= Root[qq45 /. t -> 4/5, 3]

Out[]:= $\sqrt[3]{0.720 \dots}$

In[]:= RootReduce [

1/q {-Sqrt[p71 + p72 ωω], Sqrt[p51 + p52 ωω], -Sqrt[p31 + p32 ωω], Sqrt[p11 + p12 ωω]}.
Table[(-Sqrt[Z])^j, {j, 7, 1, -2}] /. Z -> z2s /. ωω -> ω /. {{t -> 4/5}}

Out[]:= $\left\{ \sqrt[3]{0.720 \dots} \right\}$

In[]:= RootReduce [

1/q {0, 0, 0, Sqrt[p11 + p12 ωω]}.Table[(-Sqrt[Z])^j, {j, 7, 1, -2}] /. Z -> z2s /. ωω -> ω /.
{{t -> 4/5}}

Out[]:= $\left\{ \sqrt[3]{-3.50 \dots} \right\}$

In[]:= RootReduce [-Sqrt[z2s (p11 + p12 ωω)]/q /. ωω -> ω /. t -> 4/5]

Out[]:= $\sqrt[3]{-3.50 \dots}$

In[]:= rh1a =

Factor[Resultant[(((16 (-1 + t)³ t² (-1 + 5 t - 6 t² + t³))) z22 - (-6 + 25 t + 14 t² - 266 t³ + 648 t⁴ -
752 t⁵ + 464 t⁶ - 144 t⁷ + 16 t⁸) - ωω (-1 + 2 t) (2 - 3 t - 8 t² + 20 t³ - 14 t⁴ + 4 t⁵))^2 -
ωω^2 (-1 + 2 t)^2 (2 ((t (10 - 41 t - 48 t² + 590 t³ - 1482 t⁴ + 1870 t⁵ - 1328 t⁶ +
526 t⁷ - 104 t⁸ + 8 t⁹ + t (19 - 124 t + 322 t² - 422 t³ + 292 t⁴ - 100 t⁵ + 12 t⁶)
ωω))), t ωω^2 - (4 - 11 t + 8 t²), ωω]][[5]]

Out[]:= -1 + 15 t - 119 t² + 661 t³ - 2848 t⁴ + 9996 t⁵ - 29356 t⁶ + 73220 t⁷ - 156256 t⁸ + 285808 t⁹ -
446560 t¹⁰ + 591008 t¹¹ - 653440 t¹² + 591424 t¹³ - 425856 t¹⁴ + 234496 t¹⁵ - 93440 t¹⁶ +
24832 t¹⁷ - 3840 t¹⁸ + 256 t¹⁹ + 24 t³ z22 - 308 t⁴ z22 + 1872 t⁵ z22 - 6724 t⁶ z22 + 13644 t⁷ z22 -
4192 t⁸ z22 - 73824 t⁹ z22 + 294080 t¹⁰ z22 - 680448 t¹¹ z22 + 1113152 t¹² z22 -
1343232 t¹³ z22 + 1194880 t¹⁴ z22 - 765952 t¹⁵ z22 + 338944 t¹⁶ z22 - 96256 t¹⁷ z22 +
15360 t¹⁸ z22 - 1024 t¹⁹ z22 + 32 t⁵ z22² - 272 t⁶ z22² + 160 t⁷ z22² + 4992 t⁸ z22² -
17552 t⁹ z22² - 4512 t¹⁰ z22² + 180576 t¹¹ z22² - 590592 t¹² z22² + 1058112 t¹³ z22² -
1214848 t¹⁴ z22² + 923904 t¹⁵ z22² - 458240 t¹⁶ z22² + 139776 t¹⁷ z22² - 23040 t¹⁸ z22² +
1536 t¹⁹ z22² - 384 t⁸ z22³ + 3520 t⁹ z22³ - 9408 t¹⁰ z22³ - 11520 t¹¹ z22³ + 130368 t¹² z22³ -
358528 t¹³ z22³ + 536192 t¹⁴ z22³ - 487936 t¹⁵ z22³ + 273408 t¹⁶ z22³ - 90112 t¹⁷ z22³ +
15360 t¹⁸ z22³ - 1024 t¹⁹ z22³ - 256 t¹⁰ z22⁴ + 3328 t¹¹ z22⁴ - 17920 t¹² z22⁴ + 52224 t¹³ z22⁴ -
90368 t¹⁴ z22⁴ + 95488 t¹⁵ z22⁴ - 60672 t¹⁶ z22⁴ + 21760 t¹⁷ z22⁴ - 3840 t¹⁸ z22⁴ + 256 t¹⁹ z22⁴

In[]:= RootReduce [z2s /. ωω -> ω /. t -> 4/5]

Out[]:= $\sqrt[3]{0.356 \dots}$

In[]:= Solve[rh1a == 0 /. t -> 4/5]

Out[]:= $\left\{ \left\{ z22 \rightarrow \sqrt[3]{-16.2 \dots} \right\}, \left\{ z22 \rightarrow \sqrt[3]{-0.0448 \dots} \right\}, \left\{ z22 \rightarrow \sqrt[3]{0.0955 \dots} \right\}, \left\{ z22 \rightarrow \sqrt[3]{0.356 \dots} \right\} \right\}$

In[] := RootReduce[-Sqrt[z2s (p11 + p12 ωω)]/q /. ωω → ω /. t → 4/5]

Out[] :=  -3.50 ...

In[] := rh1b = Factor[Resultant[(b1 q)^2 - (z22 (p11 + p12 ωω)), t ωω^2 - (4 - 11 t + 8 t^2), ωω]][[3]]

Out[] := b1^4 - 64 b1^4 t + 1984 b1^4 t^2 - 39 680 b1^4 t^3 + 575 360 b1^4 t^4 - 6 444 032 b1^4 t^5 + 57 996 288 b1^4 t^6 -
 430 829 568 b1^4 t^7 + 2 692 684 800 b1^4 t^8 - 14 360 985 600 b1^4 t^9 + 66 060 533 760 b1^4 t^10 -
 264 242 135 040 b1^4 t^11 + 924 847 472 640 b1^4 t^12 - 2 845 684 531 200 b1^4 t^13 +
 7 724 000 870 400 b1^4 t^14 - 18 537 602 088 960 b1^4 t^15 + 39 392 404 439 040 b1^4 t^16 -
 74 150 408 355 840 b1^4 t^17 + 123 584 013 926 400 b1^4 t^18 - 182 123 809 996 800 b1^4 t^19 +
 236 760 952 995 840 b1^4 t^20 - 270 583 946 280 960 b1^4 t^21 + 270 583 946 280 960 b1^4 t^22 -
 235 290 388 070 400 b1^4 t^23 + 176 467 791 052 800 b1^4 t^24 - 112 939 386 273 792 b1^4 t^25 +
 60 813 515 685 888 b1^4 t^26 - 27 028 229 193 728 b1^4 t^27 + 9 652 938 997 760 b1^4 t^28 -
 2 662 879 723 520 b1^4 t^29 + 532 575 944 704 b1^4 t^30 - 68 719 476 736 b1^4 t^31 + 4 294 967 296 b1^4 t^32 -
 2000 b1^2 t^3 z22 + 114 080 b1^2 t^4 z22 - 3 140 704 b1^2 t^5 z22 + 55 693 168 b1^2 t^6 z22 -
 716 174 528 b1^2 t^7 z22 + 7 128 224 880 b1^2 t^8 z22 - 57 230 308 608 b1^2 t^9 z22 +
 381 345 065 824 b1^2 t^10 z22 - 2 153 063 133 312 b1^2 t^11 z22 + 10 462 436 258 192 b1^2 t^12 z22 -
 44 289 123 710 592 b1^2 t^13 z22 + 164 883 282 332 800 b1^2 t^14 z22 - 543 940 716 781 312 b1^2 t^15 z22 +
 1 599 741 382 089 472 b1^2 t^16 z22 - 4 214 930 967 298 816 b1^2 t^17 z22 +
 9 988 364 415 202 304 b1^2 t^18 z22 - 21 358 513 645 667 328 b1^2 t^19 z22 +
 41 322 786 090 166 528 b1^2 t^20 z22 - 72 499 981 573 314 560 b1^2 t^21 z22 +
 115 577 092 035 942 400 b1^2 t^22 z22 - 167 708 452 809 641 984 b1^2 t^23 z22 +
 221 863 732 471 470 080 b1^2 t^24 z22 - 267 982 717 310 480 384 b1^2 t^25 z22 +
 295 920 202 140 712 960 b1^2 t^26 z22 - 299 016 953 488 588 800 b1^2 t^27 z22 +
 276 568 034 669 826 048 b1^2 t^28 z22 - 233 982 036 714 946 560 b1^2 t^29 z22 +
 180 675 922 875 154 432 b1^2 t^30 z22 - 126 834 782 779 867 136 b1^2 t^31 z22 +
 80 468 245 874 540 544 b1^2 t^32 z22 - 45 777 581 955 350 528 b1^2 t^33 z22 +
 23 130 200 063 606 784 b1^2 t^34 z22 - 10 266 657 750 515 712 b1^2 t^35 z22 +
 3 953 827 825 844 224 b1^2 t^36 z22 - 1 302 387 822 166 016 b1^2 t^37 z22 +
 360 452 622 123 008 b1^2 t^38 z22 - 81 704 404 385 792 b1^2 t^39 z22 + 14 531 669 524 480 b1^2 t^40 z22 -
 1 864 015 806 464 b1^2 t^41 z22 + 139 318 001 664 b1^2 t^42 z22 - 1 073 741 824 b1^2 t^43 z22 -
 536 870 912 b1^2 t^44 z22 - 160 000 t^5 z22^2 + 8 992 000 t^6 z22^2 - 244 966 400 t^7 z22^2 +
 4 322 894 080 t^8 z22^2 - 55 711 506 176 t^9 z22^2 + 560 418 149 632 t^10 z22^2 -
 4 591 712 189 696 t^11 z22^2 + 31 565 527 150 080 t^12 z22^2 - 186 073 939 988 992 t^13 z22^2 +
 956 269 873 741 568 t^14 z22^2 - 4 339 944 067 916 288 t^15 z22^2 + 17 571 114 754 118 144 t^16 z22^2 -
 63 977 084 333 880 064 t^17 z22^2 + 210 838 201 898 284 800 t^18 z22^2 -
 632 116 988 514 820 352 t^19 z22^2 + 1 731 142 278 192 420 352 t^20 z22^2 -
 4 344 508 518 464 774 912 t^21 z22^2 + 10 015 935 152 162 224 128 t^22 z22^2 -
 21 251 566 164 902 695 936 t^23 z22^2 + 41 554 764 515 736 895 744 t^24 z22^2 -
 74 950 324 759 192 956 928 t^25 z22^2 + 124 761 716 261 376 186 368 t^26 z22^2 -
 191 709 020 206 542 000 128 t^27 z22^2 + 271 922 293 797 697 308 672 t^28 z22^2 -
 355 948 309 725 438 832 640 t^29 z22^2 + 429 832 584 484 632 838 144 t^30 z22^2 -
 478 590 330 134 787 047 424 t^31 z22^2 + 491 058 916 725 507 379 200 t^32 z22^2 -

