# Iteráció

*Mathematica*-ban az n-edik iterált Nest[f,x0,n]
Iteráltsorozat {x0, f[x0], ff[x0],...} NestList[f,x0,n]

```
Nest[f, x0, 3]
```

```
f[f[f[x0]]]
```

```
Nest[f, x0, 0]
```

```
x0
```

```
Nest[f, x0, 1]
```

```
f[x0]
```

```
NestList[f, x0, 3]
```

```
{x0, f[x0], f[f[x0]], f[f[f[x0]]]}
```

```
f[x_] := x^2 + 1;
```

```
Nest[f, 1, 3]
```

```
26
```

```
NestList[f, 1, 3]
```

```
{1, 2, 5, 26}
```

```
g[{x_, y_}] := {y, x + y}
```

```
NestList[g, {0, 1}, 10]
```

```
{{0, 1}, {1, 1}, {1, 2}, {2, 3}, {3, 5}, {5, 8}, {8, 13}, {13, 21}, {21, 34}, {34, 55}, {55, 89}}
```

```
NestList[g, {0, 1}, 10][[All, 1]] // TableForm
```

```
0
1
1
2
3
5
8
13
21
34
55
```

# Jacobi-Iteráció

Cél: megoldani az Ax=a lin. egyenletrendszert numerikusan. A megoldást fokozatosan közelítjük. Konvergencia kérdése, hatékonyság.
Tipikus alkalmazási terület: Parciális differenciálegyenletek numerikus megoldása.

Ezeknél az eüh. mátrix ritka.

Szüks. és elegendõ felt. (Neumann-mátrix sor-konvergencia) spektrálsugár<1
Iteráció: Ax=a helyett x=Bx+b
Szüks. és elegendõ felt. (konvergencia) spektrálsugár<1

Elegendõ: valamelyik normára ||B||<1

## Jacobi

Ha a diagonális elemek nemzérók

$A\ x = a \Longrightarrow x = B\ x + b$,ahol

$B = -D^{-1}(L + U)\ ,\ \ b = D^{-1}\ a$

Nemszinguláris együtthatómátrix.

```
Remove[A, B, a, b]
```

$$A = \begin{pmatrix} 4 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 4 \end{pmatrix};\ a = \{2, 6, 2\};$$

```
Det[A]
```

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(Pontos) Megoldásvektor (Exact Solution)

```
ES1 = LinearSolve[A, a]
```

{1, 2, 1}

Ellenôrzés

```
A.ES1
```

{2, 6, 2}

Iteráció elõkészítése

```
L = {{0, 0, 0}, {-1, 0, 0}, {0, -1, 0}}; Di = DiagonalMatrix[{4, 4, 4}];
U = {{0, -1, 0}, {0, 0, -1}, {0, 0, 0}};
Map[MatrixForm[#] &, {A, L, Di, U}]
```

$$\left\{\begin{pmatrix} 4 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 4 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}, \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}\right\}$$

```
MatrixForm[L + Di + U]
```

$$\begin{pmatrix} 4 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 4 \end{pmatrix}$$

```
B = -Inverse[Di].(L + U); MatrixForm[B]
```

$$\begin{pmatrix} 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 \end{pmatrix}$$

```
b = Inverse[Di].a
```

$\left\{\frac{1}{2}, \frac{3}{2}, \frac{1}{2}\right\}$

```
Eigenvalues[B]
```

$\left\{-\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}, 0\right\}$

```
Max[Abs[% // N]]
```

0.353553

Konvergencia garantált!

Iterációs mag

```
b
```

```
f1[x_] := B.x + b
```

B, b mindig globális!

