Modellezés — 2

Limesz, Derivált, Integrál

Direkt (normál) értékadás (=)
p legyen a 6. Chebysev polinom.

```
p = ChebyshevT[6, x]
```

$-1 + 18 x^2 - 48 x^4 + 32 x^6$

(Formális) derivált

```
D[p, x]
```

$36 x - 192 x^3 + 192 x^5$

```
D[Sin[x], x]
```

```
Cos[x]
```

```
D[Sin[x], {x, 2}]
```

```
-Sin[x]
```

```
D[Sin[x], x, x]
```

```
-Sin[x]
```

```
D[x^2 + y^2, x, y]
```

```
0
```

```
Table[D[Sin[x], {x, i}], {i, 0, 7}]
```

```
{Sin[x], Cos[x], -Sin[x], -Cos[x], Sin[x], Cos[x], -Sin[x], -Cos[x]}
```

```
TableForm[Table[D[Sin[x], {x, i}], {i, 0, 7}]]
```

```
Sin[x]
Cos[x]
-Sin[x]
-Cos[x]
Sin[x]
Cos[x]
-Sin[x]
-Cos[x]
```

```
D[x^2 + y^2, y]
```

```
2 y
```

Ugyanez, ha polinomfüggvényt definiálunk hozzárendelési szabállyal:

```
q[x_] := ChebyshevT[6, x];
```

```
q[1]
```

1

```
D[q[x], x]
```

$36 x - 192 x^3 + 192 x^5$

Határértékek

```
Limit[p, x → Infinity]
```

∞

```
Limit[(x^2 - 1) / (2 - x - 3 x^2), x → -Infinity]
```

$-\frac{1}{3}$

```
Limit[Sin[x] / x, x → 0]
```

1

```
p
```

$-1 + 18 x^2 - 48 x^4 + 32 x^6$

```
Reverse[CoefficientList[p, x]]
```

{32, 0, -48, 0, 18, 0, -1}

Integrálok

```
Integrate[x^2, x]
```

$\frac{x^3}{3}$

```
Integrate[x^2, {x, 0, 1}]
```

$\frac{1}{3}$

$\int E^{-x^2} \, dx$

$\frac{1}{2} \sqrt{\pi} \, \mathrm{Erf}[x]$

$\int_{-\infty}^{\infty} E^{-x^2} \, dx$

$\sqrt{\pi}$

$\mathrm{exp1} = \int_{-1}^{1} E^{-x^2} \, dx$

$\sqrt{\pi} \, \mathrm{Erf}[1]$

```
N[%]
```

1.49365

```
N[exp1]
```

1.49365

```
NIntegrate[E^(-x^2), {x, -1, 1}]
```

1.49365

```
Integrate[E^(-x^2), {x, -1, 1}]
```

$\sqrt{\pi}$ Erf[1]

$$\int_{-1}^{1} x^2 \, dx$$

$$\frac{2}{3}$$

Sorozatok, Függvények

Értékadás

```
A = 5;
? A
```

Global`A

A = 5

```
? B
```

Information::notfound : Symbol B not found. ≫

Table véges listák generálásához

```
Table[i^2, {i, 5}]
```

{1, 4, 9, 16, 25}

```
Random[]
```

0.125354

```
Clear[MyFunction]
```

függvény definíciója (n_ !)

```
MyFunction[n_List] := n^2 + 1
```

függvényhívás

```
MyFunction[5]
```

```
MyFunction[{1, 2, 3}]
```

{2, 5, 10}

```
Sin[{Pi / 2, Pi / 4, 0}]
```

$\left\{1, \frac{1}{\sqrt{2}}, 0\right\}$

Magyarázat az értékadáshoz '=' (Set) és ':=' (SetDelayed)
Normál és késleltetett értékadás.
Fontos ismerni a különbséget, hibák forrása lehet!

```
V1[] = Random[];
V2[] := Random[];
```

```
Table[V1[], {i, 5}]
```

{0.090674, 0.090674, 0.090674, 0.090674, 0.090674}

```
Table[V2[], {i, 5}]
```

{0.638092, 0.967085, 0.302409, 0.329627, 0.347381}

Sorozat: Hozzárendelési szabály vagy a képhalmaz egy véges szelete vagy grafikon
:= (SetDelayed, késleltetett értékadás)

