Gyakorló feladatok

- **Chebyshev polinomok szelsô értékhelyei**

Feladat: Adjuk meg a grafikon azon pontjait ahol lokális min./max. van. Színezzük is a pontokat.

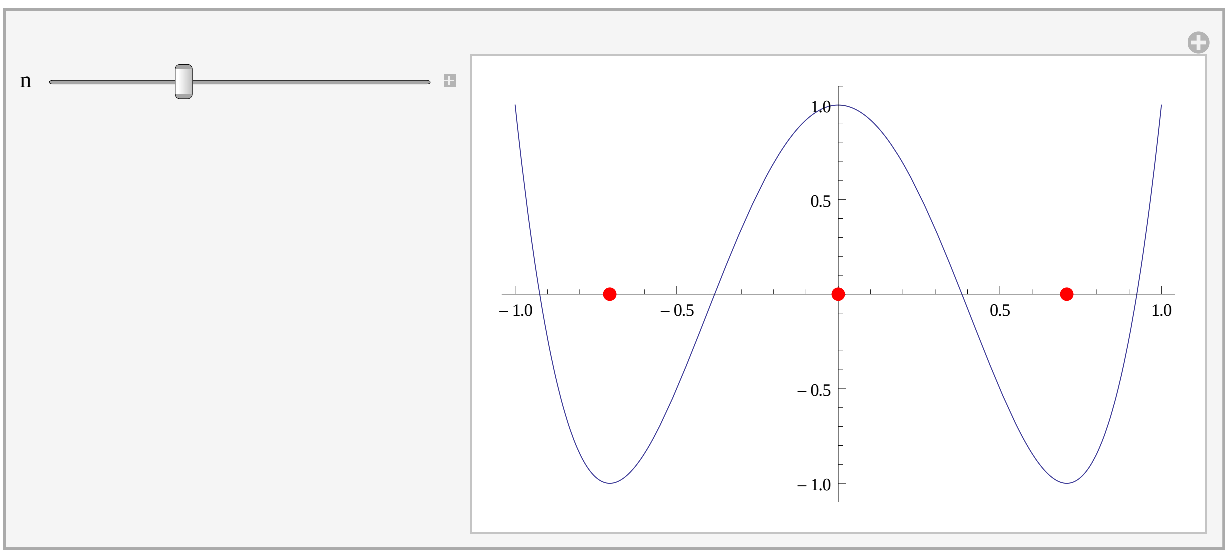
```
In[1]:= T = Table[ChebyshevT[n, x], {n, 1, 8}];

In[2]:= DZeroList = Table[x /. NSolve[D[T[[n]], x] == 0, x], {n, 2, 8}]

Out[2]= {{0.}, {-0.5, 0.5}, {-0.707107, 0., 0.707107}, {-0.809017, -0.309017, 0.309017, 0.809017},
 {-0.866025, -0.5, 0., 0.5, 0.866025}, {-0.900969, -0.62349, -0.222521, 0.222521, 0.62349, 0.900969},
 {-0.92388, -0.707107, -0.382683, 0., 0.382683, 0.707107, 0.92388}}

In[3]:= DZeroList2 = DZeroList /. {x_?NumberQ → {x, 0}};

Manipulate[Plot[T⟦n⟧, {x, -1, 1}, PlotRange → {-1.1, 1.1},
  Epilog → {Red, PointSize[.02], Point[DZeroList2[[n - 1]]]}], {n, 2, 8, 1}]
```



Most a függvényértékekre is szükségünk van.

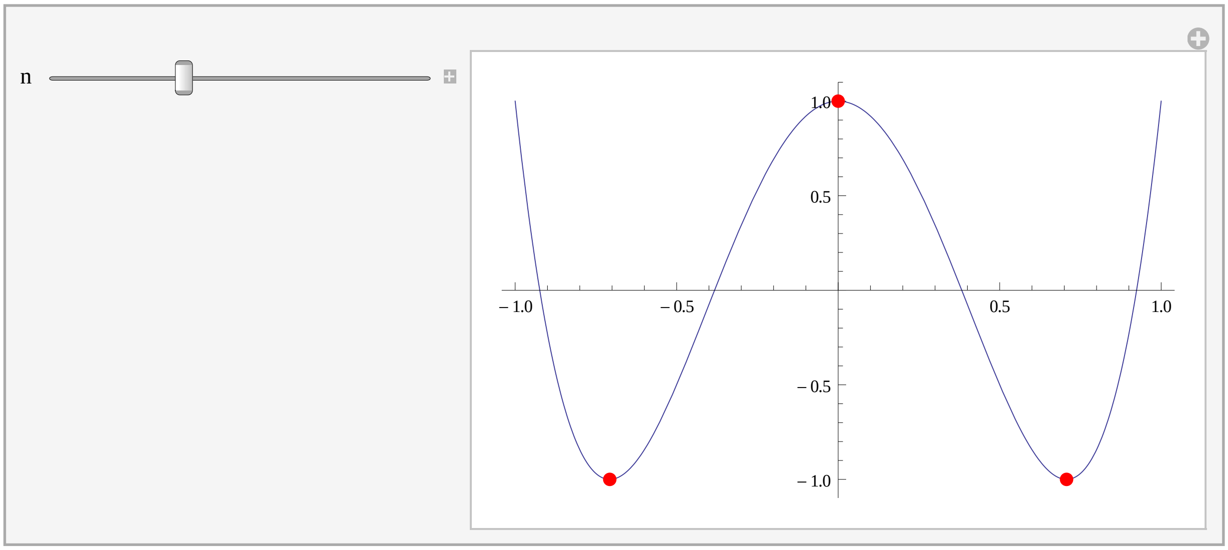
```
In[7]:=
T⟦4⟧ /. NSolve[D[T⟦4⟧, x] == 0, x]

Out[7]= {-1., 1., -1.}
```