$$\begin{aligned}
& 464\,035\,937\,294\,300\,938\,240\ t^{33} z^{22^2} + 403\,617\,626\,236\,057\,583\,616\ t^{34} z^{22^2} - \\
& 322\,973\,317\,689\,942\,081\,536\ t^{35} z^{22^2} + 237\,659\,627\,199\,746\,408\,448\ t^{36} z^{22^2} - \\
& 160\,764\,086\,128\,242\,589\,696\ t^{37} z^{22^2} + 99\,941\,176\,290\,443\,264\,000\ t^{38} z^{22^2} - \\
& 57\,080\,416\,769\,370\,226\,688\ t^{39} z^{22^2} + 29\,937\,130\,001\,337\,810\,944\ t^{40} z^{22^2} - \\
& 14\,406\,243\,322\,205\,044\,736\ t^{41} z^{22^2} + 6\,351\,748\,818\,822\,234\,112\ t^{42} z^{22^2} - \\
& 2\,560\,181\,243\,354\,284\,032\ t^{43} z^{22^2} + 940\,299\,529\,940\,893\,696\ t^{44} z^{22^2} - \\
& 313\,256\,155\,090\,518\,016\ t^{45} z^{22^2} + 94\,072\,282\,307\,100\,672\ t^{46} z^{22^2} - \\
& 25\,250\,156\,337\,168\,384\ t^{47} z^{22^2} + 5\,988\,304\,444\,981\,248\ t^{48} z^{22^2} - 1\,235\,608\,731\,123\,712\ t^{49} z^{22^2} + \\
& 217\,368\,529\,731\,584\ t^{50} z^{22^2} - 31\,758\,414\,249\,984\ t^{51} z^{22^2} + 3\,724\,005\,081\,088\ t^{52} z^{22^2} - \\
& 334\,520\,909\,824\ t^{53} z^{22^2} + 21\,474\,836\,480\ t^{54} z^{22^2} - 872\,415\,232\ t^{55} z^{22^2} + 16\,777\,216\ t^{56} z^{22^2}
\end{aligned}$$

In[]:= **rh1c = Factor[Resultant[rh1a, rh1b, z22]]][[5]]**

In[]:= **Solve[rh1c == 0 /. t -> 4/5]**

Out[]:= $\left\{ \left\{ b1 \rightarrow \sqrt{-13.9 \dots} \right\}, \left\{ b1 \rightarrow \sqrt{-3.50 \dots} \right\}, \left\{ b1 \rightarrow \sqrt{-0.730 \dots} \right\}, \left\{ b1 \rightarrow \sqrt{1.81 \dots} \right\} \right\}$

In[]:= **rh2b = Factor[Resultant[(b3 q)^2 - (z22^3 (p31 + p32 ωω)), t ωω^2 - (4 - 11 t + 8 t^2), ωω]]][[3]]**

Out[]:= $\begin{aligned}
& b^3 t^4 - 64 b^3 t^4 + 1984 b^3 t^4 t^2 - 39\,680 b^3 t^4 t^3 + 575\,360 b^3 t^4 t^4 - 6\,444\,032 b^3 t^4 t^5 + \\
& 57\,996\,288 b^3 t^4 t^6 - 430\,829\,568 b^3 t^4 t^7 + 2\,692\,684\,800 b^3 t^4 t^8 - 14\,360\,985\,600 b^3 t^4 t^9 + \\
& 66\,060\,533\,760 b^3 t^4 t^{10} - 264\,242\,135\,040 b^3 t^4 t^{11} + 924\,847\,472\,640 b^3 t^4 t^{12} - \\
& 2\,845\,684\,531\,200 b^3 t^4 t^{13} + 7\,724\,000\,870\,400 b^3 t^4 t^{14} - 18\,537\,602\,088\,960 b^3 t^4 t^{15} + \\
& 39\,392\,404\,439\,040 b^3 t^4 t^{16} - 74\,150\,408\,355\,840 b^3 t^4 t^{17} + 123\,584\,013\,926\,400 b^3 t^4 t^{18} - \\
& 182\,123\,809\,996\,800 b^3 t^4 t^{19} + 236\,760\,952\,995\,840 b^3 t^4 t^{20} - 270\,583\,946\,280\,960 b^3 t^4 t^{21} + \\
& 270\,583\,946\,280\,960 b^3 t^4 t^{22} - 235\,290\,388\,070\,400 b^3 t^4 t^{23} + 176\,467\,791\,052\,800 b^3 t^4 t^{24} - \\
& 112\,939\,386\,273\,792 b^3 t^4 t^{25} + 60\,813\,515\,685\,888 b^3 t^4 t^{26} - 27\,028\,229\,193\,728 b^3 t^4 t^{27} + \\
& 9\,652\,938\,997\,760 b^3 t^4 t^{28} - 2\,662\,879\,723\,520 b^3 t^4 t^{29} + 532\,575\,944\,704 b^3 t^4 t^{30} - \\
& 68\,719\,476\,736 b^3 t^4 t^{31} + 4\,294\,967\,296 b^3 t^4 t^{32} - 2000 b^3 t^2 t^3 z^{22^3} + 114\,080 b^3 t^2 t^4 z^{22^3} - \\
& 3\,127\,904 b^3 t^2 t^5 z^{22^3} + 55\,114\,608 b^3 t^2 t^6 z^{22^3} - 703\,916\,992 b^3 t^2 t^7 z^{22^3} + \\
& 6\,967\,487\,600 b^3 t^2 t^8 z^{22^3} - 55\,789\,076\,736 b^3 t^2 t^9 z^{22^3} + 372\,281\,911\,008 b^3 t^2 t^{10} z^{22^3} - \\
& 2\,115\,751\,488\,128 b^3 t^2 t^{11} z^{22^3} + 10\,408\,467\,543\,824 b^3 t^2 t^{12} z^{22^3} - 44\,877\,448\,613\,504 b^3 t^2 t^{13} z^{22^3} + \\
& 171\,227\,639\,975\,296 b^3 t^2 t^{14} z^{22^3} - 582\,562\,709\,408\,256 b^3 t^2 t^{15} z^{22^3} + \\
& 1\,778\,438\,627\,627\,776 b^3 t^2 t^{16} z^{22^3} - 4\,897\,036\,990\,405\,120 b^3 t^2 t^{17} z^{22^3} + \\
& 12\,217\,504\,768\,714\,752 b^3 t^2 t^{18} z^{22^3} - 27\,726\,442\,260\,625\,408 b^3 t^2 t^{19} z^{22^3} + \\
& 57\,431\,989\,500\,979\,200 b^3 t^2 t^{20} z^{22^3} - 108\,899\,443\,675\,897\,856 b^3 t^2 t^{21} z^{22^3} + \\
& 189\,469\,682\,894\,450\,688 b^3 t^2 t^{22} z^{22^3} - 303\,022\,079\,346\,184\,192 b^3 t^2 t^{23} z^{22^3} + \\
& 445\,996\,157\,062\,803\,456 b^3 t^2 t^{24} z^{22^3} - 604\,365\,096\,264\,478\,720 b^3 t^2 t^{25} z^{22^3} + \\
& 753\,706\,618\,141\,523\,968 b^3 t^2 t^{26} z^{22^3} - 863\,833\,527\,831\,281\,664 b^3 t^2 t^{27} z^{22^3} + \\
& 907\,551\,304\,065\,355\,776 b^3 t^2 t^{28} z^{22^3} - 870\,690\,628\,526\,112\,768 b^3 t^2 t^{29} z^{22^3} + \\
& 758\,897\,057\,824\,800\,768 b^3 t^2 t^{30} z^{22^3} - 597\,126\,645\,392\,998\,400 b^3 t^2 t^{31} z^{22^3} + \\
& 420\,988\,614\,128\,500\,736 b^3 t^2 t^{32} z^{22^3} - 263\,708\,256\,761\,806\,848 b^3 t^2 t^{33} z^{22^3} + \\
& 145\,397\,161\,861\,840\,896 b^3 t^2 t^{34} z^{22^3} - 69\,832\,265\,638\,608\,896 b^3 t^2 t^{35} z^{22^3} + \\
& 28\,874\,008\,704\,319\,488 b^3 t^2 t^{36} z^{22^3} - 10\,133\,962\,106\,077\,184 b^3 t^2 t^{37} z^{22^3} + \\
& 2\,963\,819\,601\,068\,032 b^3 t^2 t^{38} z^{22^3} - 703\,016\,252\,473\,344 b^3 t^2 t^{39} z^{22^3} +
\end{aligned}$