$a[n\_] := \frac{n}{n^2 + 5};$

```
a[1]
```

$\frac{1}{6}$

```
a[2]
```

$\frac{2}{9}$

```
N[a[2]]
```

0.222222

Véges sorozatok generálása

```
Table[i^2, {i, 9, 12}]
```

{81, 100, 121, 144}

```
Table[i^3, {i, 1, 5, 2}]
```

{1, 27, 125}

Az a sorozat elsõ tíz eleme

```
Table[a[n], {n, 10}]
```

$\left\{\frac{1}{6}, \frac{2}{9}, \frac{3}{14}, \frac{4}{21}, \frac{1}{6}, \frac{6}{41}, \frac{7}{54}, \frac{8}{69}, \frac{9}{86}, \frac{2}{21}\right\}$

```
t = Table[{n, a[n]}, {n, 10}]
```

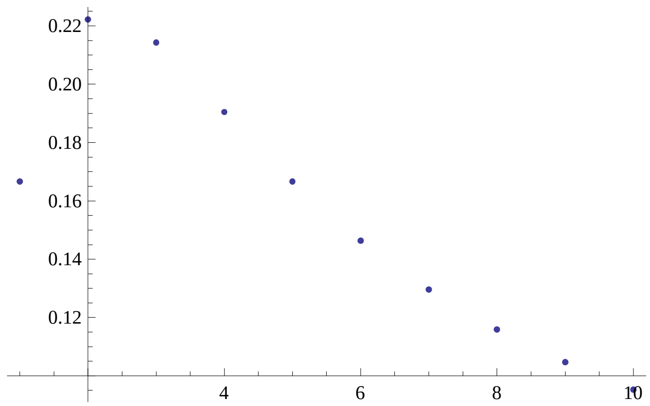
$\left\{\left\{1, \frac{1}{6}\right\}, \left\{2, \frac{2}{9}\right\}, \left\{3, \frac{3}{14}\right\}, \left\{4, \frac{4}{21}\right\}, \left\{5, \frac{1}{6}\right\}, \left\{6, \frac{6}{41}\right\}, \left\{7, \frac{7}{54}\right\}, \left\{8, \frac{8}{69}\right\}, \left\{9, \frac{9}{86}\right\}, \left\{10, \frac{2}{21}\right\}\right\}$

```
TableForm[t]
```

1	$\frac{1}{6}$
2	$\frac{2}{9}$
3	$\frac{3}{14}$
4	$\frac{4}{21}$
5	$\frac{1}{6}$
6	$\frac{6}{41}$
7	$\frac{7}{54}$
8	$\frac{8}{69}$
9	$\frac{9}{86}$
10	$\frac{2}{21}$

Ábrák

```
ListPlot[t]
```



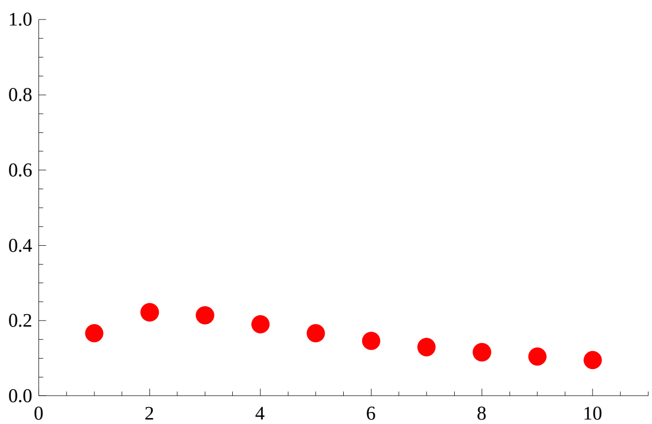

```
?ListPlot
```

ListPlot[{y_1, y_2, …}] plots points corresponding to a list of values, assumed to correspond to x coordinates 1, 2, ….
ListPlot[{{x_1, y_1}, {x_2, y_2}, …}] plots a list of points with specified x and y coordinates.
ListPlot[{$list_1$, $list_2$, …}] plots several lists of points. ≫