$x_0$kezdõvektor tetszõlegesen választható, lehet pl. ugyanaz, mint b1, de lehet akár $e_1 = \{1, 0, 0\}$ is.

```
x0 = {1, 0, 0};
```

```
B.x0 + b
```

$\left\{\frac{1}{2}, \frac{7}{4}, \frac{1}{2}\right\}$

```
b
```

$\left\{\frac{1}{2}, \frac{3}{2}, \frac{1}{2}\right\}$

```
B // MatrixForm
```

$$\begin{pmatrix} 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 \end{pmatrix}$$

```
f1[x0]
```

$\left\{\frac{1}{2}, \frac{7}{4}, \frac{1}{2}\right\}$

```
Nest[f1, x0, 0]
```

{1, 0, 0}

```
Nest[f1, x0, 1]
```

$\left\{\frac{1}{2}, \frac{7}{4}, \frac{1}{2}\right\}$

```
Nest[f1, x0, 2]
```

$\left\{\frac{15}{16}, \frac{7}{4}, \frac{15}{16}\right\}$

Az ötödik iterált

```
Nest[f1, x0, 5] // N
```

```
{0.992188, 1.99609, 0.992188}
```

Az elsõ nyolc iterált

```
(T = NestList[f1, x0, 8] // N) // ColumnForm
```

```
{1., 0., 0.}
{0.5, 1.75, 0.5}
{0.9375, 1.75, 0.9375}
{0.9375, 1.96875, 0.9375}
{0.992188, 1.96875, 0.992188}
{0.992188, 1.99609, 0.992188}
{0.999023, 1.99609, 0.999023}
{0.999023, 1.99951, 0.999023}
{0.999878, 1.99951, 0.999878}
```

Hiba az n-edik iteráció után

```
Map[Norm[# - ES1, 1] &, T] // ColumnForm
```

```
3.
1.25
0.375
0.15625
0.046875
0.0195313
0.00585938
0.00244141
0.000732422
```

A hibavektor normája gyorsan tart 0-hoz.

Hibabecslés

$\|e_m\| \leq \|B\| / (1 - \|B\|)\ \|x_m - x_{m-1}\|$ (m=1,2,...)

```
Table[Norm[B, 1] / (1 - Norm[B, 1]) Norm[T[[j + 1]] - T[[j]], 1], {j, Length[T] - 1}] // TableForm
```

```
2.75
0.875
0.21875
0.109375
0.0273438
0.0136719
0.00341797
0.00170898
```

## ■ Feladat

Ax=a, ahol A= $\begin{pmatrix} 7 & -2 & 1 & 2 \\ 2 & 8 & 3 & 1 \\ -1 & 0 & 5 & 2 \\ 0 & 2 & -1 & 4 \end{pmatrix}$ a= $\begin{pmatrix} 3 \\ -2 \\ 5 \\ 4 \end{pmatrix}$

Oldd meg a Jacobi iterációs módszerrel!

$$A = \begin{pmatrix} 7 & -2 & 1 & 2 \\ 2 & 8 & 3 & 1 \\ -1 & 0 & 5 & 2 \\ 0 & 2 & -1 & 4 \end{pmatrix}; a = \begin{pmatrix} 3 \\ -2 \\ 5 \\ 4 \end{pmatrix};$$

```
Det[A]
```

1450

```
Clear[Di, L, U]
```

$$L = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 2 & -1 & 0 \end{pmatrix};$$

$$U = \begin{pmatrix} 0 & -2 & 1 & 2 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix};$$

```
Di = DiagonalMatrix[{7, 8, 5, 4}]
```

