fokszámkorlát az unicitáshoz!

```
p1[x_] := x^2 + 1
p2[x_] := x^2 + 1 + (x + 1) (x) (x - 1)
{p1[-1], p1[0], p1[1]}
{2, 1, 2}
{p2[-1], p2[0], p2[1]}
{2, 1, 2}
```

Lagrange Interpoláció

- **Adjuk meg az Lagrange alapinterpolációs polinomokat, majd ezek segítségével állítsuk elõ a Lagrange interpolációs polinomot! Próbáljuk ki a következõ adathalmazokon:**

  x={1,2,3} , y={2,5,10}

- **Javaslat**

In[37]:= LagrBase[j_, x_List, var_] := $\left(\prod_{k=1}^{j} \frac{var - x[[k]]}{x[[j+1]] - x[[k]]}\right) \left(\prod_{k=j+2}^{Length[x]} \frac{var - x[[k]]}{x[[j+1]] - x[[k]]}\right)$

In[38]:= LagrInterp[x_List, y_List, var_] := $\sum_{j=0}^{Length[x]-1} y[[j+1]]$ LagrBase[j, x, var]

- **Példa**

```
Clear[x0, y0]
x0 = {1, 2, 3}; y0 = {2, 5, 10};
```

A bázispolinomok (j=0,1,2):

```
{LagrBase[0, x0, x], LagrBase[1, x0, x], LagrBase[2, x0, x]}
```

$\left\{\frac{1}{2} (2-x)(3-x), (3-x)(-1+x), \frac{1}{2}(-2+x)(-1+x)\right\}$

```
LagrBase[0, x0, x] /. {{x -> x0[[1]]}, {x -> x0[[2]]}, {x -> x0[[3]]}}
{1, 0, 0}
```

A Lagrange interpolációs polinom mint lineáris kombináció:

```
LagrInterp[x0, y0, x]
(2 - x) (3 - x) + 5 (3 - x) (-1 + x) + 5 (-2 + x) (-1 + x)
```

```
Expand[%]
```

$1 + x^2$

```
?InterpolatingPolynomial
```

```
Transpose[{{1, 2, 3}, {4, 5, 6}}]
```

{{1, 4}, {2, 5}, {3, 6}}

```
Expand[InterpolatingPolynomial[Transpose[{x0, y0}], x]]
```

$1 + x^2$

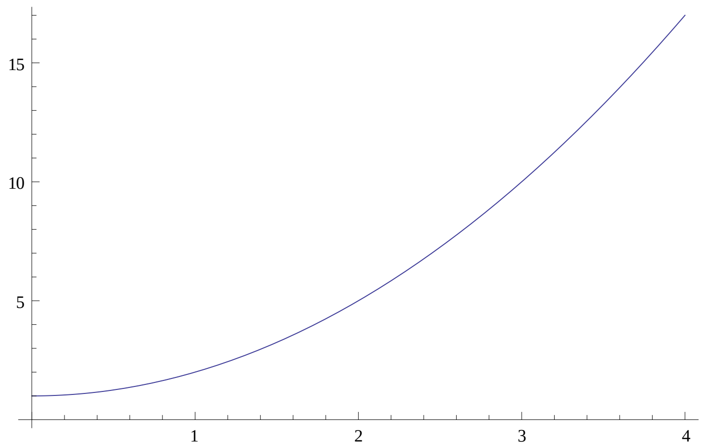
```
Expand[InterpolatingPolynomial[{{1, 2}, {2, 5}, {3, 10}}, x]]
```

```
Transpose[{x0, y0}] // TableForm
```

```
?InterpolatingPolynomial
```

Ábrázoljuk a pontokat és az interpolációs polinomot!

```
Plot[LagrInterp[x0, y0, x], {x, 0, 4}]
```