$$\begin{aligned}
& 129306483228672 b^3 t^{40} z^{22^3} - 16924318629888 b^3 t^{41} z^{22^3} + 1273189367808 b^3 t^{42} z^{22^3} - \\
& 9663676416 b^3 t^{43} z^{22^3} - 4831838208 b^3 t^{44} z^{22^3} - 160000 t^5 z^{22^6} + 8992000 t^6 z^{22^6} - \\
& 244966400 t^7 z^{22^6} + 4335694080 t^8 z^{22^6} - 56368658176 t^9 z^{22^6} + 576806040832 t^{10} z^{22^6} - \\
& 4857175608576 t^{11} z^{22^6} + 34716405255680 t^{12} z^{22^6} - 215372661502464 t^{13} z^{22^6} + \\
& 1179017532690176 t^{14} z^{22^6} - 5766121998211584 t^{15} z^{22^6} + 25428376181770752 t^{16} z^{22^6} - \\
& 101830223989285632 t^{17} z^{22^6} + 372273420534707968 t^{18} z^{22^6} - \\
& 1247379207151880448 t^{19} z^{22^6} + 3841938979935204864 t^{20} z^{22^6} - \\
& 10899643508122602240 t^{21} z^{22^6} + 28521907352016113664 t^{22} z^{22^6} - \\
& 68897319134135587840 t^{23} z^{22^6} + 153689991766468780288 t^{24} z^{22^6} - \\
& 316609366679068233728 t^{25} z^{22^6} + 602204932203871948800 t^{26} z^{22^6} - \\
& 1057157145543260749824 t^{27} z^{22^6} + 1711959073435929538560 t^{28} z^{22^6} - \\
& 2556007780135777116160 t^{29} z^{22^6} + 3516337481530179690496 t^{30} z^{22^6} - \\
& 4454747533507861676032 t^{31} z^{22^6} + 5194135862110092451840 t^{32} z^{22^6} - \\
& 5570969659876496310272 t^{33} z^{22^6} + 5493726356725024227328 t^{34} z^{22^6} - \\
& 4978958829257214459904 t^{35} z^{22^6} + 4145576935922258411520 t^{36} z^{22^6} - \\
& 3169987403346295717888 t^{37} z^{22^6} + 2225318868748778733568 t^{38} z^{22^6} - \\
& 1433387356929036386304 t^{39} z^{22^6} + 846480383958296756224 t^{40} z^{22^6} - \\
& 457703979652247191552 t^{41} z^{22^6} + 226149162144972668928 t^{42} z^{22^6} - \\
& 101807224693941338112 t^{43} z^{22^6} + 41588013269773189120 t^{44} z^{22^6} - \\
& 15330829548474335232 t^{45} z^{22^6} + 5062337849116852224 t^{46} z^{22^6} - \\
& 1482615644233924608 t^{47} z^{22^6} + 380138389197815808 t^{48} z^{22^6} - \\
& 83908104038645760 t^{49} z^{22^6} + 15610960328785920 t^{50} z^{22^6} - \\
& 2384123947646976 t^{51} z^{22^6} + 288962648211456 t^{52} z^{22^6} - 26556235776000 t^{53} z^{22^6} + \\
& 1728590118912 t^{54} z^{22^6} - 70665633792 t^{55} z^{22^6} + 1358954496 t^{56} z^{22^6}
\end{aligned}$$

In[]:= **rh2c = Factor[Resultant[rh1a, rh2b, z22]][[6]]**

Out[]:= $-625 + 38875 t - 1197650 t^2 + 24475580 t^3 - 1000 b^3 t^3 - 374568351 t^4 + 64350 b^3 t^4 +$
 $4590299513 t^5 - 2023440 b^3 t^5 + 200 b^3 t^5 - 47002640856 t^6 + 41334950 b^3 t^6 - 8820 b^3 t^6 +$
 $414042407206 t^7 - 615401084 b^3 t^7 + 151748 b^3 t^7 - 3204532355987 t^8 + 7099875890 b^3 t^8 -$
 $667160 b^3 t^8 + 160 b^3 t^8 + 22138137463085 t^9 - 65828941890 b^3 t^9 - 23344496 b^3 t^9 -$
 $9240 b^3 t^9 - 138182087993230 t^{10} + 501033071938 b^3 t^{10} + 612175196 b^3 t^{10} +$
 $253864 b^3 t^{10} - 16 b^3 t^{10} + 786758870284500 t^{11} - 3156685298440 b^3 t^{11} -$
 $8434477308 b^3 t^{11} - 4365768 b^3 t^{11} + 1072 b^3 t^{11} - 4117364545670237 t^{12} +$
 $16327250578252 b^3 t^{12} + 81851665904 b^3 t^{12} + 51589768 b^3 t^{12} - 34864 b^3 t^{12} +$
 $19927829853144971 t^{13} - 66375427932198 b^3 t^{13} - 605995222296 b^3 t^{13} -$
 $423241920 b^3 t^{13} + 733200 b^3 t^{13} - 89649763746015368 t^{14} + 177780113122312 b^3 t^{14} +$
 $3500110806968 b^3 t^{14} + 2114970352 b^3 t^{14} - 11206656 b^3 t^{14} + 376431825893833074 t^{15} +$
 $60430453394096 b^3 t^{15} - 15385247590416 b^3 t^{15} + 63687824 b^3 t^{15} + 132658176 b^3 t^{15} -$
 $1480342260943951152 t^{16} - 4636262068084792 b^3 t^{16} + 44338711864256 b^3 t^{16} -$
 $127018955424 b^3 t^{16} - 1265506304 b^3 t^{16} + 546784222738672596 t^{17} +$
 $37771523835896688 b^3 t^{17} - 1337129171824 b^3 t^{17} + 1562754279392 b^3 t^{17} +$
 $9995614208 b^3 t^{17} - 19014299392013918192 t^{18} - 216962853813848752 b^3 t^{18} -$
 $1013084389884896 b^3 t^{18} - 12623204602496 b^3 t^{18} - 66649702400 b^3 t^{18} +$