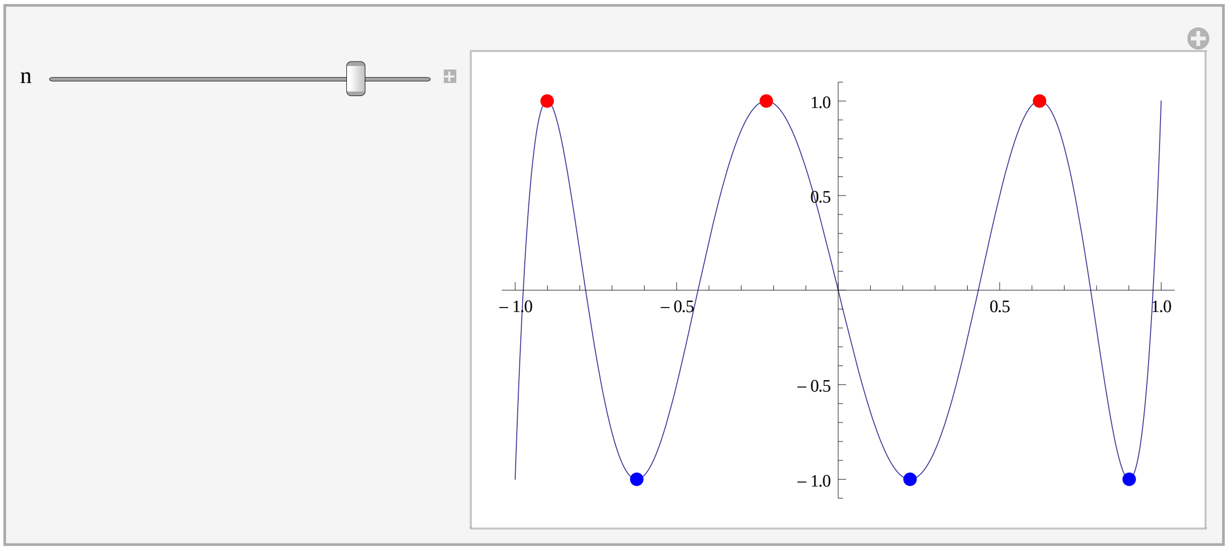
In[4]:=
```
DZeroList3 = Table[{x, T[[n]]} /. NSolve[D[T[[n]], x] == 0, x], {n, 2, 8}]
```

Out[4]=
```
{{{0., -1.}}, {{-0.5, 1.}, {0.5, -1.}}, {{-0.707107, -1.}, {0., 1.}, {0.707107, -1.}},
 {{-0.809017, 1.}, {-0.309017, -1.}, {0.309017, 1.}, {0.809017, -1.}},
 {{-0.866025, -1.}, {-0.5, 1.}, {0., -1.}, {0.5, 1.}, {0.866025, -1.}},
 {{-0.900969, 1.}, {-0.62349, -1.}, {-0.222521, 1.}, {0.222521, -1.}, {0.62349, 1.}, {0.900969, -1.}},
 {{-0.92388, -1.}, {-0.707107, 1.}, {-0.382683, -1.},
  {0., 1.}, {0.382683, -1.}, {0.707107, 1.}, {0.92388, -1.}}}
```

```
Manipulate[Plot[T[[n]], {x, -1, 1}, PlotRange → {-1.1, 1.1},
  Epilog → {Red, PointSize[.02], Point[DZeroList3[[n - 1]]]}], {n, 2, 8, 1}]
```




```
Manipulate[Plot[T[[n]], {x, -1, 1}, PlotRange → {-1.1, 1.1},
  Epilog → {PointSize[.02], Map[{If[#[[2]] == 1, Red, Blue], Point[#]} &, DZeroList3[[n - 1]]]}], {n,
  2, 8, 1}]
```



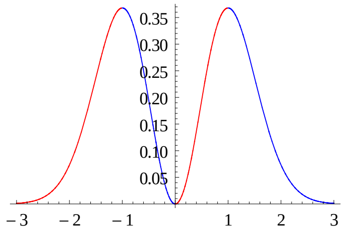
- **függvénygrafikon pontkainak színezése**

In[9]:=
```
f[x_] = x^2 Exp[-x^2]
```

Out[9]= $e^{-x^2}\, x^2$

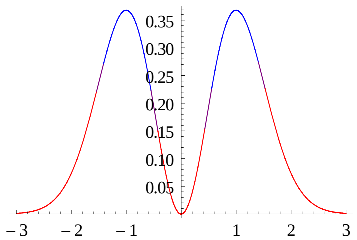
In[10]:=
```
Plot[f[x], {x, -3, 3}, ColorFunctionScaling → False,
 ColorFunction → (If[(D[f[x], x] /. x → #) > 0, Red, Blue] &)]
```

