Ábrázolások 2

■ 4

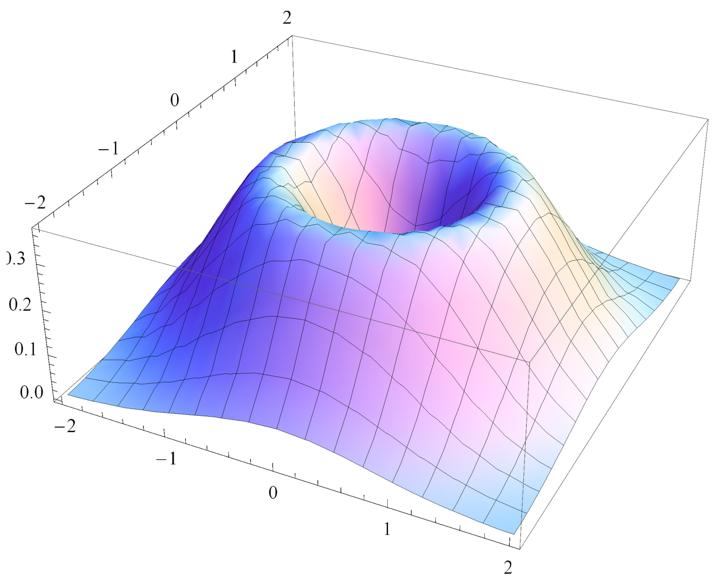
Ábrázoljuk egy kétváltozós függvény szintvonalait, ill. parciális deriváltjainak 0-szintvonalait! Oldjuk meg szimbolikusan/numerikusan a stacionárius pontokat karakterizáló egyenletrendszert! (ÉT!)

F[x_ , y_]:=(x^2 + y^2) Exp[-x^2 - y^2]
G[x_ , y_]:=(10x y+x^2+3y^2) Exp[1-x^2-y^2]
H[x_, y_]:=-2 Sqrt[x^2+y^2]+x^4+y^2 - 0.4 x

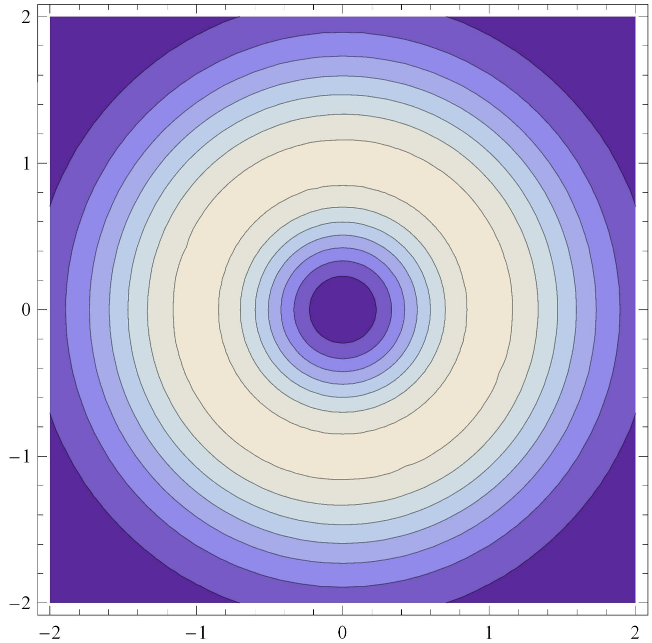
In[2]:=
```
F[x_ , y_ ] := (x^2 + y^2) Exp[-x^2 - y^2]
```