$62\,375\,641\,942\,695\,874\,792\ t^{19} + 1\,029\,806\,241\,630\,324\,320\ b_3 t^{19} + 8\,332\,570\,458\,774\,880\ b_3^2 t^{19} +$
 $79\,965\,339\,647\,744\ b_3^3 t^{19} + 380\,632\,399\,872\ b_3^4 t^{19} - 193\,345\,535\,581\,151\,095\,648\ t^{20} -$
 $4\,259\,593\,245\,611\,296\,576\ b_3 t^{20} - 46\,544\,746\,339\,031\,040\ b_3^2 t^{20} - 421\,825\,602\,427\,520\ b_3^3 t^{20} -$
 $1\,882\,437\,992\,448\ b_3^4 t^{20} + 567\,057\,789\,429\,670\,848\,720\ t^{21} + 15\,744\,168\,128\,496\,985\,536\ b_3 t^{21} +$
 $210\,592\,265\,001\,187\,648\ b_3^2 t^{21} + 1\,907\,745\,944\,023\,680\ b_3^3 t^{21} + 8\,131\,190\,046\,720\ b_3^4 t^{21} -$
 $1\,575\,349\,016\,369\,992\,505\,600\ t^{22} - 52\,740\,976\,337\,888\,449\,664\ b_3 t^{22} -$
 $817\,177\,104\,228\,272\,256\ b_3^2 t^{22} - 7\,523\,896\,012\,147\,456\ b_3^3 t^{22} - 30\,881\,863\,434\,240\ b_3^4 t^{22} +$
 $4\,149\,264\,709\,453\,371\,034\,368\ t^{23} + 161\,548\,136\,552\,775\,907\,584\ b_3 t^{23} +$
 $2\,792\,486\,515\,600\,393\,216\ b_3^2 t^{23} + 26\,162\,077\,082\,988\,800\ b_3^3 t^{23} + 103\,664\,222\,208\,000\ b_3^4 t^{23} -$
 $10\,368\,513\,519\,444\,537\,519\,424\ t^{24} - 455\,163\,165\,558\,923\,250\,816\ b_3 t^{24} -$
 $8\,527\,245\,762\,585\,845\,248\ b_3^2 t^{24} - 80\,814\,768\,969\,734\,656\ b_3^3 t^{24} - 308\,797\,424\,271\,360\ b_3^4 t^{24} +$
 $24\,594\,929\,954\,242\,026\,782\,016\ t^{25} + 1\,184\,575\,594\,220\,643\,016\,960\ b_3 t^{25} +$
 $23\,476\,365\,041\,846\,565\,632\ b_3^2 t^{25} + 222\,962\,555\,202\,286\,080\ b_3^3 t^{25} + 818\,744\,092\,262\,400\ b_3^4 t^{25} -$
 $55\,401\,978\,393\,429\,928\,009\,728\ t^{26} - 2\,856\,322\,480\,618\,965\,228\,544\ b_3 t^{26} -$
 $58\,607\,437\,732\,457\,815\,040\ b_3^2 t^{26} - 551\,538\,083\,958\,179\,840\ b_3^3 t^{26} -$
 $1\,936\,366\,365\,573\,120\ b_3^4 t^{26} + 118\,539\,401\,452\,342\,742\,183\,936\ t^{27} +$
 $6\,395\,469\,591\,454\,915\,920\,896\ b_3 t^{27} + 133\,182\,114\,045\,211\,138\,048\ b_3^2 t^{27} +$
 $1\,226\,649\,510\,995\,302\,400\ b_3^3 t^{27} + 4\,090\,630\,860\,963\,840\ b_3^4 t^{27} -$
 $240\,940\,966\,389\,578\,811\,759\,104\ t^{28} - 13\,319\,238\,793\,880\,219\,378\,176\ b_3 t^{28} -$
 $276\,218\,449\,429\,440\,904\,192\ b_3^2 t^{28} - 2\,457\,500\,382\,873\,927\,680\ b_3^3 t^{28} -$
 $7\,724\,000\,870\,400\,000\ b_3^4 t^{28} + 465\,234\,164\,663\,806\,382\,711\,808\ t^{29} +$
 $25\,832\,108\,951\,935\,683\,811\,328\ b_3 t^{29} + 523\,793\,251\,794\,188\,532\,736\ b_3^2 t^{29} +$
 $4\,440\,378\,794\,872\,463\,360\ b_3^3 t^{29} + 13\,035\,511\,700\,520\,960\ b_3^4 t^{29} -$
 $853\,300\,089\,580\,466\,958\,185\,472\ t^{30} - 46\,697\,916\,513\,973\,116\,453\,888\ b_3 t^{30} -$
 $909\,300\,851\,409\,475\,915\,776\ b_3^2 t^{30} - 7\,240\,543\,324\,412\,805\,120\ b_3^3 t^{30} -$
 $19\,648\,557\,329\,940\,480\ b_3^4 t^{30} + 1\,486\,326\,879\,360\,491\,076\,760\,576\ t^{31} +$
 $78\,732\,922\,519\,611\,906\,834\,432\ b_3 t^{31} + 1\,446\,307\,566\,593\,913\,464\,832\ b_3^2 t^{31} +$
 $10\,656\,071\,682\,456\,125\,440\ b_3^3 t^{31} + 26\,413\,155\,986\,964\,480\ b_3^4 t^{31} -$
 $2\,458\,001\,561\,667\,532\,401\,808\,384\ t^{32} - 123\,851\,369\,452\,295\,708\,459\,008\ b_3 t^{32} -$
 $2\,108\,912\,325\,202\,302\,095\,360\ b_3^2 t^{32} - 14\,149\,749\,635\,272\,540\,160\ b_3^3 t^{32} -$
 $31\,595\,879\,265\,730\,560\ b_3^4 t^{32} + 3\,857\,808\,700\,971\,003\,301\,682\,176\ t^{33} +$
 $181\,806\,625\,955\,719\,186\,472\,960\ b_3 t^{33} + 2\,820\,019\,180\,263\,983\,607\,808\ b_3^2 t^{33} +$
 $16\,939\,135\,492\,147\,118\,080\ b_3^3 t^{33} + 33\,528\,880\,300\,032\,000\ b_3^4 t^{33} -$
 $5\,743\,681\,846\,227\,027\,346\,812\,928\ t^{34} - 249\,050\,112\,469\,381\,335\,908\,352\ b_3 t^{34} -$
 $3\,458\,861\,435\,545\,635\,987\,456\ b_3^2 t^{34} - 18\,260\,414\,633\,339\,781\,120\ b_3^3 t^{34} -$
 $31\,434\,795\,846\,205\,440\ b_3^4 t^{34} + 8\,107\,726\,810\,514\,788\,977\,788\,928\ t^{35} +$
 $318\,322\,872\,929\,644\,003\,344\,384\ b_3 t^{35} + 3\,891\,890\,218\,130\,076\,033\,024\ b_3^2 t^{35} +$
 $17\,697\,514\,235\,894\,956\,032\ b_3^3 t^{35} + 25\,900\,765\,918\,789\,632\ b_3^4 t^{35} -$
 $10\,844\,377\,675\,480\,701\,438\,849\,024\ t^{36} - 379\,512\,862\,793\,521\,040\,998\,400\ b_3 t^{36} -$
 $4\,017\,756\,195\,715\,755\,671\,552\ b_3^2 t^{36} - 15\,388\,947\,033\,331\,073\,024\ b_3^3 t^{36} -$
 $18\,629\,206\,971\,777\,024\ b_3^4 t^{36} + 13\,734\,631\,351\,484\,590\,373\,707\,776\ t^{37} +$
 $421\,868\,756\,329\,135\,507\,095\,552\ b_3 t^{37} + 3\,805\,949\,128\,198\,979\,567\,616\ b_3^2 t^{37} +$
 $11\,975\,500\,020\,175\,601\,664\ b_3^3 t^{37} + 11\,596\,075\,618\,009\,088\ b_3^4 t^{37} -$