{{7, 0, 0, 0}, {0, 8, 0, 0}, {0, 0, 5, 0}, {0, 0, 0, 4}}

```
{L + U + Di // MatrixForm, A // MatrixForm}
```

$$\left\{ \begin{pmatrix} 7 & -2 & 1 & 2 \\ 2 & 8 & 3 & 1 \\ -1 & 0 & 5 & 2 \\ 0 & 2 & -1 & 4 \end{pmatrix}, \begin{pmatrix} 7 & -2 & 1 & 2 \\ 2 & 8 & 3 & 1 \\ -1 & 0 & 5 & 2 \\ 0 & 2 & -1 & 4 \end{pmatrix} \right\}$$

```
B = -Inverse[Di].(L + U); b = Inverse[Di].a;

f[x_] := B.x + b

B
```

$$\left\{\left\{0, \frac{2}{7}, -\frac{1}{7}, -\frac{2}{7}\right\}, \left\{-\frac{1}{4}, 0, -\frac{3}{8}, -\frac{1}{8}\right\}, \left\{\frac{1}{5}, 0, 0, -\frac{2}{5}\right\}, \left\{0, -\frac{1}{2}, \frac{1}{4}, 0\right\}\right\}$$

```
b = Flatten[b, 1]
```

$$\left\{\frac{3}{7}, -\frac{1}{4}, 1, 1\right\}$$

```
NestList[f, b, 15] // N
```

{{0.428571, -0.25, 1., 1.}, {-0.0714286, -0.857143, 0.685714, 1.375},
{-0.307143, -0.661161, 0.435714, 1.6}, {-0.279719, -0.536607, 0.298571, 1.43951},
{-0.178686, -0.471973, 0.368253, 1.34295}, {-0.142585, -0.511291, 0.427084, 1.32805},
{-0.157967, -0.540517, 0.440263, 1.36242}, {-0.178019, -0.545909, 0.42344, 1.38032},
{-0.182272, -0.536826, 0.412267, 1.37881}, {-0.17765, -0.531384, 0.41202, 1.37148},
{-0.173964, -0.53153, 0.415878, 1.3687}, {-0.173762, -0.53355, 0.417729, 1.36973},
{-0.1749, -0.534425, 0.417354, 1.37121}, {-0.175517, -0.534184, 0.416537, 1.37155},
{-0.175429, -0.533766, 0.416276, 1.37123}, {-0.17518, -0.53365, 0.416424, 1.37095}}

Utolsó eleme a listának, utolsó iterált.

```
Last[%]
```

{-0.17518, -0.53365, 0.416424, 1.37095}

```
LinearSolve[A, a] // N
```

{{-0.175172}, {-0.533793}, {0.416552}, {1.37103}}

```
LinearSolve[A, a] // TableForm
```

# GS-Iteráció

- **Feladat**

Ax=a, ahol A= $\begin{pmatrix} 7 & -2 & 1 & 2 \\ 2 & 8 & 3 & 1 \\ -1 & 0 & 5 & 2 \\ 0 & 2 & -1 & 4 \end{pmatrix}$ a= $\begin{pmatrix} 3 \\ -2 \\ 5 \\ 4 \end{pmatrix}$

Oldd meg a Gauss-Seidel iterációs módszerrel!

```
A = ( 7  -2  1  2
      2   8  3  1
     -1   0  5  2
      0   2 -1  4 ); a = ( 3
                          -2
                           5
                           4 );

Det[A]

Clear[Di, L, U]

L = ( 0  0  0  0
      2  0  0  0
     -1  0  0  0
      0  2 -1  0 );

U = ( 0 -2  1  2
      0  0  3  1
      0  0  0  2
      0  0  0  0 );

Di = DiagonalMatrix[{7, 8, 5, 4}]

{L + U + Di // MatrixForm, A // MatrixForm}

B = -Inverse[Di + L].(U); b = Inverse[Di + L].a;


f[x_] := B.x + b

B

b = Flatten[b, 1]

NestList[f, b, 8] // N
```

Összevetés Jacobival.