Out[10]=



In[11]:=
```
Plot[f[x], {x, -3, 3}, ColorFunctionScaling → False,
 ColorFunction → (If[(D[f[x], {x, 2}] /. x → #) > 0, Red, Blue] &)]
```

Out[11]=



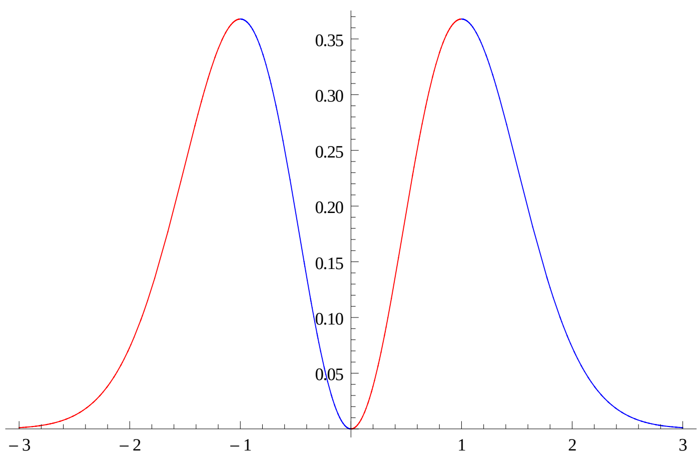
In[12]:=
```
MyFirstColorFunction[x_] := If[(D[f[y], y] /. y → x) > 0, Red, Blue]
```

In[13]:=
```
MyFirstColorFunction[-2]
```

Out[13]=
```
RGBColor[1, 0, 0]
```

```
Plot[f[x], {x, -3, 3}, ColorFunctionScaling → False, ColorFunction → (MyFirstColorFunction[#] &)]
```

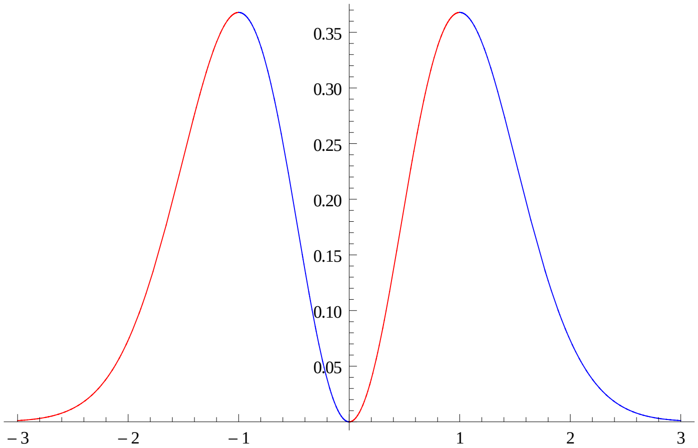
Out[15]=



In[16]:=
```
MySecondColorFunction[x_, y_] := If[(D[f[z], z] /. z → x) > 0, Red, Blue]
```

In[18]:=
```
Plot[f[x], {x, -3, 3}, ColorFunctionScaling → False,
 ColorFunction → (MySecondColorFunction[#1, #2] &)]
```

Out[18]=



Egyváltozós függvénydiszkusszió

Függvédiszkusszió számítógéppel (ÉT, zéróhelyek, limeszek (lokális/globális/aszimpt viselkedés), monotonitás, szimb+num+viz)

```
f[x_] = (x^6 - 3 x^4 + x^3 - 5 x^2 - 2) / (x^7 - x^5 + x^3 - 3 x^2 + x - 2)
```

$$\frac{-2 - 5 x^2 + x^3 - 3 x^4 + x^6}{-2 + x - 3 x^2 + x^3 - x^5 + x^7}$$

ET: ℝ\ {x0}, ahol x0~ 1.34

```
x /. NSolve[Denominator[f[x]] == 0, x, Reals]
```

{1.34871}

```
Length[%]
```

7

```
Head[%[[1]]]
```

```
%[[1]] // FullForm
```

```
Cases[{1, I, 2 + I, Sqrt[2]}, Complex[__]]
```

```
DeleteCases[x /. NSolve[Denominator[f[x]] == 0, x], Complex[__]]
```

Complex::argr : Complex called with 1 argument; 2 arguments are expected. ≫

{1.34871}

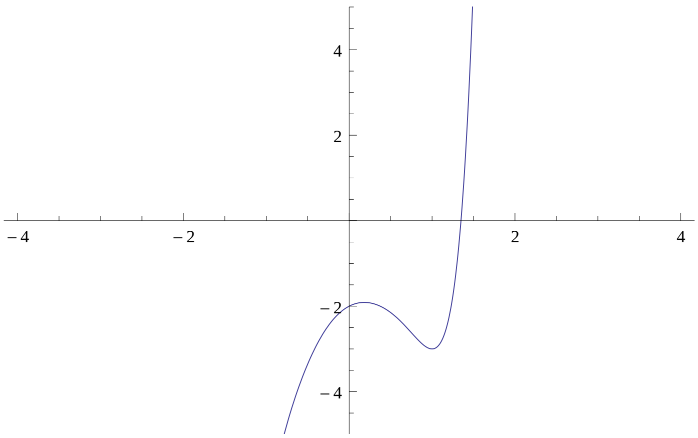
```
x /. NSolve[Denominator[f[x]] == 0, x, Reals]
```

```
x /. Solve[Denominator[f[x]] == 0, x, Reals]
```