$$\begin{aligned}
& 16\,459\,667\,304\,835\,558\,363\,136\,000\ t^{38} - 436\,996\,922\,728\,242\,226\,872\,320\ b_3 t^{38} - \\
& 3\,308\,921\,265\,410\,953\,347\,072\ b_3^2 t^{38} - 8\,313\,607\,525\,341\,593\,600\ b_3^3 t^{38} - \\
& 6\,177\,880\,958\,566\,400\ b_3^4 t^{38} + 18\,650\,002\,932\,810\,444\,456\,968\,192\ t^{39} + \\
& 421\,532\,781\,313\,910\,501\,769\,216\ b_3 t^{39} + 2\,641\,018\,805\,519\,335\,456\,768\ b_3^2 t^{39} + \\
& 5\,128\,172\,352\,135\,233\,536\ b_3^3 t^{39} + 2\,776\,318\,399\,741\,952\ b_3^4 t^{39} - \\
& 19\,963\,309\,385\,306\,269\,436\,821\,504\ t^{40} - 378\,338\,681\,001\,788\,933\,111\,808\ b_3 t^{40} - \\
& 1\,935\,738\,439\,153\,465\,491\,456\ b_3^2 t^{40} - 2\,796\,278\,504\,570\,748\,928\ b_3^3 t^{40} - \\
& 1\,032\,132\,180\,836\,352\ b_3^4 t^{40} + 20\,170\,014\,975\,759\,793\,080\,877\,056\ t^{41} + \\
& 315\,653\,193\,150\,224\,523\,264\,000\ b_3 t^{41} + 1\,303\,164\,173\,767\,899\,676\,672\ b_3^2 t^{41} + \\
& 1\,338\,717\,427\,118\,112\,768\ b_3^3 t^{41} + 308\,928\,407\,666\,688\ b_3^4 t^{41} - \\
& 19\,218\,023\,426\,317\,838\,237\,696\,000\ t^{42} - 244\,532\,168\,944\,074\,834\,116\,608\ b_3 t^{42} - \\
& 805\,713\,186\,218\,863\,755\,264\ b_3^2 t^{42} - 557\,535\,295\,264\,784\,384\ b_3^3 t^{42} - 71\,536\,975\,282\,176\ b_3^4 t^{42} + \\
& 17\,251\,821\,769\,513\,791\,184\,699\,392\ t^{43} + 175\,669\,602\,850\,620\,785\,426\,432\ b_3 t^{43} + \\
& 457\,164\,005\,031\,735\,721\,984\ b_3^2 t^{43} + 199\,407\,568\,265\,674\,752\ b_3^3 t^{43} + \\
& 12\,025\,908\,428\,800\ b_3^4 t^{43} - 14\,576\,953\,953\,258\,988\,486\,262\,784\ t^{44} - \\
& 116\,852\,094\,529\,266\,460\,917\,760\ b_3 t^{44} - 237\,643\,850\,518\,361\,014\,272\ b_3^2 t^{44} - \\
& 60\,138\,665\,728\,278\,528\ b_3^3 t^{44} - 1\,305\,670\,057\,984\ b_3^4 t^{44} + 11\,581\,694\,366\,726\,965\,359\,149\,056\ t^{45} + \\
& 71\,842\,699\,421\,503\,399\,788\,544\ b_3 t^{45} + 112\,827\,939\,710\,429\,036\,544\ b_3^2 t^{45} + \\
& 14\,891\,958\,895\,902\,720\ b_3^3 t^{45} + 68\,719\,476\,736\ b_3^4 t^{45} - 8\,643\,743\,014\,706\,418\,216\,599\,552\ t^{46} - \\
& 40\,739\,092\,420\,856\,022\,564\,864\ b_3 t^{46} - 48\,698\,648\,827\,514\,585\,088\ b_3^2 t^{46} - \\
& 2\,908\,669\,964\,451\,840\ b_3^3 t^{46} + 6\,053\,181\,065\,687\,364\,580\,671\,488\ t^{47} + \\
& 21\,252\,124\,015\,033\,732\,038\,656\ b_3 t^{47} + 18\,985\,364\,688\,604\,233\,728\ b_3^2 t^{47} + \\
& 420\,002\,704\,392\,192\ b_3^3 t^{47} - 3\,972\,934\,825\,329\,100\,755\,566\,592\ t^{48} - \\
& 10\,166\,620\,991\,748\,764\,598\,272\ b_3 t^{48} - 6\,628\,644\,904\,803\,237\,888\ b_3^2 t^{48} - \\
& 39\,814\,346\,833\,920\ b_3^3 t^{48} + 2\,440\,786\,155\,466\,067\,312\,640\,000\ t^{49} + \\
& 4\,442\,362\,632\,307\,854\,016\,512\ b_3 t^{49} + 2\,049\,770\,201\,926\,336\,512\ b_3^2 t^{49} + \\
& 1\,855\,425\,871\,872\ b_3^3 t^{49} - 1\,401\,562\,307\,461\,280\,584\,171\,520\ t^{50} - \\
& 1\,764\,188\,848\,098\,071\,543\,808\ b_3 t^{50} - 553\,038\,503\,965\,884\,416\ b_3^2 t^{50} + \\
& 750\,991\,743\,117\,609\,447\,981\,056\ t^{51} + 632\,720\,312\,614\,483\,132\,416\ b_3 t^{51} + \\
& 127\,442\,471\,046\,086\,656\ b_3^2 t^{51} - 374\,750\,111\,566\,030\,596\,734\,976\ t^{52} - \\
& 203\,276\,852\,939\,301\,322\,752\ b_3 t^{52} - 24\,284\,547\,445\,161\,984\ b_3^2 t^{52} + \\
& 173\,741\,696\,179\,554\,287\,091\,712\ t^{53} + 57\,895\,669\,533\,763\,436\,544\ b_3 t^{53} + \\
& 3\,629\,249\,244\,168\,192\ b_3^2 t^{53} - 74\,623\,355\,271\,683\,325\,820\,928\ t^{54} - \\
& 14\,422\,661\,953\,241\,481\,216\ b_3 t^{54} - 385\,516\,264\,488\,960\ b_3^2 t^{54} + \\
& 29\,589\,083\,573\,551\,500\,361\,728\ t^{55} + 3\,088\,088\,475\,522\,564\,096\ b_3 t^{55} + 22\,690\,312\,224\,768\ b_3^2 t^{55} - \\
& 10\,784\,707\,641\,587\,382\,878\,208\ t^{56} - 555\,327\,075\,410\,509\,824\ b_3 t^{56} + 77\,309\,411\,328\ b_3^2 t^{56} + \\
& 3\,594\,274\,336\,279\,034\,331\,136\ t^{57} + 81\,284\,952\,678\,727\,680\ b_3 t^{57} - 77\,309\,411\,328\ b_3^2 t^{57} - \\
& 1\,088\,210\,423\,945\,084\,534\,784\ t^{58} - 9\,261\,948\,931\,670\,016\ b_3 t^{58} + 296\,910\,617\,803\,594\,137\,600\ t^{59} + \\
& 766\,917\,816\,090\,624\ b_3 t^{59} - 72\,285\,108\,756\,623\,130\,624\ t^{60} - 40\,833\,864\,695\,808\ b_3 t^{60} + \\
& 15\,512\,481\,752\,911\,183\,872\ t^{61} + 1\,043\,677\,052\,928\ b_3 t^{61} - 2\,890\,527\,043\,139\,665\,920\ t^{62} + \\
& 458\,969\,907\,682\,344\,960\ t^{63} - 60\,638\,588\,043\,264\,000\ t^{64} + 6\,460\,045\,680\,181\,248\ t^{65} - \\
& 531\,115\,655\,823\,360\ t^{66} + 31\,506\,001\,035\,264\ t^{67} - 1\,195\,879\,956\,480\ t^{68} + 21\,743\,271\,936\ t^{69}
\end{aligned}$$

```
In[ ]:= Solve[rh2c == 0 /. t -> 4/5]
```

```
Out[ ]:= {{b3 -> -671. ...}, {b3 -> -1.23 ...}, {b3 -> -0.0972 ...}, {b3 -> 8.85 ...}}
```

```
In[ ]:= RootReduce[Sqrt[z2s ^ 3 (p31 + p32 ωω)]/q /. ωω -> ω /. t -> 4/5]
```

```
Out[ ]:= 8.85 ...
```

```
In[ ]:= RootReduce [
```

```
1/q {0, 0, -Sqrt[p31 + p32 ωω], Sqrt[p11 + p12 ωω]}.Table[(-Sqrt[Z])^j, {j, 7, 1, -2}] /.  
Z -> z2s /. ωω -> ω /. {t -> 4/5}]
```

```
Out[ ]:= {5.35 ...}
```

```
In[ ]:= rh2d = Factor[Resultant[b1 + b3 - b13, rh1c, b1]];
```

```
In[ ]:= rh2e = Factor[Resultant[rh2d, rh2c, b3]][[2]]
```

```
In[ ]:= Solve[rh2e == 0 /. t -> 4/5]
```

```
Out[ ]:= {{b13 -> -685. ...}, {b13 -> -0.828 ...}, {b13 -> 0.581 ...}, {b13 -> 5.35 ...}}
```

```
In[ ]:= rh3b = Factor[Resultant[(b5 q)^2 - (z22 ^ 5 (p51 + p52 ωω)), t ωω ^ 2 - (4 - 11 t + 8 t^2), ωω]][[3]];
```

```
In[ ]:= rh3c = Factor[Resultant[rh1a, rh3b, z22]][[5]]
```

```
In[ ]:= Solve[rh3c == 0 /. t -> 4/5]
```

```
Out[ ]:= {{b5 -> -7.38 ... x 10^3}, {b5 -> -5.70 ...}, {b5 -> -2.95 ... x 10^-3}, {b5 -> 0.212 ...}}
```

```
In[ ]:= RootReduce[-Sqrt[z2s ^ 5 (p51 + p52 ωω)]/q /. ωω -> ω /. t -> 4/5]
```

```
Out[ ]:= -5.70 ...
```

```
In[ ]:= rh4b = Factor[Resultant[(b7 q)^2 - (z22 ^ 7 (p71 + p72 ωω)), t ωω ^ 2 - (4 - 11 t + 8 t^2), ωω]][[3]];
```

```
In[ ]:= rh4c = Factor[Resultant[rh1a, rh4b, z22]][[6]]
```

```
Out[ ]:= 1 - 58 t + 1702 t^2 - 33 684 t^3 - 8 b7 t^3 + 505 568 t^4 - 10 b7 t^4 - 6 134 800 t^5 + 10 936 b7 t^5 - 8 b7^2 t^5 +  
62 649 048 t^6 - 350 684 b7 t^6 - 452 b7^2 t^6 - 553 381 760 t^7 + 6 098 670 b7 t^7 + 28 088 b7^2 t^7 +  
4 312 553 702 t^8 - 71 006 700 b7 t^8 - 556 836 b7^2 t^8 + 32 b7^3 t^8 - 30 096 665 516 t^9 +  
585 361 892 b7 t^9 + 4 407 828 b7^2 t^9 - 1720 b7^3 t^9 + 190 282 870 428 t^10 - 3 265 937 362 b7 t^10 +  
23 442 424 b7^2 t^10 + 43 400 b7^3 t^10 + 16 b7^4 t^10 - 1 099 937 566 840 t^11 + 7 917 944 102 b7 t^11 -  
1 045 146 304 b7^2 t^11 - 668 016 b7^3 t^11 - 1120 b7^4 t^11 + 5 856 678 006 564 t^12 +  
68 772 238 824 b7 t^12 + 14 177 534 044 b7^2 t^12 + 6 626 504 b7^3 t^12 + 38 128 b7^4 t^12 -  
28 900 176 000 248 t^13 - 1 136 333 784 054 b7 t^13 - 120 414 735 968 b7^2 t^13 - 35 924 192 b7^3 t^13 -  
841 024 b7^4 t^13 + 132 836 890 014 192 t^14 + 9 742 263 986 648 b7 t^14 + 698 056 662 424 b7^2 t^14 -  
86 904 840 b7^3 t^14 + 13 511 920 b7^4 t^14 - 571 156 355 636 272 t^15 - 62 778 228 984 280 b7 t^15 -  
2 527 523 397 384 b7^2 t^15 + 4 500 479 096 b7^3 t^15 - 168 512 608 b7^4 t^15 + 2 305 542 905 620 129 t^16 +  
331 883 320 068 598 b7 t^16 + 1 586 185 084 352 b7^2 t^16 - 60 216 293 136 b7^3 t^16 +  
1 697 834 000 b7^4 t^16 - 8 764 027 694 315 082 t^17 - 1 489 081 940 845 132 b7 t^17 +
```