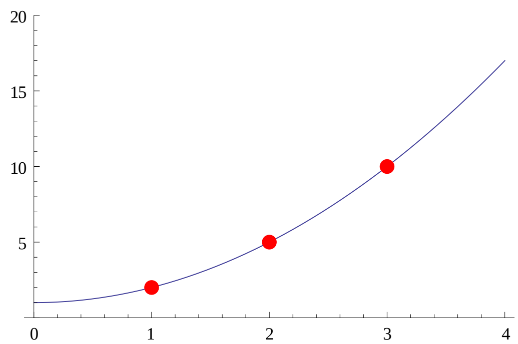

```
Transpose[{x0, y0}]
```

{{1, 2}, {2, 5}, {3, 10}}

```
Map[Point[#] &, Transpose[{x0, y0}]]
```

{Point[{1, 2}], Point[{2, 5}], Point[{3, 10}]}

```
Plot[LagrInterp[x0, y0, x], {x, 0, 4}, ImageSize → {300, 300}, Epilog →
  {RGBColor[1, 0, 0], PointSize[.03], Map[Point[#] &, Transpose[{x0, y0}]]}, PlotRange → {0, 20}]
```



- **Feladat**

x1={1,2,3,4,5}; y1={0,6,24,60,120}

Adjuk meg a bázispolinomokat, az interpolációs polinomokat és ábrázoljuk a pontokat és az interpolációs polinomot!

Oldjuk meg a feladatot a határoztlan együtthatók módszerével is!

In[39]:=
```
x1 = {1, 2, 3, 4, 5}; y1 = {0, 6, 24, 60, 120};
```

In[40]:=
```
{LagrBase[0, x1, x], LagrBase[1, x1, x],
  LagrBase[2, x1, x], LagrBase[3, x1, x], LagrBase[4, x1, x]} // TableForm
```

Out[40]//TableForm=

$\frac{1}{24} (2-x)(3-x)(4-x)(5-x)$

$\frac{1}{6} (3-x)(4-x)(5-x)(-1+x)$

$\frac{1}{4} (4-x)(5-x)(-2+x)(-1+x)$

$\frac{1}{6} (5-x)(-3+x)(-2+x)(-1+x)$

$\frac{1}{24} (-4+x)(-3+x)(-2+x)(-1+x)$

In[41]:=
```
LagrInterp[x1, y1, x] // Expand
```

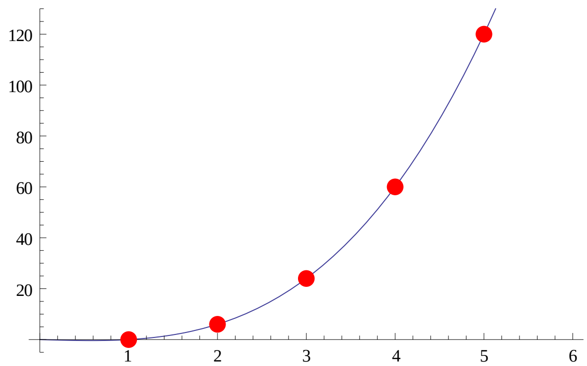
Out[41]= $-x + x^3$

In[42]:=
```
y1
```

Out[42]= {0, 6, 24, 60, 120}

In[43]:=
```
Plot[LagrInterp[x1, y1, x], {x, 0, 6}, ImageSize → {300, 300}, Epilog →
  {RGBColor[1, 0, 0], PointSize[.03], Map[Point[#] &, Transpose[{x1, y1}]]}, PlotRange → {-5, 130}]
```

Out[43]=



Lagrange Interpoláció, hibabecslés

- **x/(x^2+1) [-1,1]-en L4-gyel,**

  x0={-1,-1/2,0,1/2,1}
  y0={-1/2,-2/5,0,2/5,1/2}
  Milyen hibabecslé st adhatunk?