{Root[-2 + #1 - 3 #1^2 + #1^3 - #1^5 + #1^7 &, 1]}

```
Factor[Denominator[f[x]]]
```

```
Plot[Evaluate[Denominator[f[x]]], {x, -4, 4}, PlotRange → {-5, 5}]
```



Zéróhely 2 zéróhely ~ -2.2 1.98

```
x /. NSolve[Numerator[f[x]] == 0, x]
```

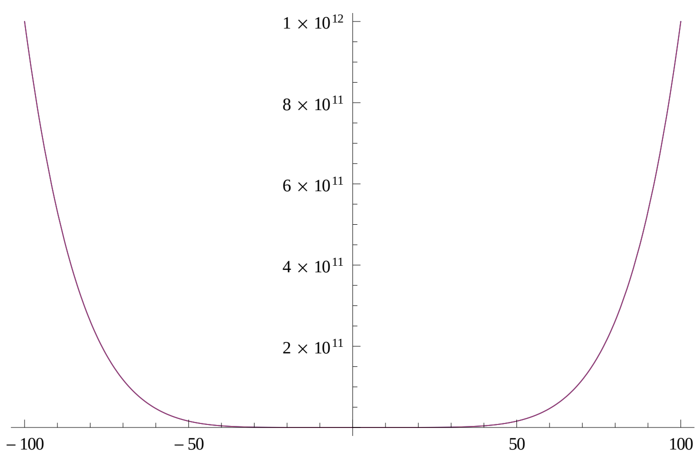
{-2.15314, -0.146099 - 0.706609 i, -0.146099 + 0.706609 i,
0.234566 - 0.920744 i, 0.234566 + 0.920744 i, 1.97621}

```
DeleteCases[%, Complex[__]]
```

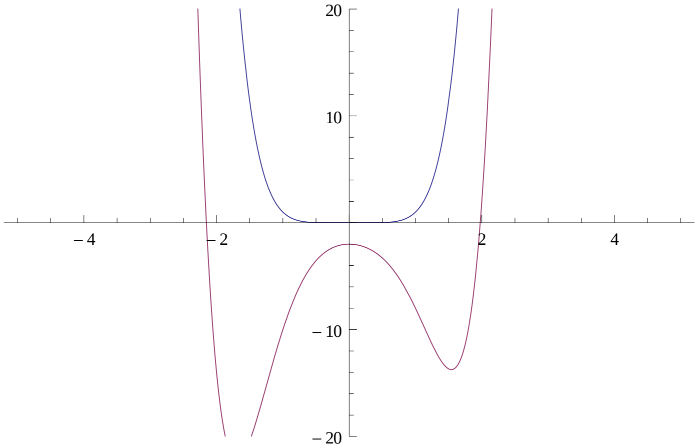
```
x /. NSolve[Numerator[f[x]] == 0, x, Reals]
```

{-2.15314, 1.97621}

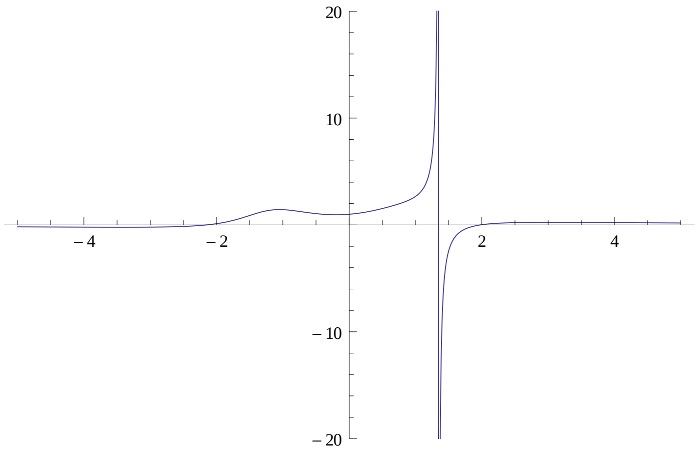
```
Plot[Evaluate[{x^6, Numerator[f[x]]}], {x, -100, 100}]
```



```
Plot[Evaluate[{x^6, Numerator[f[x]]}], {x, -5, 5}, PlotRange → {-20, 20}]
```




```
Plot[f[x], {x, -5, 5}, PlotRange → {-20, 20}]
```




```
ccp = DeleteCases[x /. NSolve[D[f[x], x] == 0], Complex[__]]
```