$52\,071\,602\,511\,416\,b^7t^{17} + 558\,160\,755\,040\,b^7t^{17} - 14\,201\,314\,304\,b^7t^{17} +$
 $31\,454\,553\,475\,425\,486\,t^{18} + 5\,745\,880\,167\,446\,160\,b^7t^{18} - 439\,558\,446\,003\,064\,b^7t^{18} -$
 $4\,119\,141\,486\,176\,b^7t^{18} + 100\,565\,722\,112\,b^7t^{18} - 106\,828\,427\,686\,954\,260\,t^{19} -$
 $19\,056\,001\,563\,478\,592\,b^7t^{19} + 2\,201\,025\,831\,560\,608\,b^7t^{19} + 25\,470\,377\,076\,800\,b^7t^{19} -$
 $611\,833\,856\,000\,b^7t^{19} + 343\,993\,028\,288\,108\,500\,t^{20} + 53\,341\,916\,633\,530\,480\,b^7t^{20} -$
 $7\,764\,115\,544\,889\,952\,b^7t^{20} - 135\,297\,211\,727\,424\,b^7t^{20} + 3\,234\,279\,913\,472\,b^7t^{20} -$
 $1\,051\,935\,046\,662\,742\,184\,t^{21} - 118\,898\,625\,939\,040\,448\,b^7t^{21} + 18\,370\,436\,554\,395\,936\,b^7t^{21} +$
 $626\,060\,916\,984\,576\,b^7t^{21} - 14\,987\,050\,926\,080\,b^7t^{21} + 3\,059\,272\,858\,693\,125\,448\,t^{22} +$
 $167\,745\,807\,706\,891\,136\,b^7t^{22} - 14\,937\,332\,869\,477\,440\,b^7t^{22} - 2\,544\,389\,494\,701\,376\,b^7t^{22} +$
 $61\,303\,379\,951\,616\,b^7t^{22} - 8\,471\,496\,081\,610\,598\,384\,t^{23} + 127\,025\,838\,240\,442\,880\,b^7t^{23} -$
 $110\,809\,866\,226\,757\,312\,b^7t^{23} + 9\,125\,329\,704\,995\,072\,b^7t^{23} - 222\,585\,820\,643\,328\,b^7t^{23} +$
 $22\,359\,212\,565\,076\,545\,312\,t^{24} - 2\,024\,412\,956\,901\,346\,528\,b^7t^{24} + 722\,922\,158\,938\,530\,560\,b^7t^{24} -$
 $28\,946\,004\,712\,899\,456\,b^7t^{24} + 720\,566\,871\,244\,800\,b^7t^{24} - 56\,295\,911\,695\,761\,999\,520\,t^{25} +$
 $9\,303\,321\,482\,689\,377\,408\,b^7t^{25} - 2\,706\,663\,750\,661\,943\,296\,b^7t^{25} +$
 $81\,211\,939\,795\,431\,936\,b^7t^{25} - 2\,087\,010\,895\,134\,720\,b^7t^{25} + 135\,308\,488\,420\,410\,373\,600\,t^{26} -$
 $31\,493\,061\,027\,355\,014\,976\,b^7t^{26} + 7\,634\,078\,013\,109\,384\,320\,b^7t^{26} -$
 $201\,034\,008\,085\,854\,976\,b^7t^{26} + 5\,422\,655\,137\,382\,400\,b^7t^{26} - 310\,631\,078\,149\,246\,207\,808\,t^{27} +$
 $89\,306\,117\,239\,623\,036\,160\,b^7t^{27} - 17\,268\,401\,306\,448\,626\,432\,b^7t^{27} +$
 $436\,427\,338\,506\,115\,072\,b^7t^{27} - 12\,664\,759\,658\,741\,760\,b^7t^{27} +$
 $681\,435\,575\,903\,734\,238\,464\,t^{28} - 221\,756\,049\,619\,691\,266\,176\,b^7t^{28} +$
 $31\,624\,840\,169\,387\,159\,296\,b^7t^{28} - 820\,762\,020\,244\,605\,440\,b^7t^{28} +$
 $26\,623\,736\,642\,273\,280\,b^7t^{28} - 1\,428\,898\,282\,750\,054\,142\,592\,t^{29} +$
 $491\,872\,022\,732\,272\,549\,888\,b^7t^{29} - 45\,434\,531\,503\,282\,988\,032\,b^7t^{29} +$
 $1\,303\,534\,041\,444\,020\,224\,b^7t^{29} - 50\,415\,773\,260\,185\,600\,b^7t^{29} +$
 $2\,864\,588\,186\,362\,268\,804\,224\,t^{30} - 984\,011\,848\,096\,767\,288\,576\,b^7t^{30} +$
 $44\,908\,787\,539\,947\,970\,048\,b^7t^{30} - 1\,643\,968\,897\,597\,085\,696\,b^7t^{30} +$
 $86\,017\,725\,903\,667\,200\,b^7t^{30} - 5\,490\,965\,898\,010\,076\,049\,664\,t^{31} +$
 $1\,782\,403\,630\,872\,326\,166\,016\,b^7t^{31} - 9\,333\,780\,152\,739\,737\,600\,b^7t^{31} +$
 $1\,323\,188\,691\,707\,846\,656\,b^7t^{31} - 132\,189\,363\,948\,748\,800\,b^7t^{31} +$
 $10\,063\,698\,445\,996\,949\,809\,664\,t^{32} - 2\,921\,739\,362\,611\,359\,999\,488\,b^7t^{32} -$
 $77\,781\,112\,600\,307\,707\,904\,b^7t^{32} + 414\,758\,544\,307\,036\,160\,b^7t^{32} +$
 $182\,816\,530\,916\,966\,400\,b^7t^{32} - 17\,633\,647\,375\,832\,273\,074\,688\,t^{33} +$
 $4\,311\,098\,465\,957\,327\,602\,688\,b^7t^{33} + 208\,887\,495\,242\,505\,754\,624\,b^7t^{33} -$
 $4\,291\,010\,805\,155\,364\,864\,b^7t^{33} - 227\,204\,543\,388\,057\,600\,b^7t^{33} +$
 $29\,533\,392\,182\,386\,004\,709\,888\,t^{34} - 5\,653\,455\,777\,209\,103\,752\,192\,b^7t^{34} -$
 $335\,242\,487\,629\,484\,298\,240\,b^7t^{34} + 10\,516\,518\,228\,498\,874\,368\,b^7t^{34} +$
 $253\,222\,230\,530\,457\,600\,b^7t^{34} - 47\,265\,069\,628\,267\,115\,971\,584\,t^{35} +$
 $6\,407\,284\,833\,683\,362\,514\,944\,b^7t^{35} + 370\,657\,381\,018\,723\,721\,216\,b^7t^{35} -$
 $18\,351\,613\,606\,415\,695\,872\,b^7t^{35} - 252\,387\,673\,623\,232\,512\,b^7t^{35} +$
 $72\,252\,115\,970\,798\,812\,224\,512\,t^{36} - 5\,847\,196\,496\,082\,500\,558\,848\,b^7t^{36} -$
 $226\,929\,963\,654\,690\,856\,960\,b^7t^{36} + 26\,060\,293\,767\,821\,000\,704\,b^7t^{36} +$
 $224\,164\,772\,566\,794\,240\,b^7t^{36} - 105\,445\,944\,186\,788\,395\,495\,424\,t^{37} +$
 $3\,257\,601\,261\,381\,263\,130\,624\,b^7t^{37} - 131\,227\,586\,312\,532\,918\,272\,b^7t^{37} -$