In[3]:=
```
Plot3D[F[x, y], {x, -2, 2}, {y, -2, 2}]
```

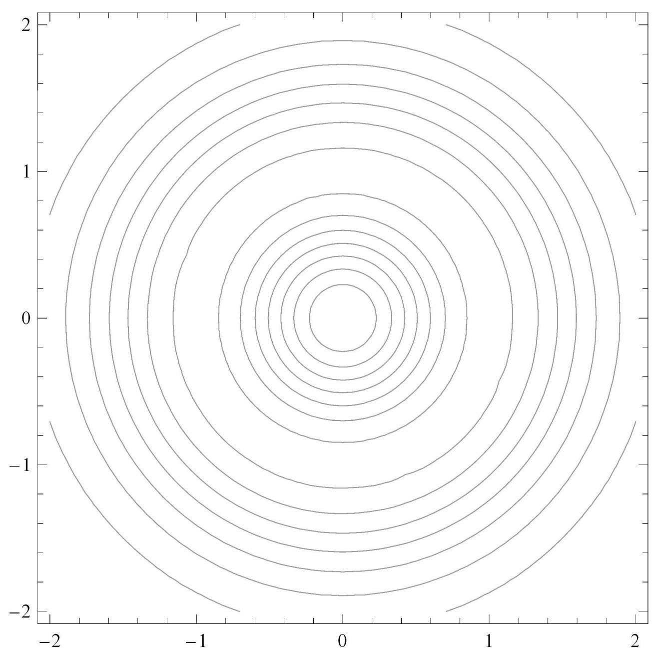
Out[3]=



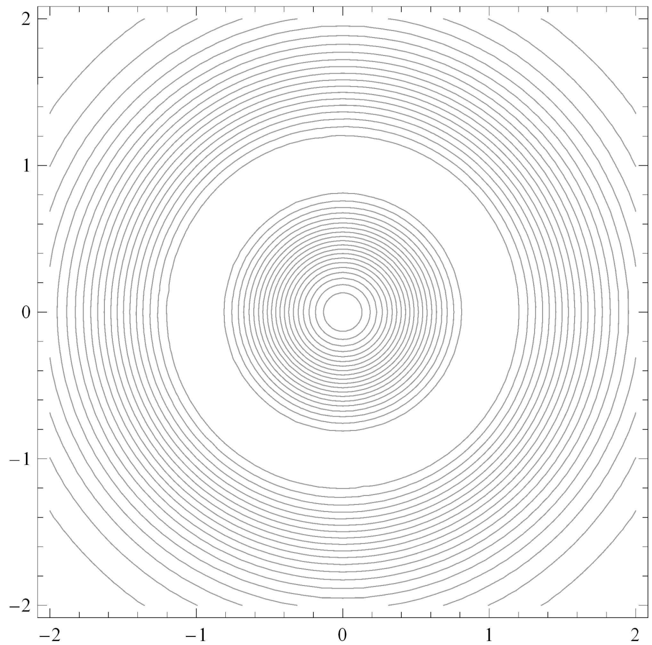
```
ContourPlot[F[x, y], {x, -2, 2}, {y, -2, 2}]
```




```
ContourPlot[F[x, y], {x, -2, 2}, {y, -2, 2}, ContourShading → False]
```



```
ContourPlot[F[x, y], {x, -2, 2}, {y, -2, 2}, ContourShading → False, Contours → 20]
```