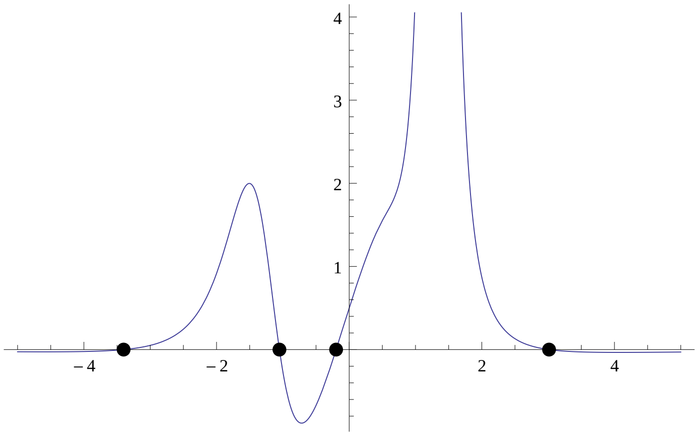
Complex::argr : Complex called with 1 argument; 2 arguments are expected. ≫

{-3.40133, 3.01403, -1.05169, -0.197297}

```
Sort[ccp]
```

{-3.40133, -1.05169, -0.197297, 3.01403}

```
Plot[Evaluate[D[f[x], x]], {x, -5, 5}, Epilog → {PointSize[.02], Map[Point[{#, 0}] &, ccp]}]
```



Limeszek vizsgálata

```
Limit[f[x], x → Infinity]
```

0

```
Limit[f[x], x → -Infinity]
```

0

Gyökök Rendezése: mindig a valósak kerülnek elôre, ezen belül normál rendezés

```
(x /. Solve[Denominator[f[x]] == 0, x][[1]])
```

Root$\left[-2 + \#1 - 3\,\#1^2 + \#1^3 - \#1^5 + \#1^7\,\&, 1\right]$

```
N[%]
```

```
Limit[f[x], x → (x /. Solve[Denominator[f[x]] == 0, x][[1]]), Direction → 1]
```

∞

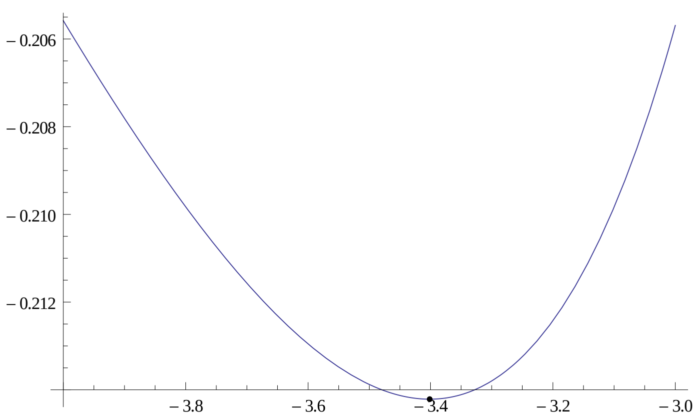
```
Limit[f[x], x → (x /. Solve[Denominator[f[x]] == 0, x][[1]]), Direction → -1]
```

$-\infty$

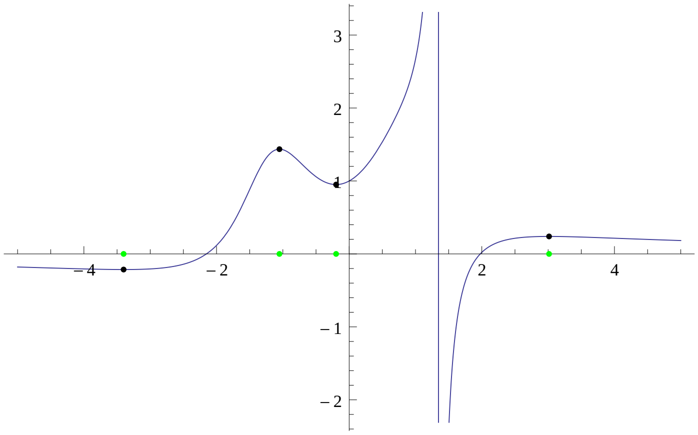
```
Map[Point[{#, f[#]}] &, ccp]
```

ccp

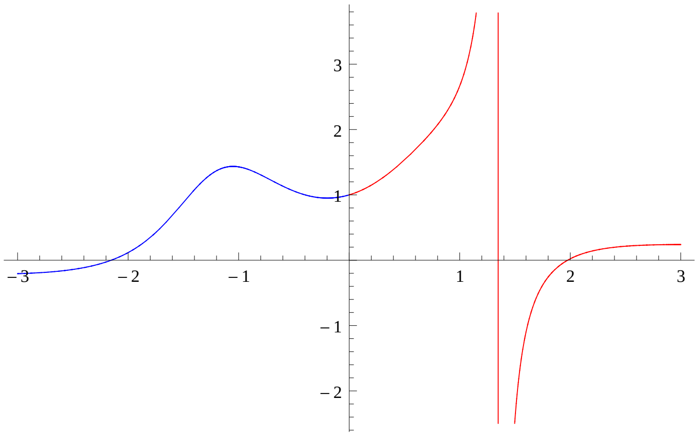
```
Plot[f[x], {x, -4, -3},
 Epilog → {Black, Map[Point[{#, f[#]}] &, ccp], Green, Map[Point[{#, 0}] &, ccp]}]
```