$$\begin{aligned}
& 31\,466\,869\,748\,324\,630\,528\ b^7 t^{37} - 176\,620\,790\,135\,914\,496\ b^7 t^{37} + \\
& 146\,832\,353\,358\,569\,652\,236\,288\ t^{38} + 1\,768\,935\,826\,557\,424\,496\,640\ b^7 t^{38} + \\
& 646\,406\,125\,932\,546\,613\,248\ b^7 t^{38} + 32\,917\,058\,293\,364\,424\,704\ b^7 t^{38} + \\
& 122\,754\,494\,646\,714\,368\ b^7 t^{38} - 194\,951\,586\,005\,312\,643\,842\,048\ t^{39} - \\
& 9\,036\,240\,431\,991\,553\,802\,240\ b^7 t^{39} - 1\,177\,630\,003\,780\,308\,975\,616\ b^7 t^{39} - \\
& 30\,085\,345\,052\,180\,086\,784\ b^7 t^{39} - 74\,727\,395\,101\,245\,440\ b^7 t^{39} + \\
& 246\,604\,764\,217\,589\,686\,616\,064\ t^{40} + 17\,606\,928\,160\,921\,574\,096\,896\ b^7 t^{40} + \\
& 1\,561\,550\,828\,018\,730\,614\,784\ b^7 t^{40} + 24\,101\,179\,181\,607\,419\,904\ b^7 t^{40} + \\
& 39\,490\,805\,873\,770\,496\ b^7 t^{40} - 296\,931\,988\,325\,592\,746\,549\,248\ t^{41} - \\
& 25\,944\,997\,600\,156\,385\,935\,360\ b^7 t^{41} - 1\,690\,307\,673\,804\,833\,161\,216\ b^7 t^{41} - \\
& 16\,920\,764\,776\,882\,110\,464\ b^7 t^{41} - 17\,912\,161\,107\,968\,000\ b^7 t^{41} + \\
& 339\,983\,176\,584\,235\,475\,918\,848\ t^{42} + 32\,340\,985\,335\,899\,651\,424\,256\ b^7 t^{42} + \\
& 1\,557\,176\,686\,427\,904\,311\,296\ b^7 t^{42} + 10\,384\,087\,851\,582\,095\,360\ b^7 t^{42} + \\
& 6\,871\,037\,140\,533\,248\ b^7 t^{42} - 369\,758\,155\,472\,920\,019\,976\,192\ t^{43} - \\
& 35\,469\,403\,502\,892\,167\,200\,768\ b^7 t^{43} - 1\,245\,048\,110\,077\,294\,673\,920\ b^7 t^{43} - \\
& 5\,543\,455\,391\,993\,561\,088\ b^7 t^{43} - 2\,185\,554\,238\,111\,744\ b^7 t^{43} + \\
& 381\,506\,068\,880\,004\,013\,096\,960\ t^{44} + 34\,835\,803\,206\,223\,656\,550\,400\ b^7 t^{44} + \\
& 873\,079\,255\,522\,965\,520\,384\ b^7 t^{44} + 2\,555\,865\,799\,990\,444\,032\ b^7 t^{44} + \\
& 560\,922\,728\,857\,600\ b^7 t^{44} - 372\,921\,666\,065\,312\,030\,851\,072\ t^{45} - \\
& 30\,907\,725\,087\,632\,165\,175\,296\ b^7 t^{45} - 540\,181\,949\,143\,956\,586\,496\ b^7 t^{45} - \\
& 1\,007\,681\,411\,177\,512\,960\ b^7 t^{45} - 111\,600\,430\,219\,264\ b^7 t^{45} + \\
& 344\,840\,078\,143\,831\,081\,943\,040\ t^{46} + 24\,879\,079\,979\,923\,211\,681\,792\ b^7 t^{46} + \\
& 295\,946\,636\,302\,116\,454\,400\ b^7 t^{46} + 335\,168\,147\,571\,081\,216\ b^7 t^{46} + 16\,149\,077\,032\,960\ b^7 t^{46} - \\
& 301\,156\,190\,293\,307\,544\,961\,024\ t^{47} - 18\,199\,486\,273\,925\,067\,571\,200\ b^7 t^{47} - \\
& 143\,909\,374\,477\,304\,070\,144\ b^7 t^{47} - 92\,320\,219\,485\,175\,808\ b^7 t^{47} - 1\,511\,828\,488\,192\ b^7 t^{47} + \\
& 247\,949\,817\,830\,337\,871\,216\,640\ t^{48} + 12\,098\,582\,662\,742\,222\,700\,544\ b^7 t^{48} + \\
& 62\,214\,174\,569\,854\,402\,560\ b^7 t^{48} + 20\,513\,790\,839\,750\,656\ b^7 t^{48} + 68\,719\,476\,736\ b^7 t^{48} - \\
& 192\,081\,914\,760\,517\,159\,616\,512\ t^{49} - 7\,299\,770\,360\,243\,501\,137\,920\ b^7 t^{49} - \\
& 23\,939\,984\,040\,189\,231\,104\ b^7 t^{49} - 3\,536\,879\,798\,452\,224\ b^7 t^{49} + \\
& 139\,712\,458\,817\,817\,308\,954\,624\ t^{50} + 3\,988\,257\,840\,637\,229\,137\,920\ b^7 t^{50} + \\
& 8\,202\,372\,083\,146\,031\,104\ b^7 t^{50} + 444\,462\,543\,142\,912\ b^7 t^{50} - \\
& 95\,191\,961\,727\,827\,165\,577\,216\ t^{51} - 1\,966\,731\,522\,254\,136\,934\,400\ b^7 t^{51} - \\
& 2\,498\,298\,324\,721\,860\,608\ b^7 t^{51} - 36\,258\,113\,912\,832\ b^7 t^{51} + 60\,600\,871\,231\,511\,420\,207\,104\ t^{52} + \\
& 871\,703\,119\,125\,131\,821\,056\ b^7 t^{52} + 672\,737\,448\,366\,702\,592\ b^7 t^{52} + 1\,443\,109\,011\,456\ b^7 t^{52} - \\
& 35\,947\,097\,544\,282\,148\,962\,304\ t^{53} - 345\,442\,379\,495\,668\,449\,280\ b^7 t^{53} - \\
& 158\,217\,588\,235\,567\,104\ b^7 t^{53} + 19\,807\,574\,456\,001\,005\,879\,296\ t^{54} + \\
& 121\,608\,279\,162\,204\,717\,056\ b^7 t^{54} + 31\,788\,349\,769\,383\,936\ b^7 t^{54} - \\
& 10\,104\,737\,257\,555\,038\,830\,592\ t^{55} - 37\,730\,044\,734\,726\,995\,968\ b^7 t^{55} - \\
& 5\,261\,526\,600\,515\,584\ b^7 t^{55} + 4\,754\,853\,357\,688\,900\,812\,800\ t^{56} + \\
& 10\,216\,156\,816\,415\,391\,744\ b^7 t^{56} + 676\,289\,576\,960\,000\ b^7 t^{56} - \\
& 2\,055\,341\,493\,084\,737\,241\,088\ t^{57} - 2\,384\,625\,634\,656\,649\,216\ b^7 t^{57} - 60\,597\,693\,579\,264\ b^7 t^{57} + \\
& 812\,411\,022\,081\,347\,551\,232\ t^{58} + 472\,329\,367\,880\,466\,432\ b^7 t^{58} + 2\,847\,563\,317\,248\ b^7 t^{58} - \\
& 292\,130\,908\,439\,868\,080\,128\ t^{59} - 77\,759\,288\,784\,715\,776\ b^7 t^{59} + 34\,359\,738\,368\ b^7 t^{59} +
\end{aligned}$$

$$95\,007\,496\,406\,879\,436\,800\ t^{60} + 10\,341\,817\,358\,745\,600\ b_7 t^{60} - 8\,589\,934\,592\ b_7^2 t^{60} -$$

$$27\,760\,166\,140\,075\,048\,960\ t^{61} - 1\,066\,301\,598\,466\,048\ b_7 t^{61} + 7\,231\,365\,573\,995\,659\,264\ t^{62} +$$

$$79\,862\,366\,732\,288\ b_7 t^{62} - 1\,664\,260\,892\,717\,481\,984\ t^{63} - 3\,859\,564\,986\,368\ b_7 t^{63} +$$

$$334\,758\,827\,641\,536\,512\ t^{64} + 90\,194\,313\,216\ b_7 t^{64} - 58\,080\,476\,765\,618\,176\ t^{65} +$$

$$8\,549\,765\,809\,700\,864\ t^{66} - 1\,045\,287\,732\,772\,864\ t^{67} + 103\,119\,346\,204\,672\ t^{68} -$$

$$7\,875\,090\,972\,672\ t^{69} + 436\,207\,616\,000\ t^{70} - 15\,569\,256\,448\ t^{71} + 268\,435\,456\ t^{72}$$

```
In[ ]:= Solve[rh4c == 0 /. t -> 4/5]
```

```
Out[ ]:= {{b7 -> -2.52... x 10^3}, {b7 -> -0.0106...}, {b7 -> -2.79... x 10^-6}, {b7 -> 1.06...}}
```

```
In[ ]:= RootReduce[Sqrt[z2s ^ 7 (p71 + p72 ωω)] / q /. ωω -> ω /. t -> 4/5]
```

```
Out[ ]:= 1.06...
```

```
In[ ]:= rh4d = Factor[Resultant[b5 + b7 - b57, rh3c, b5]];
```

```
In[ ]:= rh4e = Factor[Resultant[rh4d, rh4c, b7]]
```

```
In[ ]:= Length[rh4e]
```

```
Out[ ]:= 7
```

```
In[ ]:= rh4e[[5]]
```

```
In[ ]:= Solve[rh4e[[5]] == 0 /. t -> 4/5]
```

```
Out[ ]:= {{b57 -> -9.90... x 10^3}, {b57 -> -4.63...}, {b57 -> -2.96... x 10^-3}, {b57 -> 0.202...}}
```

```
In[ ]:= RootReduce [
```

$$1/q \{-\text{Sqrt}[p71 + p72 \omega\omega], \text{Sqrt}[p51 + p52 \omega\omega], 0, 0\}.\text{Table}[(-\text{Sqrt}[Z])^j, \{j, 7, 1, -2\}] /.$$

$$Z \rightarrow z2s /. \omega\omega \rightarrow \omega /. \{t \rightarrow 4/5\}]$$

```
Out[ ]:= {-4.63...}
```

```
In[ ]:= rh5a = Factor[Resultant[b13 + b57 - y, rh2e, b13]];
```

```
In[ ]:= rh5b = Factor[Resultant[rh5a, rh4e[[5]], b57]]
```

```
In[ ]:= rh5b[[5]]
```

```
Out[ ]:= -4096 + 148480 t - 2750720 t^2 + 34749824 t^3 - 336526192 t^4 + 2659653476 t^5 -
```

$$17816570327 t^6 + 103669805970 t^7 - 532673407533 t^8 + 2444555432428 t^9 -$$

$$10101516992278 t^{10} + 37808103046200 t^{11} - 128736341031482 t^{12} + 400097778377888 t^{13} -$$

$$1137744898086151 t^{14} + 2965536850326350 t^{15} - 7093301577214869 t^{16} +$$