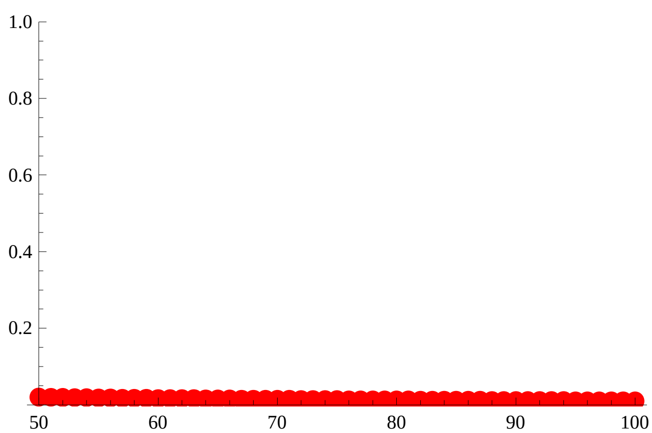
```
Options[ListPlot]
```

```
{AlignmentPoint → Center, AspectRatio → 1/GoldenRatio, Axes → True, AxesLabel → None,
 AxesOrigin → Automatic, AxesStyle → {}, Background → None, BaselinePosition → Automatic,
 BaseStyle → {}, ClippingStyle → None, ColorFunction → Automatic,
 ColorFunctionScaling → True, ColorOutput → Automatic, ContentSelectable → Automatic,
 DataRange → Automatic, DisplayFunction :→ $DisplayFunction, Epilog → {}, Filling → None,
 FillingStyle → Automatic, FormatType :→ TraditionalForm, Frame → False, FrameLabel → None,
 FrameStyle → {}, FrameTicks → Automatic, FrameTicksStyle → {}, GridLines → None,
 GridLinesStyle → {}, ImageMargins → 0., ImagePadding → All, ImageSize → Automatic,
 InterpolationOrder → None, Joined → False, LabelStyle → {}, MaxPlotPoints → ∞, Mesh → None,
 MeshFunctions → {#1 &}, MeshShading → None, MeshStyle → Automatic, Method → Automatic,
 PerformanceGoal :→ $PerformanceGoal, PlotLabel → None, PlotMarkers → None,
 PlotRange → Automatic, PlotRangeClipping → True, PlotRangePadding → Automatic,
 PlotRegion → Automatic, PlotStyle → Automatic, PreserveImageOptions → Automatic,
 Prolog → {}, RotateLabel → True, Ticks → Automatic, TicksStyle → {}}
```

```
ListPlot[t, PlotStyle → {RGBColor[1, 0, 0], PointSize[.03]},
 AxesOrigin → {0, 0}, PlotRange → {{0, 11}, {0, 1}}]
```




```
g = ListPlot[Table[{n, a[n]}, {n, 50, 100}],
  PlotStyle → {RGBColor[1, 0, 0], PointSize[.03]}, PlotRange → {{49, 101}, {0, 1}}]
```




```
g // InputForm
```

```
Graphics[{Hue[0.67, 0.6,
   0.6], RGBColor[1, 0, 0],
```

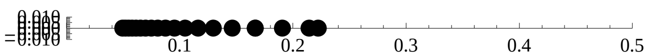
```
PointSize[0.03],
Point[{{50.,
 0.01996007984031936},
 {51.,
 0.019570222563315427},
 {52.,
 0.019195275009228498},
 {53.,
 0.018834399431414357},
 {54.,
 0.018486819582334817},
 {55.,
 0.018151815181518153},
 {56.,
 0.017828716969118114},
 {57.,
 0.017516902274124155},
 {58.,
 0.017215791035915702},
 {59.,
 0.016924842226047045},
 {60.,
 0.016643550624133148},
 {61.,
 0.01637144390767579},
 {62.,
 0.01610808002078462},
 {63.,
 0.015853044791142426},
 {64.,
 0.015605949768349184},
 {65.,
 0.015366430260047281},
 {66.,
 0.015134143545058473},
 {67.,
 0.014908767245215844},
 {68., 0.0146899978397062},
 {69.,
 0.014477549307595467},
 {70.,
 0.014271151885830785},
 {71.,
 0.014070550931430836},
 {72.,
 0.013875505877818462},
 {73.,
 0.013685789276340458},
 {74.,
 0.013501185914979019},
 {75.,
 0.013321492007104795},
 {76.,
 0.013146514443867843},
 {77.,
 0.012976070104482642},
 {78.,
 0.012809985219247824},
 {79.,
 0.012648094780659622},
 {80.,
 0.01249024199843872},
 {81.,
 0.012336277794699969},
 {82.,
```

```
      0.012186060335859712},
      {83.,
      0.012039454598201334},
      {84.,
      0.011896331964311004},
      {85.,
      0.011756569847856155},
      {86.,
      0.011620051344412918},
      {87.,
      0.011486664906258251},
      {88.,
      0.011356304039230868},
      {89.,
      0.011228867019934393},
      {90.,
      0.011104256631708822},
      {91.,
      0.010982379917933865},
      {92.,
      0.01086314795135199},
      {93.,
      0.010746475618211232},
      {94.,
      0.010632281416129397},
      {95.,
      0.010520487264673311},
      {96.,
      0.01041101832773018},
      {97.,
      0.01030380284682388},
      {98.,
      0.010198771984597774},
      {99.,
      0.010095859677748318},
      {100.,
      0.009995002498750625}}]},
    {AspectRatio -> GoldenRatio^
       (-1), Axes -> True,
     PlotRange -> {{49, 101},
       {0, 1}},
     PlotRangeClipping -> True}]
```