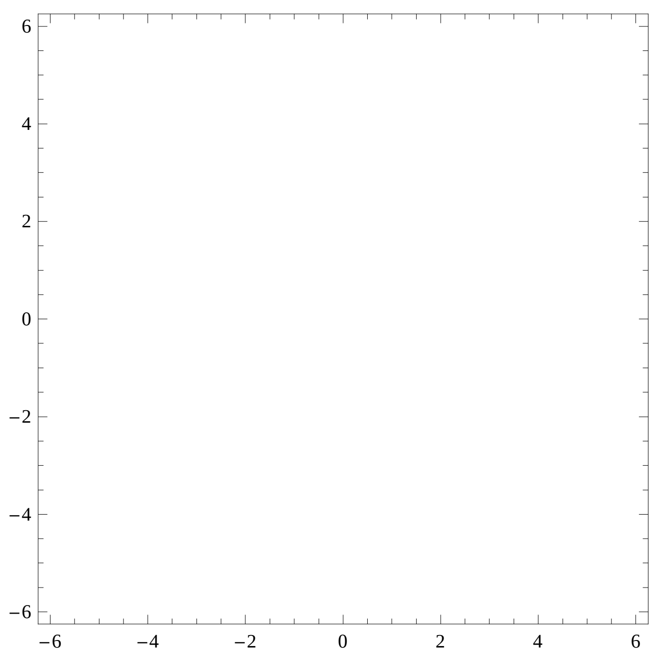


Vigyázat!

```
ContourPlot[F[x, y], {x, -3, 3}, {y, -3, 3}, ContourShading -> False, Contours → {1 / 10, 1 / 20}]
```

In[4]:=
```
ContourPlot[F[x, y], {x, -6, 6}, {y, -6, 6}, ContourShading -> False, Contours → {1 / 10, 1 / 20}]
```

Out[4]=



Parciális deriváltak szimbolikusan

```
D[F[x, y], x]
```

$2\, e^{-x^2-y^2} x - 2\, e^{-x^2-y^2} x \left(x^2 + y^2\right)$

```
Solve[{D[F[x, y], x] == 0, D[F[x, y], y] == 0}, {x, y}]
```

InverseFunction::ifun : Inverse functions are being used. Values may be lost for multivalued inverses. ≫

InverseFunction::ifun : Inverse functions are being used. Values may be lost for multivalued inverses. ≫

Solve::incnst : Inconsistent or redundant transcendental equation. After reduction, the bad equation is −InverseFunction[♯1 ℯ^♯1 &, 1, 1][ℯ^≪1≫ (≪1≫)] == 0. ≫

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. ≫

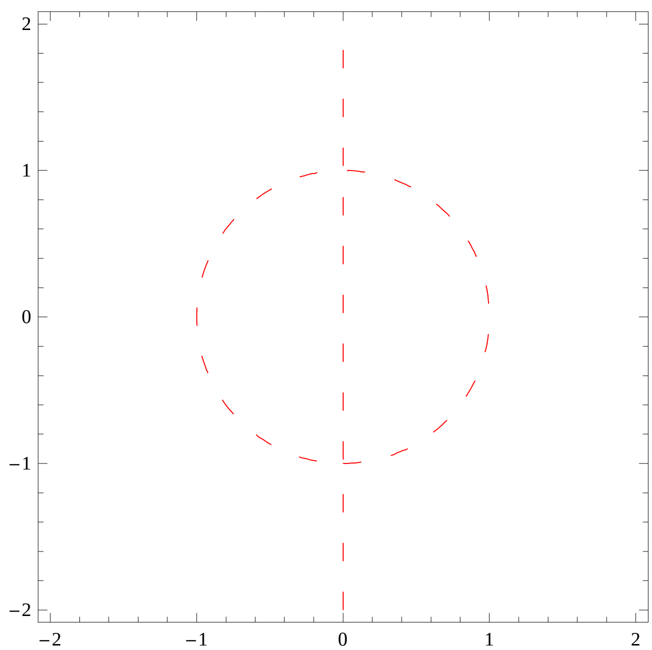
Solve::svars : Equations may not give solutions for all "solve" variables. ≫

$\{\{x \to -1, y \to 0\}, \{x \to 0, y \to 0\}, \{x \to 1, y \to 0\}, \{y \to -1, x \to 0\}, \{y \to 1, x \to 0\}, \{x \to -\sqrt{1 - y^2}\}, \{x \to \sqrt{1 - y^2}\}\}$