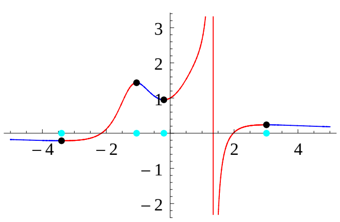
```
Plot[f[x], {x, -5, 5},
 Epilog → {Black, Map[Point[{#, f[#]}] &, ccp], Green, Map[Point[{#, 0}] &, ccp]}]
```



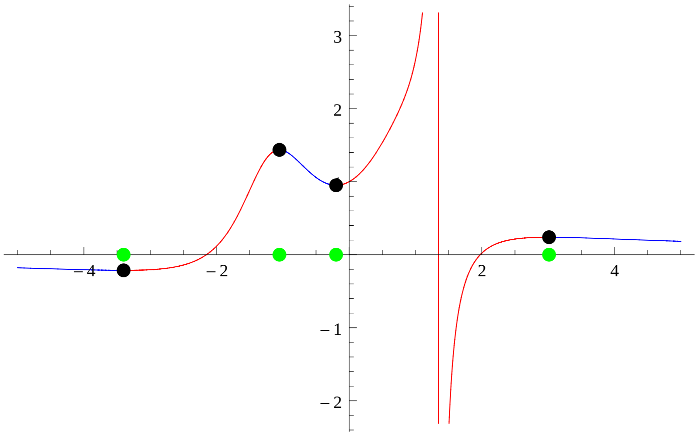

```
Plot[f[x], {x, -3, 3}, ColorFunctionScaling → False, ColorFunction → (If[# > 0, Red, Blue] &)]
```



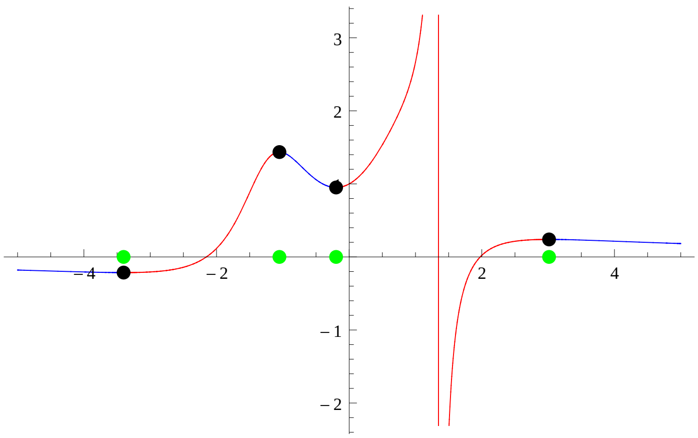

```
Plot[f[x], {x, -5, 5},
 Epilog → {PointSize[.02], Black, Map[Point[{#, f[#]}] &, ccp], Cyan, Map[Point[{#, 0}] &, ccp]},
 ColorFunctionScaling → False, ColorFunction -> (If[(D[f[x], x] /. x → #) > 0, Red, Blue] &)]
```



```
Plot[f[x], {x, -5, 5},
 Epilog → {PointSize[.02], Black, Map[Point[{#, f[#]}] &, ccp], Green, Map[Point[{#, 0}] &, ccp]},
 ColorFunctionScaling → False, ColorFunction -> (If[(D[f[x], x] /. x → #1) > 0, Red, Blue] &)]
```




```
Plot[f[x], {x, -5, 5},
 Epilog → {PointSize[.02], Black, Map[Point[{#, f[#]}] &, ccp], Green, Map[Point[{#, 0}] &, ccp]},
 ColorFunctionScaling → False,
 ColorFunction -> (If[(Evaluate[D[f[x], x]] /. x → #1) > 0, Red, Blue] &)]
```



Egzakt (mechanikus) elôjelvizsgálat rac. törtfgv. esetén

```
CylindricalDecomposition[D[f[x], x] > 0, x] // N
```

$-3.40133 < x < -1.05169 \,||\, -0.197297 < x < 1.34871 \,||\, 1.34871 < x < 3.01403$

```
Reduce[D[f[x], x] > 0, x] // N
```

$-3.40133 < x < -1.05169 \,||\, -0.197297 < x < 1.34871 \,||\, 1.34871 < x < 3.01403$

```
CylindricalDecomposition[D[f[x], x] < 0, x] // N
```