$$15579473043083968 t^{17} - 31424558991009676 t^{18} + 58188795780620280 t^{19} -$$

$$98832142780309764 t^{20} + 153771297026767552 t^{21} - 218772834488230896 t^{22} +$$

$$283964518819782336 t^{23} - 335334923634828768 t^{24} + 359082661480751680 t^{25} -$$

$$347305120815149760 t^{26} + 302022806671910528 t^{27} - 234884594325708288 t^{28} +$$

$$162340823171777024 t^{29} - 98977513449947392 t^{30} + 52763299284114944 t^{31} -$$

$$24330463229814016 t^{32} + 9577133976019968 t^{33} - 3164554413297664 t^{34} +$$

$$858818914918400 t^{35} - 185838351155200 t^{36} + 30727222591488 t^{37} -$$

$$\begin{aligned}
& 3\,631\,831\,056\,384\,t^{38} + 272\,092\,889\,088\,t^{39} - 9\,663\,676\,416\,t^{40} - 512\,t^3y + 36\,480\,t^4y - \\
& 862\,368\,t^5y + 10\,578\,472\,t^6y - 71\,258\,236\,t^7y + 129\,341\,869\,t^8y + 2\,786\,256\,320\,t^9y - \\
& 38\,165\,474\,712\,t^{10}y + 295\,155\,126\,592\,t^{11}y - 1\,688\,825\,202\,801\,t^{12}y + 7\,745\,491\,543\,804\,t^{13}y - \\
& 29\,539\,529\,842\,228\,t^{14}y + 95\,567\,084\,428\,168\,t^{15}y - 265\,211\,205\,189\,724\,t^{16}y + \\
& 634\,568\,767\,617\,900\,t^{17}y - 1\,308\,675\,104\,905\,300\,t^{18}y + 2\,309\,673\,007\,928\,456\,t^{19}y - \\
& 3\,422\,964\,145\,264\,132\,t^{20}y + 4\,063\,930\,244\,975\,440\,t^{21}y - 3\,328\,525\,537\,082\,832\,t^{22}y + \\
& 362\,287\,730\,145\,472\,t^{23}y + 5\,031\,272\,957\,570\,880\,t^{24}y - 11\,916\,590\,513\,322\,944\,t^{25}y + \\
& 18\,346\,637\,582\,050\,816\,t^{26}y - 22\,202\,048\,974\,222\,720\,t^{27}y + 22\,291\,811\,606\,586\,368\,t^{28}y - \\
& 18\,958\,202\,256\,457\,728\,t^{29}y + 13\,762\,753\,646\,345\,216\,t^{30}y - 8\,537\,042\,285\,750\,272\,t^{31}y + \\
& 4\,508\,930\,756\,182\,016\,t^{32}y - 2\,012\,599\,688\,642\,560\,t^{33}y + 750\,296\,622\,743\,552\,t^{34}y - \\
& 229\,579\,662\,491\,648\,t^{35}y + 56\,196\,285\,202\,432\,t^{36}y - 10\,581\,868\,544\,000\,t^{37}y + \\
& 1\,438\,010\,834\,944\,t^{38}y - 125\,275\,471\,872\,t^{39}y + 5\,234\,491\,392\,t^{40}y + 128\,t^5y^2 + 944\,t^6y^2 - \\
& 121\,024\,t^7y^2 + 2\,435\,295\,t^8y^2 - 25\,309\,618\,t^9y^2 + 150\,214\,572\,t^{10}y^2 - 338\,717\,930\,t^{11}y^2 - \\
& 2\,716\,061\,235\,t^{12}y^2 + 35\,586\,205\,432\,t^{13}y^2 - 228\,758\,901\,298\,t^{14}y^2 + 1\,038\,778\,158\,408\,t^{15}y^2 - \\
& 3\,641\,811\,626\,338\,t^{16}y^2 + 10\,209\,673\,867\,420\,t^{17}y^2 - 23\,229\,240\,646\,864\,t^{18}y^2 + \\
& 43\,067\,989\,979\,784\,t^{19}y^2 - 64\,999\,863\,846\,764\,t^{20}y^2 + 80\,441\,318\,627\,968\,t^{21}y^2 - \\
& 87\,334\,674\,313\,120\,t^{22}y^2 + 105\,688\,325\,071\,488\,t^{23}y^2 - 180\,692\,976\,848\,352\,t^{24}y^2 + \\
& 356\,158\,390\,490\,240\,t^{25}y^2 - 627\,743\,179\,812\,288\,t^{26}y^2 + 917\,571\,987\,183\,616\,t^{27}y^2 - \\
& 1\,106\,630\,331\,519\,232\,t^{28}y^2 + 1\,109\,792\,927\,015\,936\,t^{29}y^2 - 931\,896\,661\,092\,352\,t^{30}y^2 + \\
& 657\,693\,440\,925\,696\,t^{31}y^2 - 390\,390\,706\,241\,536\,t^{32}y^2 + 194\,478\,675\,722\,240\,t^{33}y^2 - \\
& 80\,920\,576\,049\,152\,t^{34}y^2 + 27\,904\,990\,773\,248\,t^{35}y^2 - 7\,880\,314\,781\,696\,t^{36}y^2 + \\
& 1\,787\,806\,613\,504\,t^{37}y^2 - 315\,265\,908\,736\,t^{38}y^2 + 40\,653\,291\,520\,t^{39}y^2 - 3\,388\,997\,632\,t^{40}y^2 + \\
& 134\,217\,728\,t^{41}y^2 + 8\,t^8y^3 - 202\,t^9y^3 - 273\,t^{10}y^3 + 87\,928\,t^{11}y^3 - 1\,942\,927\,t^{12}y^3 + \\
& 24\,965\,964\,t^{13}y^3 - 227\,100\,424\,t^{14}y^3 + 1\,579\,091\,256\,t^{15}y^3 - 8\,743\,747\,008\,t^{16}y^3 + \\
& 39\,530\,964\,420\,t^{17}y^3 - 148\,289\,139\,060\,t^{18}y^3 + 466\,280\,733\,784\,t^{19}y^3 - 1\,235\,973\,648\,972\,t^{20}y^3 + \\
& 2\,765\,663\,780\,976\,t^{21}y^3 - 5\,205\,285\,494\,432\,t^{22}y^3 + 8\,148\,562\,651\,392\,t^{23}y^3 - \\
& 10\,336\,766\,132\,480\,t^{24}y^3 + 9\,948\,814\,128\,128\,t^{25}y^3 - 5\,692\,194\,596\,864\,t^{26}y^3 - \\
& 1\,905\,393\,430\,528\,t^{27}y^3 + 10\,212\,250\,810\,368\,t^{28}y^3 - 15\,862\,040\,682\,496\,t^{29}y^3 + \\
& 16\,889\,421\,414\,400\,t^{30}y^3 - 13\,848\,250\,941\,440\,t^{31}y^3 + 9\,035\,515\,887\,616\,t^{32}y^3 - \\
& 4\,720\,780\,378\,112\,t^{33}y^3 + 1\,959\,608\,975\,360\,t^{34}y^3 - 633\,610\,960\,896\,t^{35}y^3 + 154\,131\,496\,960\,t^{36}y^3 - \\
& 26\,576\,158\,720\,t^{37}y^3 + 2\,900\,361\,216\,t^{38}y^3 - 150\,994\,944\,t^{39}y^3 - t^{10}y^4 + 50\,t^{11}y^4 - 1203\,t^{12}y^4 + \\
& 18\,544\,t^{13}y^4 - 205\,715\,t^{14}y^4 + 1\,748\,874\,t^{15}y^4 - 11\,847\,485\,t^{16}y^4 + 65\,652\,532\,t^{17}y^4 - \\
& 303\,151\,596\,t^{18}y^4 + 1\,182\,149\,760\,t^{19}y^4 - 3\,931\,550\,832\,t^{20}y^4 + 11\,232\,064\,320\,t^{21}y^4 - \\
& 27\,706\,743\,616\,t^{22}y^4 + 59\,214\,199\,808\,t^{23}y^4 - 109\,855\,142\,400\,t^{24}y^4 + 177\,017\,808\,896\,t^{25}y^4 - \\
& 247\,564\,564\,480\,t^{26}y^4 + 299\,858\,362\,368\,t^{27}y^4 - 313\,427\,288\,064\,t^{28}y^4 + 281\,219\,727\,360\,t^{29}y^4 - \\
& 215\,000\,383\,488\,t^{30}y^4 + 138\,664\,345\,600\,t^{31}y^4 - 74\,425\,892\,864\,t^{32}y^4 + 32\,629\,850\,112\,t^{33}y^4 - \\
& 11\,380\,981\,760\,t^{34}y^4 + 3\,036\,676\,096\,t^{35}y^4 - 581\,959\,680\,t^{36}y^4 + 71\,303\,168\,t^{37}y^4 - 4\,194\,304\,t^{38}y^4
\end{aligned}$$

In[]:= Solve[rh5b[[5]] == 0 /. t -> 4 / 5]

Out[]:= {{y -> $-1.06 \dots \times 10^4$ }, {y -> $-0.831 \dots$ }, {y -> $0.720 \dots$ }, {y -> $0.783 \dots$ }}

```
In[ * ]:= Expand[rh5b[[5]] - qq45]
```

```
Out[ * ]= 0
```