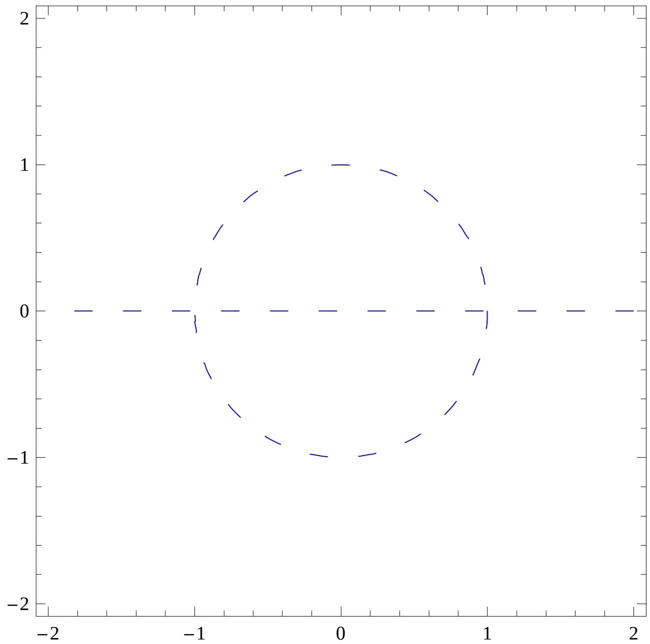
```
Factor[D[F[x, y], x]]
```

$-2\, e^{-x^2-y^2} x \left(-1 + x^2 + y^2\right)$

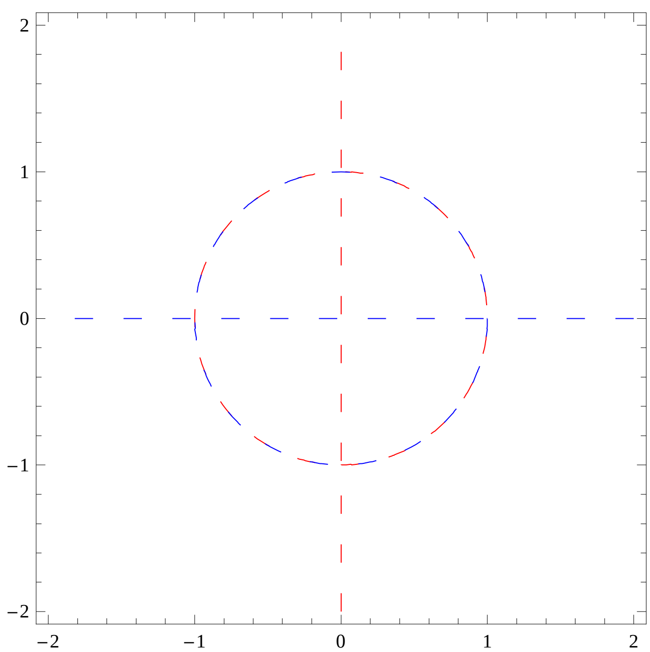
```
g1 = ContourPlot[Evaluate[D[F[x, y], x]], {x, -2, 2}, {y, -2, 2}, Contours → {0},
  ContourShading → False, ContourStyle → {Red, Dashing[{0.03, 0.05}]}]
```



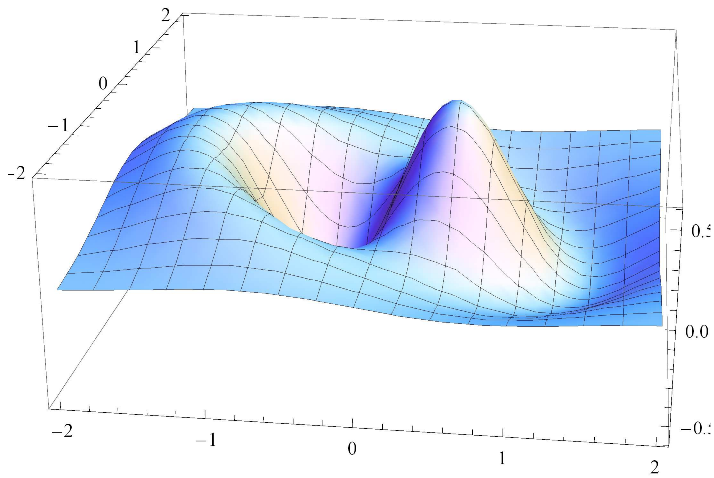
```
g2 = ContourPlot[Evaluate[D[F[x, y], y]], {x, -2, 2}, {y, -2, 2}, Contours → {0},
  ContourShading → False, ContourStyle → {Blue, Dashing[{0.03, 0.05}]}]
```




```
Show[g1, g2]
```



```
Plot3D[Evaluate[D[F[x, y], x]], {x, -2, 2}, {y, -2, 2}]
```




```
? FindMinimum
```