$x < -3.40133 \,||\, -1.05169 < x < -0.197297 \,||\, x > 3.01403$

Függvénydiszkusszió

$f(x) = x^3 - x$

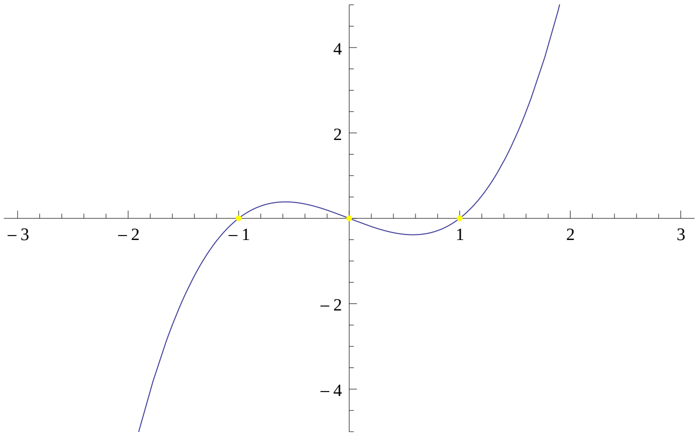
$g(x) = x^2 e^{-x^2}$

```
f[x_] := x^3 - x;
```

```
zhl = x /. Solve[f[x] == 0, x]
```

```
{-1, 0, 1}
```

```
Plot[f[x], {x, -3, 3}, Epilog → {Yellow, Map[Point[{#, 0}] &, zhl]}, PlotRange → {-5, 5}]
```




```
D[f[x], {x, 2}]
```

```
6 x
```

```
? Function
```

```
MyFunction[x_, y_] := If[x > 0, Red, Blue]
```

```
MyFunctionb[x_, y_] := If[(D[f[z], {z, 2}] /. z → x) > 0, Red, Blue]
```

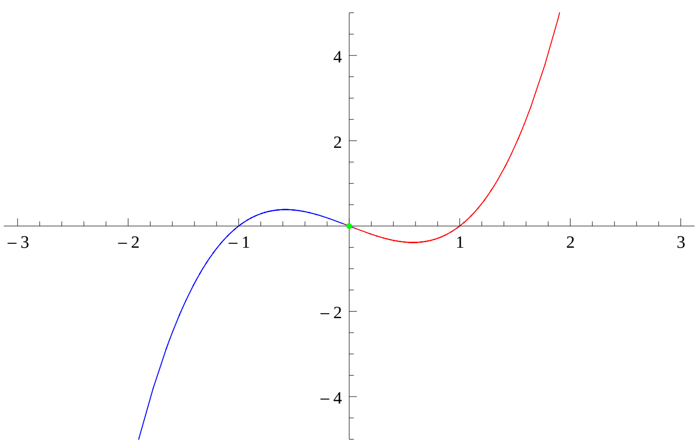
```
MyFunctionb[1, 0]
```

```
RGBColor[1, 0, 0]
```

```
MyFunctionb[-1, 0]
```

```
RGBColor[0, 0, 1]
```

```
Plot[f[x], {x, -3, 3}, Epilog → {Green, Point[{0, 0}]}, PlotRange → {-5, 5},
 ColorFunctionScaling → False, ColorFunction → (MyFunction[#1, #2] &)]
```



```
g[x_] = x^2 Exp[-x^2]
```

$e^{-x^2} x^2$

```
g2[x_] = x^2 E^(-x^2);
Solve[g[x] == 0, x]
```

Solve::ifun: Inverse functions are being used by Solve, so
some solutions may not be found; use Reduce for complete solution information. ≫

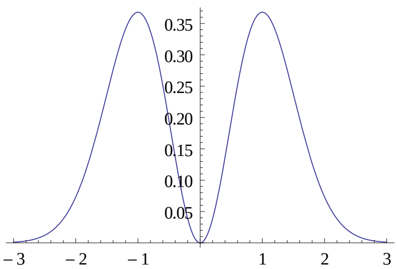
{{x → 0}}

```
? g2
```

Global`g2

g2[x_] = $e^{-x^2} x^2$

```
Plot[g2[x], {x, -3, 3}]
```




```
h[x_] = x^3 - 5 x^2 + 7 x + 8;
D[h[x], x]
```

$7 - 10 x + 3 x^2$

```
h'[x]
```

$7 - 10 x + 3 x^2$

```
D[h[x], {x, 2}]
```

$-10 + 6 x$

```
h''[x]
```

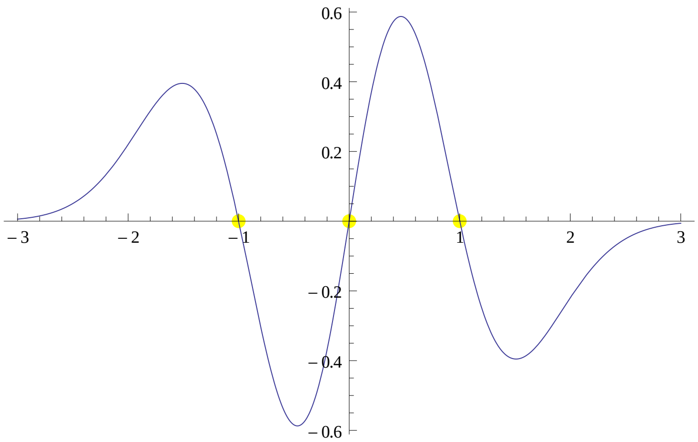
$-10 + 6 x$

```
D[h[x], x, x]
```

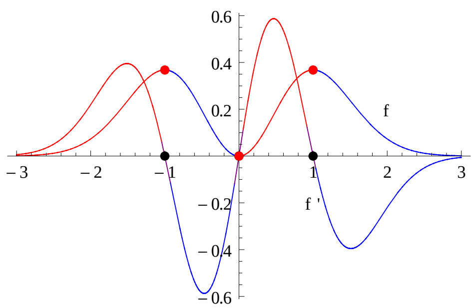
$-10 + 6 x$

```
DZeroList1 = {x, 0} /. Solve[D[g2[x], x] == 0, x]
DZeroList2 = {x, g2[x]} /. Solve[D[g2[x], x] == 0, x]
```