- **Javaslat**

  Hibaképlet: $\omega(x) / n!\, f^{(n)}(\xi)$,n=5

```
x0 = {-1, -1 / 2, 0, 1 / 2, 1}
```

$\left\{-1, -\frac{1}{2}, 0, \frac{1}{2}, 1\right\}$

```
f[x_] := x / (x^2 + 1)

f[1 / 2]
```

$\frac{2}{5}$

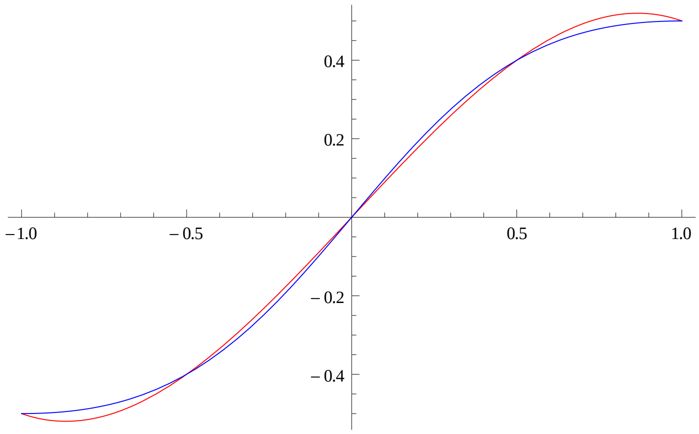
```
Table[f[x0[[i]]], {i, 5}]
```

$\left\{-\frac{1}{2}, -\frac{2}{5}, 0, \frac{2}{5}, \frac{1}{2}\right\}$

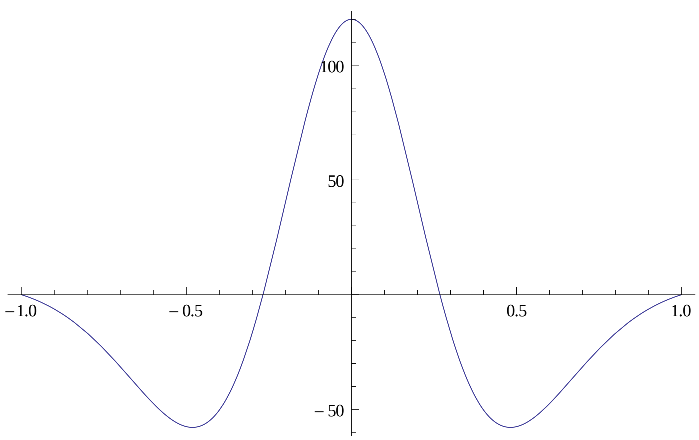
```
p =
 Expand[InterpolatingPolynomial[{{-1, -1/2}, {-1/2, -2/5}, {0, 0}, {1/2, 2/5}, {1, 1/2}}, x]]
```

$$\frac{9x}{10} - \frac{2x^3}{5}$$

```
Plot[{%, f[x]}, {x, -1, 1}, PlotStyle → {{RGBColor[1, 0, 0]}, {RGBColor[0, 0, 1]}}]
```



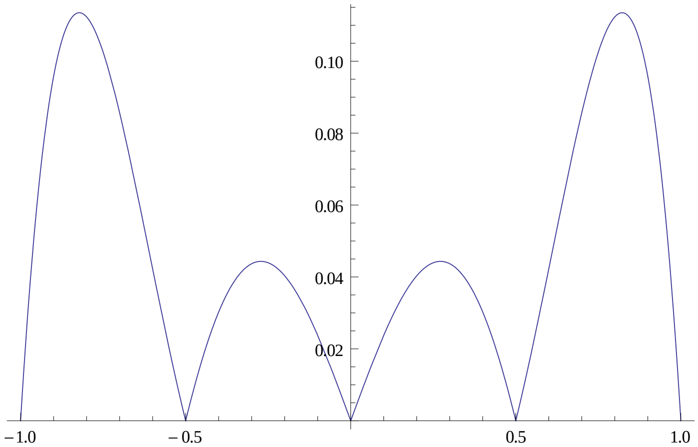

```
Plot[Evaluate[D[f[x], {x, 5}]], {x, -1, 1}]
```




```
D[f[x], {x, 5}] /. x → 0
```