FindMinimum[f, {x, x_0}] searches for a local minimum in f, starting from the point $x = x_0$.
FindMinimum[f, {{x, x_0}, {y, y_0}, …}] searches for a local minimum in a function of several variables.
FindMinimum[{f, *cons*}, {{x, x_0}, {y, y_0}, …}] searches for a local minimum subject to the constraints *cons*.
FindMinimum[{f, *cons*}, {x, y, …}] starts from a point within the region defined by the constraints. ≫

```
FindMinimum[F[x, y], {{x, 1 / 2}, {y, 1 / 2}}] // Chop
```

```
{0, {x → 0, y → 0}}
```

```
? Chop
```

Chop[*expr*] replaces approximate real numbers in *expr* that are close to zero by the exact integer 0. ≫

■ 5

Programozás: listák, minták, helyettesítési szabályok, scoping
Példa: Adjuk meg/konstruáljuk a Taylor polinomot a deriváltak segítségével

```
5 !
```

```
120
```

```
D[x^3, {x, 2}]
```

```
6 x
```

```
x^3 /. x → 3
```

```
27
```

```
Sum[k, {k, 1, 10}]
```

55

```
Sum[k, {k, 1, n}]
```

$\frac{1}{2} n (1 + n)$

```
Sum[1/k^2, {k, 2, Infinity}] // Expand
```

$-1 + \frac{\pi^2}{6}$

$\sum_{k=1}^{n} k$

$\frac{1}{2} n (1 + n)$

Függvénydef.

In[6]:=

```
MyTaylor[expr_, x0_, n_, var_] := Sum[(D[expr, {var, k}] /. var → x0) / k! (var - x0)^k, {k, 0, n}]
```

Hívások

```
MyTaylor[Sin[x], 0, 5, x]
```

$x - \frac{x^3}{6} + \frac{x^5}{120}$

In[7]:= `MyTaylor[Exp[x], 0, 5, x]`

Out[7]= $1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120}$

■ 6

(Lineáris) egyenlet, egyenletrendszer, differencia és differeciálegyenlet, differencia és differeciálegyenletrendszer

```
Solve[x^2 - 5 x + 6 == 0, x]
```

{{x → 2}, {x → 3}}

Feladat

$x_1 + x_2 + x_3 = 7$
$4 x_1 + 3 x_2 + 4 x_3 = 8$
$9 x_1 + 3 x_2 + 4 x_3 = 3$

```
Solve[{x1 + x2 + x3 == 7, 4 x1 + 3 x2 + 4 x3 == 8, 9 x1 + 3 x2 + 4 x3 == 3}, {x1, x2, x3}]
```

$\{\{x_1 \to -1,\ x_2 \to 20,\ x_3 \to -12\}\}$

```
In[12]:= A = {{1, 1, 1}, {4, 3, 4}, {9, 3, 4}}

Out[12]= {{1, 1, 1}, {4, 3, 4}, {9, 3, 4}}
```