Grafikus objektumok ábrázolása

```
Show[Graphics[{PointSize[.02], Red, Point[{0, 0}]}], Graphics[Point[{1, 1}]]]
```

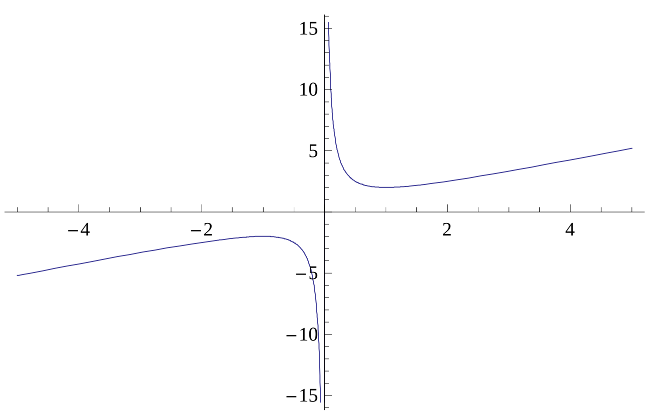
```
Show[Table[Graphics[{PointSize[.03], Point[{a[n], 0}]}], {n, 20}],
 Axes → True, PlotRange → {{0, 1 / 2}, {-.01, .01}}]
```



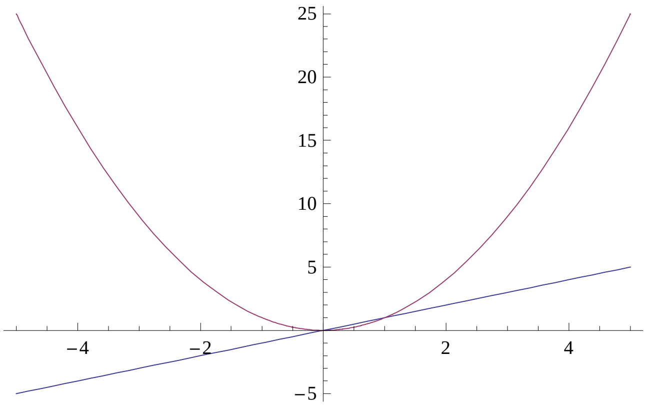
Függvény: Hozzárendelési szabály vagy grafikon

```
f[x_] := x + 1 / x;
```

```
Plot[f[x], {x, -5, 5}]
```



```
Plot[{x, x^2}, {x, -5, 5}]
```




```
MyFun[x_] := ArcTan[x];
```

```
MyDFun[x_] := D[MyFun[x], x];
```