{-0.182272, -0.536826, 0.412267, 1.37881}

Összevetés a Mathematica bép. megoldó rutinjával.

```
LinearSolve[A, a] // N

LinearSolve[A, a]
```

# diag dom, partició

In[7]:=
```
SDM[n_] := {PadRight[{4, -1}, n],
  Sequence @@ Table[ReplacePart[Table[0, {n}], {j - 1 → -1, j -> 4, j + 1 → -1}], {j, 2, n - 1}],
  PadLeft[{-1, 4}, n]}
Table[MatrixForm[SDM[n]], {n, 3, 6}]
```

$$\left\{\begin{pmatrix} 4 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 4 \end{pmatrix}, \begin{pmatrix} 4 & -1 & 0 & 0 \\ -1 & 4 & -1 & 0 \\ 0 & -1 & 4 & -1 \\ 0 & 0 & -1 & 4 \end{pmatrix}, \begin{pmatrix} 4 & -1 & 0 & 0 & 0 \\ -1 & 4 & -1 & 0 & 0 \\ 0 & -1 & 4 & -1 & 0 \\ 0 & 0 & -1 & 4 & -1 \\ 0 & 0 & 0 & -1 & 4 \end{pmatrix}, \begin{pmatrix} 4 & -1 & 0 & 0 & 0 & 0 \\ -1 & 4 & -1 & 0 & 0 & 0 \\ 0 & -1 & 4 & -1 & 0 & 0 \\ 0 & 0 & -1 & 4 & -1 & 0 \\ 0 & 0 & 0 & -1 & 4 & -1 \\ 0 & 0 & 0 & 0 & -1 & 4 \end{pmatrix}\right\}$$

```
Table[Det[SDM[n]], {n, 3, 10}]
Table[Eigenvalues[SDM[n]], {n, 3, 10}]
```

$$\left\{\left\{4+\sqrt{2}, 4, 4-\sqrt{2}\right\},\right.$$
$$\left\{\frac{1}{2}\left(9+\sqrt{5}\right), \frac{1}{2}\left(7+\sqrt{5}\right), \frac{1}{2}\left(9-\sqrt{5}\right), \frac{1}{2}\left(7-\sqrt{5}\right)\right\}, \left\{4+\sqrt{3}, 5, 4, 3, 4-\sqrt{3}\right\},$$

```
{Root[-71 + 54 #1 - 13 #1^2 + #1^3 &, 3], Root[-41 + 38 #1 - 11 #1^2 + #1^3 &, 3],
 Root[-71 + 54 #1 - 13 #1^2 + #1^3 &, 2], Root[-41 + 38 #1 - 11 #1^2 + #1^3 &, 2],
 Root[-71 + 54 #1 - 13 #1^2 + #1^3 &, 1], Root[-41 + 38 #1 - 11 #1^2 + #1^3 &, 1]},
```

$$\left\{4+\sqrt{2+\sqrt{2}}, 4+\sqrt{2}, 4+\sqrt{2-\sqrt{2}}, 4, 4-\sqrt{2-\sqrt{2}}, 4-\sqrt{2}, 4-\sqrt{2+\sqrt{2}}\right\},$$

```
{Root[-53 + 45 #1 - 12 #1^2 + #1^3 &, 3], Root[-51 + 45 #1 - 12 #1^2 + #1^3 &, 3],
 5, Root[-51 + 45 #1 - 12 #1^2 + #1^3 &, 2], Root[-53 + 45 #1 - 12 #1^2 + #1^3 &, 2],
 3, Root[-53 + 45 #1 - 12 #1^2 + #1^3 &, 1], Root[-51 + 45 #1 - 12 #1^2 + #1^3 &, 1]},
```