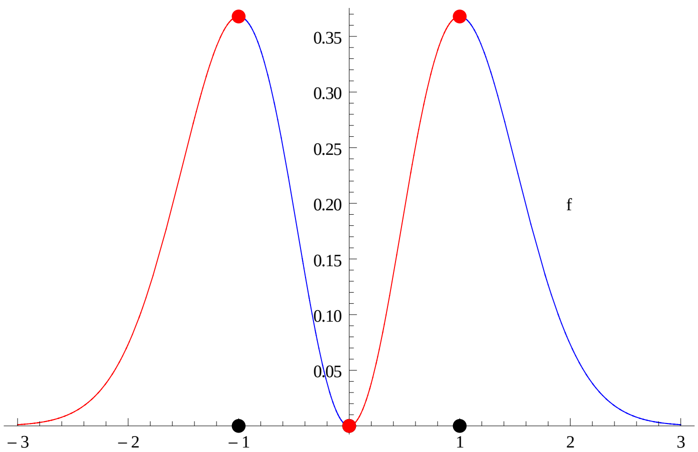
```
Plot[Evaluate[D[g2[x], x]], {x, -3, 3}, Prolog → {Yellow, PointSize[.02], Point[DZeroList1]}]
```



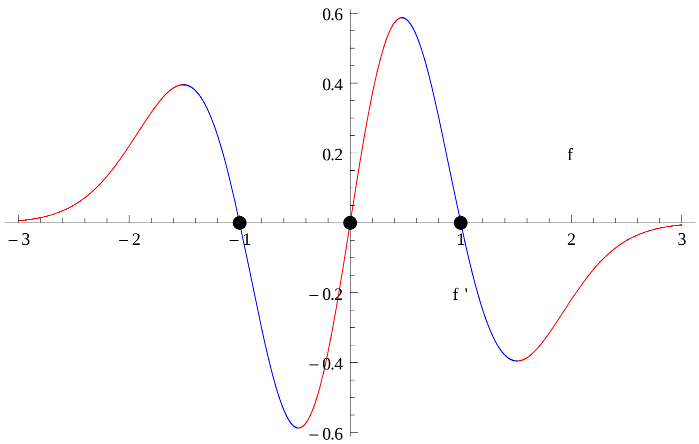
```
Plot[Evaluate[{g2[x], D[g2[x], x]}], {x, -3, 3},
 Epilog → {Black, PointSize[.02], Point[DZeroList1], Red, Point[DZeroList2],
   Black, Text["f", {2, .2}], Text["f'", {1, -.2}]}, ColorFunctionScaling → False,
 ColorFunction -> (If[(Evaluate[D[g2[x], x]] /. x → #1) > 0, Red, Blue] &)]
```



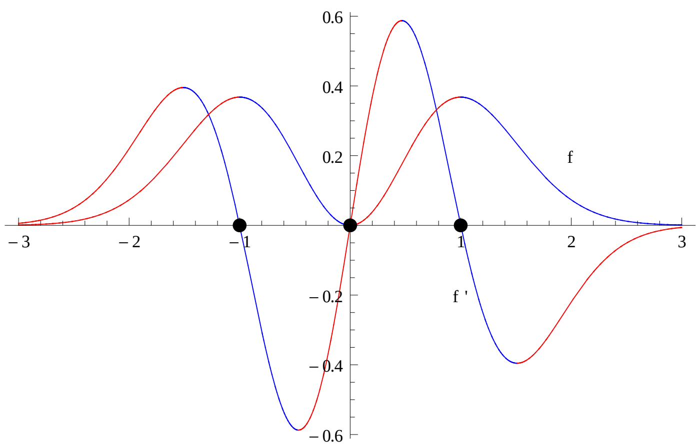

```
G1 = Plot[g2[x], {x, -3, 3},
  Epilog → {Black, PointSize[.02], Point[DZeroList1], Red, Point[DZeroList2],
    Black, Text["f", {2, .2}], Text["f'", {1, -.2}]}, ColorFunctionScaling → False,
  ColorFunction -> (If[(Evaluate[D[g2[x], x]] /. x → #1) > 0, Red, Blue] &)]
```



```
G2 = Plot[Evaluate[D[g2[x], x]], {x, -3, 3}, Epilog → {Black, PointSize[.02], Point[DZeroList1],
    Black, Text["f", {2, .2}], Text["f'", {1, -.2}]}, ColorFunctionScaling → False,
  ColorFunction -> (If[(Evaluate[D[g2[x], {x, 2}]] /. x → #1) > 0, Red, Blue] &)]
```




```
Show[G2, G1]
```




```
N[E]
```

2.71828

```
N[I]
```

0. + 1. ⅈ

```
N[Pi]
```

3.14159

```
Plot[{x, x^2}, {x, -4, 4}, PlotRange → {{-2, 2}, {-3, 3}}]
```