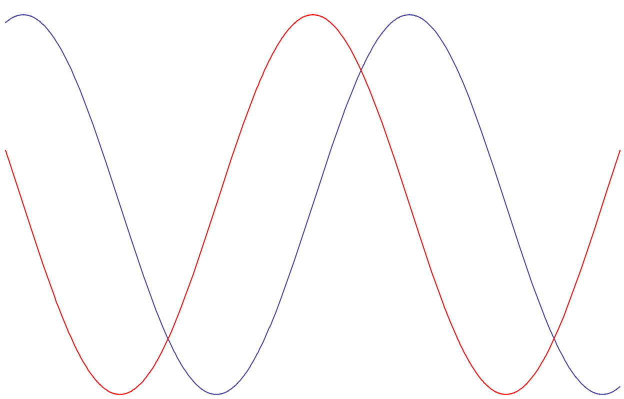
```
MyDFun[x]
```

$$\frac{1}{1 + x^2}$$

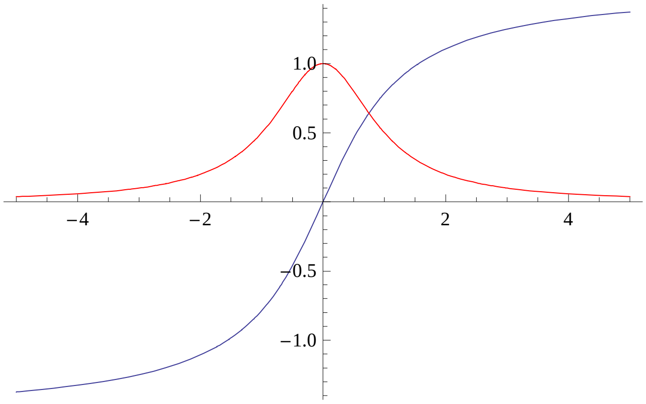
```
? Plot
```

```
?? Plot
```

```
Plot[{Sin[x], Cos[x]}, {x, -5, 5}, PlotStyle → {{}, {RGBColor[1, 0, 0]}}, Axes → False]
```



```
Plot[Evaluate[{MyFun[x], MyDFun[x]}], {x, -5, 5}, PlotStyle → {{}, {RGBColor[1, 0, 0]}}]
```



Feladatok

■ 1

?Hogyan lehetne az x^2+2y^2==1 és 2x^2+y^2==1 ellipszisek grafikonját felrajzolni és a metszetük területét kiszámolni?
Hint: pl. ContourPlot, RegionPlot,Impr. integrál, Boole

```
==

And[expr1, expr2]

Integrate[Integrate[Boole[2 x^2 + y^2 < 1 ∧ x^2 + 2 y^2 < 1], {y, -Infinity, Infinity}],
 {x, -Infinity, Infinity}]
```

$\sqrt{2}\ \mathrm{ArcCos}\left[\sqrt{\frac{2}{3}}\right] + \sqrt{2}\ \mathrm{ArcSin}\left[\frac{1}{\sqrt{3}}\right]$