$$\left\{\frac{1}{2}\left(8+\sqrt{2\left(5+\sqrt{5}\right)}\right), \frac{1}{2}\left(9+\sqrt{5}\right), \frac{1}{2}\left(8+\sqrt{2\left(5-\sqrt{5}\right)}\right), \frac{1}{2}\left(7+\sqrt{5}\right), 4,\right.$$
$$\left.\frac{1}{2}\left(9-\sqrt{5}\right), \frac{1}{2}\left(8-\sqrt{2\left(5-\sqrt{5}\right)}\right), \frac{1}{2}\left(7-\sqrt{5}\right), \frac{1}{2}\left(8-\sqrt{2\left(5+\sqrt{5}\right)}\right)\right\},$$

```
{4 + √(16 + Root[564719 + 200623 #1 + 28445 #1^2 + 2012 #1^3 + 71 #1^4 + #1^5 &, 5]),
 4 + √(16 + Root[564719 + 200623 #1 + 28445 #1^2 + 2012 #1^3 + 71 #1^4 + #1^5 &, 4]),
 4 + √(16 + Root[564719 + 200623 #1 + 28445 #1^2 + 2012 #1^3 + 71 #1^4 + #1^5 &, 3]),
 4 + √(16 + Root[564719 + 200623 #1 + 28445 #1^2 + 2012 #1^3 + 71 #1^4 + #1^5 &, 2]),
 4 + √(16 + Root[564719 + 200623 #1 + 28445 #1^2 + 2012 #1^3 + 71 #1^4 + #1^5 &, 1]),
 4 - √(16 + Root[564719 + 200623 #1 + 28445 #1^2 + 2012 #1^3 + 71 #1^4 + #1^5 &, 1]),
 4 - √(16 + Root[564719 + 200623 #1 + 28445 #1^2 + 2012 #1^3 + 71 #1^4 + #1^5 &, 2]),
 4 - √(16 + Root[564719 + 200623 #1 + 28445 #1^2 + 2012 #1^3 + 71 #1^4 + #1^5 &, 3]),
 4 - √(16 + Root[564719 + 200623 #1 + 28445 #1^2 + 2012 #1^3 + 71 #1^4 + #1^5 &, 4]),
 4 - √(16 + Root[564719 + 200623 #1 + 28445 #1^2 + 2012 #1^3 + 71 #1^4 + #1^5 &, 5])}}
```

```
N[%]
```

```
{{5.41421, 4., 2.58579}, {5.61803, 4.61803, 3.38197, 2.38197},
 {5.73205, 5., 4., 3., 2.26795}, {5.80194, 5.24698, 4.44504, 3.55496, 2.75302, 2.19806},
 {5.84776, 5.41421, 4.76537, 4., 3.23463, 2.58579, 2.15224},
 {5.87939, 5.53209, 5., 4.3473, 3.6527, 3., 2.46791, 2.12061},
 {5.90211, 5.61803, 5.17557, 4.61803, 4., 3.38197, 2.82443, 2.38197, 2.09789},
 {5.91899, 5.68251, 5.30972, 4.83083, 4.28463, 3.71537, 3.16917, 2.69028, 2.31749, 2.08101}}
```

In[8]:=
```
PartJac[A_] := {UpperTriangularize[A, 1],
  LowerTriangularize[A, -1], DiagonalMatrix[Table[A[[j, j]], {j, Length[A]}]]}
```

```
(A = Table[w[j, k], {j, 4}, {k, 4}]) // MatrixForm
```

$$\begin{pmatrix} w[1,1] & w[1,2] & w[1,3] & w[1,4] \\ w[2,1] & w[2,2] & w[2,3] & w[2,4] \\ w[3,1] & w[3,2] & w[3,3] & w[3,4] \\ w[4,1] & w[4,2] & w[4,3] & w[4,4] \end{pmatrix}$$

```
MatrixForm[UpperTriangularize[A, 4]]
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```
Map[MatrixForm, PartJac[SDM[5]]]
```

$$\left\{\begin{pmatrix} 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \end{pmatrix}, \begin{pmatrix} 4 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{pmatrix}\right\}$$

In[13]:=
```
PartJac[SDM[7]]
```

Out[13]=
```
{{{0, -1, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0},
  {0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, -1}, {0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0},
  {0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, -1, 0}},
 {{4, 0, 0, 0, 0, 0, 0}, {0, 4, 0, 0, 0, 0, 0}, {0, 0, 4, 0, 0, 0, 0}, {0, 0, 0, 4, 0, 0, 0},
  {0, 0, 0, 0, 4, 0, 0}, {0, 0, 0, 0, 0, 4, 0}, {0, 0, 0, 0, 0, 0, 4}}}
```