In[9]:= $B = \begin{pmatrix} 1 & 1 & 1 \\ 4 & 3 & 4 \\ 9 & 3 & 4 \end{pmatrix};$

```
In[13]:= LinearSolve[A, {7, 8, 3}]

Out[13]= {-1, 20, -12}

A2 = {{1, 1, 1, 7}, {4, 3, 4, 8}, {9, 3, 4, 3}}

{{1, 1, 1, 7}, {4, 3, 4, 8}, {9, 3, 4, 3}}

RowReduce[A2] // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 20 \\ 0 & 0 & 1 & -12 \end{pmatrix}$$

```
MatrixForm[A2]
```

$$\begin{pmatrix} 1 & 1 & 1 & 7 \\ 4 & 3 & 4 & 8 \\ 9 & 3 & 4 & 3 \end{pmatrix}$$

```
Det[A]

5

RowReduce[A] // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
LinearSolve[A, {7, 8, 3}]

{-1, 20, -12}

NSolve[x^2 + 2 == 0, x]
```

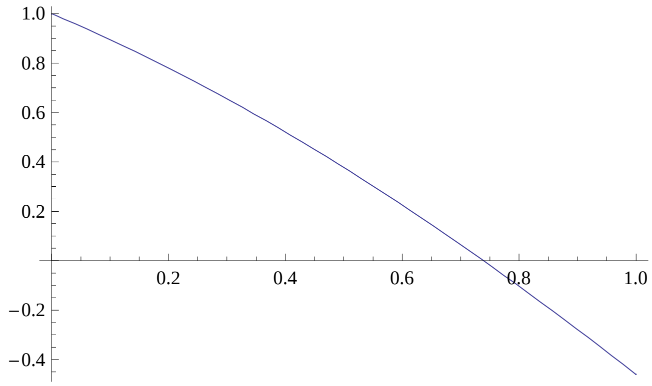
$\{\{x \to -3.64678 \times 10^{-27} - 1.41421\, i\},\ \{x \to -3.64678 \times 10^{-27} + 1.41421\, i\}\}$

```
NSolve[Cos[x] == x, x]
```

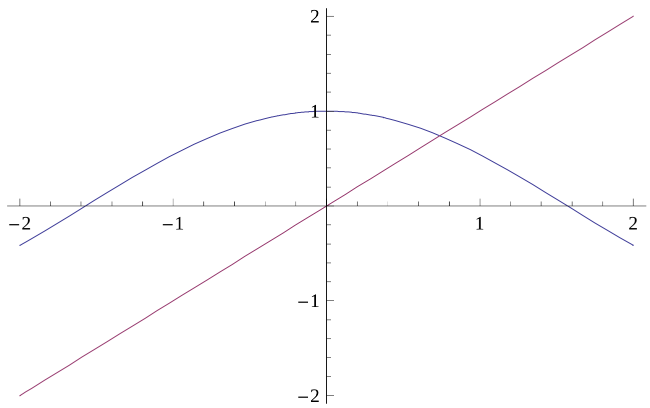
Solve::tdep : The equations appear to involve the variables to be solved for in an essentially non−algebraic way. ≫

```
NSolve[Cos[x] == x, x]

FindRoot[Cos[x] - x, {x, 1}]