120

```
Plot[Abs[(x + 1) (x + 1/2) x (x - 1/2) (x - 1)], {x, -1, 1}]
```




```
Solve[D[(x + 1) (x + 1/2) x (x - 1/2) (x - 1), x] == 0]
```

$$\left\{\left\{x \to -\frac{1}{2}\sqrt{\frac{1}{10}\left(15-\sqrt{145}\right)}\right\}, \left\{x \to \frac{1}{2}\sqrt{\frac{1}{10}\left(15-\sqrt{145}\right)}\right\}, \left\{x \to -\frac{1}{2}\sqrt{\frac{1}{10}\left(15+\sqrt{145}\right)}\right\}, \left\{x \to \frac{1}{2}\sqrt{\frac{1}{10}\left(15+\sqrt{145}\right)}\right\}\right\}$$

```
N[%]
```

{{x → −0.271956}, {x → 0.271956}, {x → −0.822216}, {x → 0.822216}}

```
{{x → -0.8222164340791343`}, {x → 0.8222164340791343`},
 {x → -0.271956127951169`}, {x → 0.271956127951169`}}
```

```
(x + 1) (x + 1/2) x (x - 1/2) (x - 1) /. x -> 0.8222164340791343`
```

−0.113482

A hiba mértéke tesztpontokban

```
Table[Abs[f[t] - p /. x → t], {t, -1, 1, .1}]
```

{0., 0.0211624, 0.0273951, 0.0230013, 0.0124235, 1.38778×10^{-17}, 0.0104276, 0.0160294, 0.0155077, 0.0094099, 5.55112×10^{-18}, 0.0094099, 0.0155077, 0.0160294, 0.0104276, 4.85723×10^{-17}, 0.0124235, 0.0230013, 0.0273951, 0.0211624, 0.}

- **Sin[x] [0,π/2]-en L2-vel,**

  x0={0, π/6, .π/2}
  y0={0,1/2,1}
  Milyen hibabecslé st adhatunk?

- **Javaslat**

Hibaképlet: $\omega(x)\,/\,n\,!\,f^{(n)}(\xi)$,n=3

In[1]:= `x0 = {0, π / 6, π / 2}`

Out[1]= $\left\{0, \frac{\pi}{6}, \frac{\pi}{2}\right\}$

In[2]:= `f[x_] := Sin[x]`

In[3]:= `f[π / 6]`

Out[3]= $\frac{1}{2}$

In[4]:= `Table[f[x0[[i]]], {i, 3}]`

Out[4]= $\left\{0, \frac{1}{2}, 1\right\}$

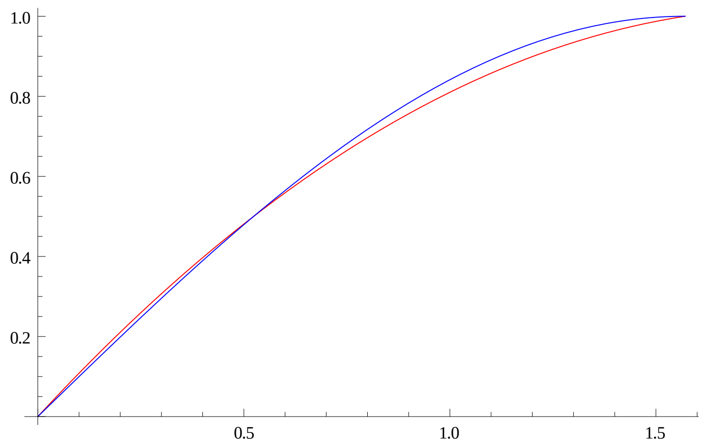
In[5]:=

`p = Expand[InterpolatingPolynomial[{{0, 0}, {π / 6, 1 / 2}, {π / 2, 1}}, x]]`

Out[5]= $\frac{7\,x}{2\,\pi} - \frac{3\,x^2}{\pi^2}$

In[6]:= `Plot[{%, f[x]}, {x, 0, π / 2}, PlotStyle → {{RGBColor[1, 0, 0]}, {RGBColor[0, 0, 1]}}]`