In[14]:=
```
Map[MatrixForm, %]
```

Out[14]=

$$\left\{\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}, \begin{pmatrix} 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 \end{pmatrix}\right\}$$

In[10]:=
```
Eigenvalues[-Inverse[%[[3]]].(%[[1]] + %[[2]])]
```

Out[10]=

$$\left\{-\frac{1}{4}\sqrt{2+\sqrt{2}}, \frac{\sqrt{2+\sqrt{2}}}{4}, -\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}, -\frac{1}{4}\sqrt{2-\sqrt{2}}, \frac{\sqrt{2-\sqrt{2}}}{4}, 0\right\}$$

In[11]:=
```
N[%]
```

Out[11]=
```
{-0.46194, 0.46194, -0.353553, 0.353553, -0.191342, 0.191342, 0.}
```

GS explicit (n=3) [$x_j = x_j^{(m-1)}$, $y_j = x_j^{(m)}$ átnevezéssel]

In[15]:=
```
-DiagonalMatrix[1 / {a11, a22, a33}].({{0, a12, a13}, {0, 0, a23}, {0, 0, 0}}.{y1, y2, y3} +
    {{0, 0, 0}, {a21, 0, 0}, {a31, a32, 0}}.{x1, x2, x3} - {a14, a24, a34})
```

Out[15]=

$$\left\{-\frac{-a14 + a12\ y2 + a13\ y3}{a11}, -\frac{-a24 + a21\ x1 + a23\ y3}{a22}, -\frac{-a34 + a31\ x1 + a32\ x2}{a33}\right\}$$

Jacobi explicit

In[16]:=
```
% /. {y1 → x1, y2 -> x2, y3 -> x3}
```

Out[16]=

$$\left\{-\frac{-a14 + a12\ x2 + a13\ x3}{a11}, -\frac{-a24 + a21\ x1 + a23\ x3}{a22}, -\frac{-a34 + a31\ x1 + a32\ x2}{a33}\right\}$$

Példa ||A|| definíció alapján közelítve, ill mint felt. optimalizálási probléma megoldása

```
In[1]:= A = ({{12, -2, 1}, {-2, 15, 3}, {1, -2, 10}});

In[2]:= T = Table[Table[Random[Real, {-1, 1}], {3}], {10^5}];

In[3]:= Max[Map[Norm[A.#] / Norm[#] &, T]]

Out[3]= 16.193

In[4]:= NMaximize[{Norm[A.{x1, x2, x3}], x1^2 + x2^2 + x3^2 == 1}, {x1, x2, x3}]

Out[4]= {16.1931, {x1 → -0.43126, x2 → 0.897697, x3 → 0.0903124}}

In[5]:= Maximize[{Norm[A.{x1, x2, x3}], x1^2 + x2^2 + x3^2 == 1}, {x1, x2, x3}]
```

Out[5]= $\left\{\sqrt{-\text{Root}\left[3294225 + 72816\ \#1 + 492\ \#1^2 + \#1^3\ \&,\ 1\right]}\ ,\right.$
$\left\{x1 \to \text{Root}\left[-3459321856 + 37626759296\ \#1^2 - 125676752925\ \#1^4 + 125676752925\ \#1^6\ \&,\ 3\right],\right.$
$x2 \to \text{Root}\left[-939299904 + 20827845521\ \#1^2 - 125676752925\ \#1^4 + 125676752925\ \#1^6\ \&,\ 6\right],$
$\left.\left. x3 \to \text{Root}\left[-9486400 + 1203778745\ \#1^2 - 5027070117\ \#1^4 + 5027070117\ \#1^6\ \&,\ 4\right]\right\}\right\}$