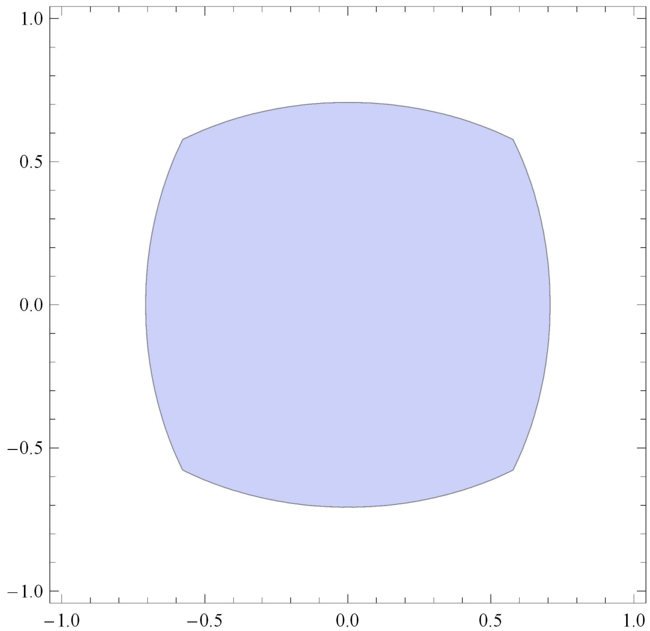
```
FullSimplify[%]
```

$\sqrt{2}\ \mathrm{ArcSec}[3]$

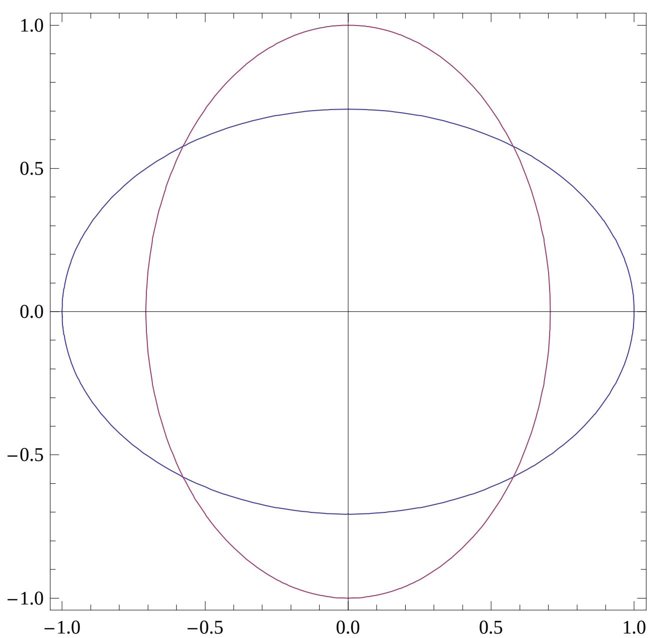
```
N[%]
```

1.74084

```
RegionPlot[x^2 + 2 y^2 <= 1 ∧ 2 x^2 + y^2 <= 1, {x, -1, 1}, {y, -1, 1}]
```




```
ContourPlot[{x^2 + 2 y^2 == 1, 2 x^2 + y^2 == 1}, {x, -1, 1}, {y, -1, 1}, Axes -> True]
```




```
8 (1 / 6 + Integrate[Integrate[1, {y, 0, Sqrt[1 - 2 x^2]}], {x, 1 / Sqrt[3], 1 / Sqrt[2]}])
```

$$8\left(\frac{1}{6}+\frac{1}{12}\left(-2+3\sqrt{2}\ \mathrm{ArcCos}\left[\sqrt{\frac{2}{3}}\right]\right)\right)$$

```
N[%]
```

1.74084

```
Solve[{x^2 + 2 y^2 == 1, 2 x^2 + y^2 == 1}, {x, y}]
```

$$\left\{\left\{x \to -\frac{1}{\sqrt{3}}, y \to -\frac{1}{\sqrt{3}}\right\}, \left\{x \to -\frac{1}{\sqrt{3}}, y \to \frac{1}{\sqrt{3}}\right\}, \left\{x \to \frac{1}{\sqrt{3}}, y \to -\frac{1}{\sqrt{3}}\right\}, \left\{x \to \frac{1}{\sqrt{3}}, y \to \frac{1}{\sqrt{3}}\right\}\right\}$$

■ 2

f[x]:=x/(x^2-5x+6) Ábrázoljuk, lok. glob, limeszek derivált mon., szimb. szám is! stb.

```
In[1]:= Clear[f]

In[2]:= f[x_] := x / (x^2 - 5 x + 6)

In[3]:= Solve[f[x] == 0, x]

Out[3]= {{x → 0}}
```

```
Options[Limit]
```

{Analytic → False, Assumptions :→ $Assumptions, Direction → Automatic}

féloldali limeszek

```
Limit[f[x], x → 3, Direction → -1]
```

∞

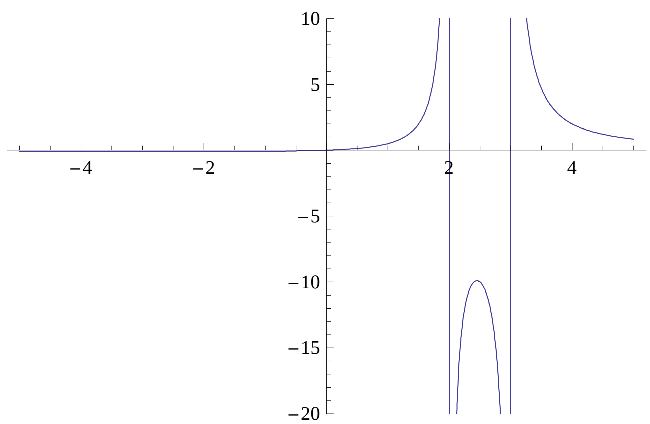
```
Limit[f[x], x → 3, Direction → 1]
```