{x → 0.739085}
```

In[14]:=
```
Plot[Cos[x] - x, {x, 0, 1}]
```

Out[14]=


In[15]:=
```
Plot[{Cos[x], x}, {x, -2, 2}]
```

Out[15]=


In[16]:=
```
a[1] = 1;
a[n_] := 2 a[n - 1];
```

In[18]:=
```
a[5]
```

Out[18]= 16

In[19]:=
```
On[a]
```

In[20]:= a[5]

a::trace : a[5] --> 2 a[5 - 1]. ≫

a::trace : a[5 - 1] --> a[4]. ≫

a::trace : a[4] --> 2 a[4 - 1]. ≫

a::trace : a[4 - 1] --> a[3]. ≫

a::trace : a[3] --> 2 a[3 - 1]. ≫

a::trace : a[3 - 1] --> a[2]. ≫

a::trace : a[2] --> 2 a[2 - 1]. ≫

a::trace : a[2 - 1] --> a[1]. ≫

a::trace : a[1] --> 1. ≫

Out[20]= 16

In[21]:= Clear[a]

In[22]:= Off[a]

In[23]:= RSolve[{a[n] == 2 a[n - 1] + a[n - 2], a[1] == 1, a[2] == 3}, a[n], n]

Out[23]= $\left\{\left\{a[n] \to \frac{1}{2}\left(\left(1-\sqrt{2}\right)^n+\left(1+\sqrt{2}\right)^n\right)\right\}\right\}$

In[24]:= DSolve[y'[x] == y[x], y[x], x]

Out[24]= $\{\{y[x] \to e^x\, C[1]\}\}$

In[25]:= DSolve[{y'[x] == y[x], y[0] == 1}, y[x], x]

Out[25]= $\{\{y[x] \to e^x\}\}$

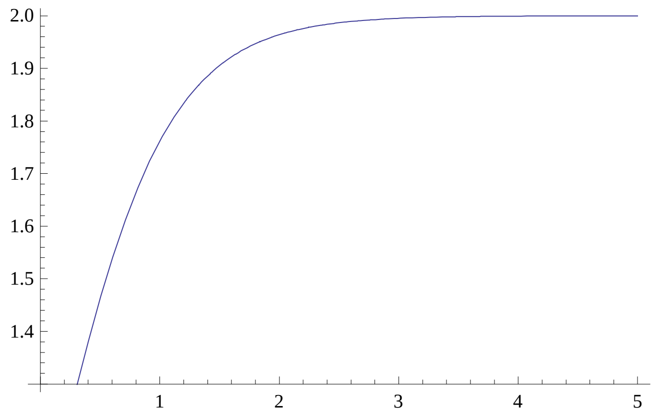
In[26]:= y[x] /. DSolve[{y'[x] == y[x] (2 - y[x]), y[0] == 1}, y[x], x][[1]]

Solve::ifun : Inverse functions are being used by Solve, so
some solutions may not be found; use Reduce for complete solution information. ≫

Out[26]= $\frac{2\, e^{2x}}{1+e^{2x}}$

In[27]:= `Plot[%, {x, 0, 5}]`

Out[27]=

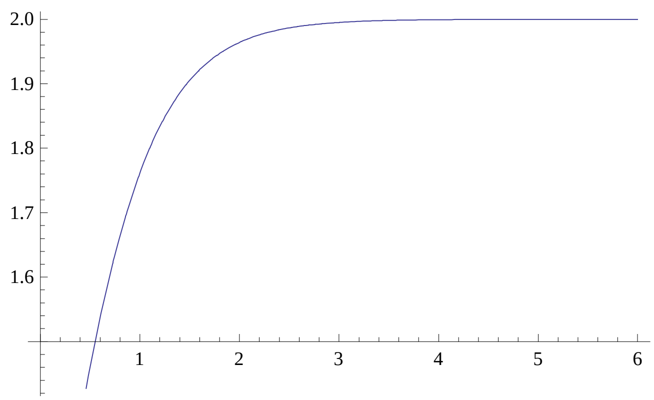


In[28]:= `y[x] /. NDSolve[{y'[x] == y[x] (2 - y[x]), y[0] == 1}, y[x], {x, 0, 6}][[1]]`

Out[28]= `InterpolatingFunction[{{0., 6.}}, <>][x]`

In[29]:= `Plot[%, {x, 0, 6}]`

Out[29]=



In[30]:= `{4, 5, 6}[[3]]`

Out[30]= 6

In[31]:=
```
Manipulate[
 Plot[Evaluate[y[x] /. NDSolve[{y'[x] == y[x] (2 - y[x]), y[0] == y0}, y[x], {x, 0, 6}][[1]]],
  {x, 0, 6}, PlotRange → {0, 3}], {y0, 0, 2, .1}]
```

Out[31]=