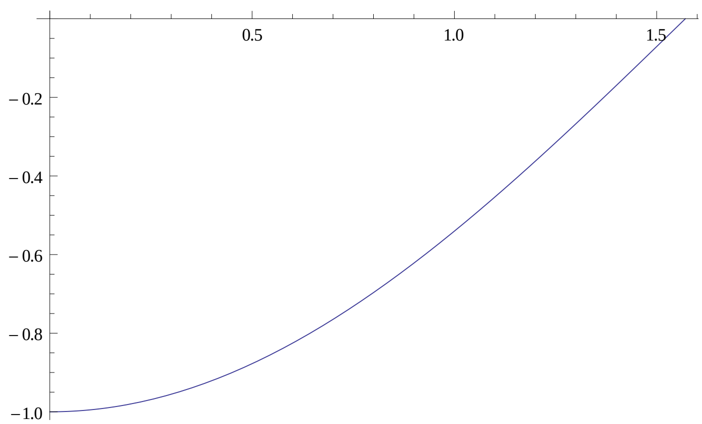
Out[6]=



In[11]:=

`Plot[Evaluate[D[f[x], {x, 3}]], {x, 0, π / 2}]`

Out[11]=

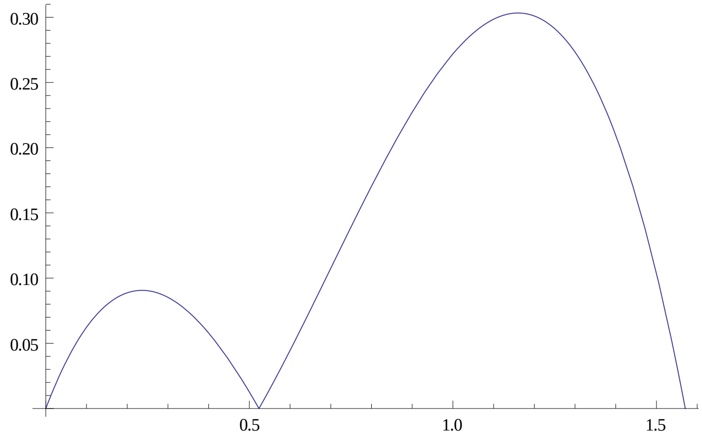


In[12]:= `D[f[x], {x, 3}] /. x → 0`

Out[12]= −1

In[14]:=
```
Plot[Abs[(x) (x - π / 6) (x - π / 2)], {x, 0, π / 2}]
```

Out[14]=



In[22]:=
```
Solve[D[(x) (x - π / 6) (x - π / 2), x] == 0]
```

Out[22]= $\left\{\left\{x\to\frac{1}{18}\left(4-\sqrt{7}\right)\pi\right\},\left\{x\to\frac{1}{18}\left(4+\sqrt{7}\right)\pi\right\}\right\}$

In[23]:=
```
N[%]
```

Out[23]= {{x → 0.236361}, {x → 1.1599}}

In[29]:=
```
x (x - π / 6) (x - π / 2) /. x -> 1.1599024164680696`
```

Out[29]= -0.30326

In[30]:=
```
Abs[% / 6]
```

Out[30]= 0.0505434

In[31]:=
```
1 / 6 × 31 / 100.
```

Out[31]= 0.0516667

A hiba $|f - L| \leq 1/6 \times 31/100 \sim 0.052$

A hiba mértéke tesztpontokban

In[28]:=
```
Table[Abs[f[t] - p /. x → t], {t, 0, π / 2, .1}]
```

Out[28]= {0., 0.00853541, 0.011989, 0.0113485, 0.00758133, 0.00162587, 0.00561859, 0.0133006, 0.0206251, 0.0268612, 0.0313499, 0.0335102, 0.0328451, 0.0289466, 0.0214998, 0.0102861}

Black-box one stroke:

In[34]:=
```
NMaximize[{Abs[x (x - π / 6) (x - π / 2)] / 6, 0 ≤ x ≤ π / 2}, x]
```

Out[34]= {0.0505434, {x → 1.1599}}

In[32]:=
```
NMaximize[{Abs[f[x] - p], 0 ≤ x ≤ π / 2}, x]
```

Out[32]= {0.033628, {x → 1.12842}}