$-\infty$

```
Solve[Denominator[f[x]] == 0, x]
```

{{x → 2}, {x → 3}}

```
Plot[f[x], {x, -5, 5}, PlotRange → {-20, 10}]
```




```
x /. Solve[D[f[x], x] == 0]
```

$\{-\sqrt{6}, \sqrt{6}\}$

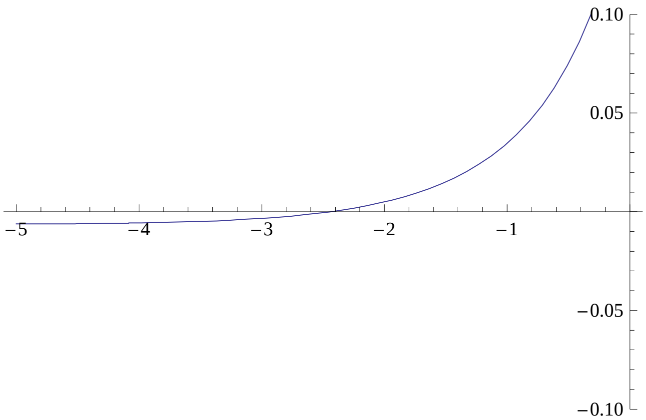
```
Reduce[D[f[x], x] > 0, x]
```

$-\sqrt{6} < x < 2 \,||\, 2 < x < \sqrt{6}$

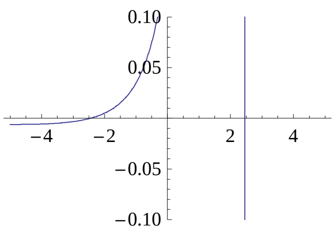
```
N[-Sqrt[6]]
```

-2.44949

```
Plot[Evaluate[D[f[x], x]], {x, -5, 0}, PlotRange → {-.1, .1}]
```

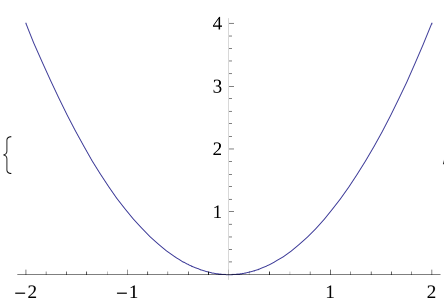
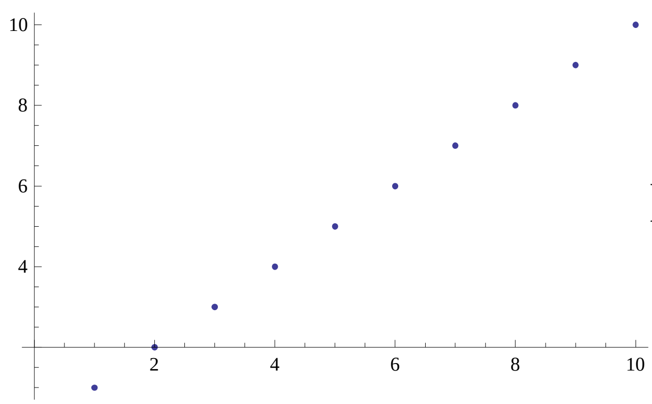



```
Plot[Evaluate[D[f[x], x]], {x, -5, 5}, PlotRange → {-.1, .1}]
```



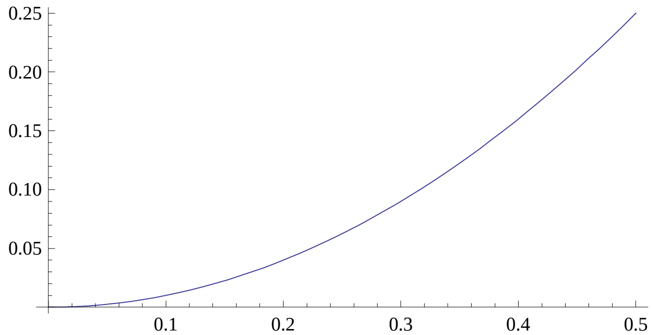
Ábrázolások

```
{Plot[x^2, {x, -2, 2}, ImageSize → {200, 200}], ListPlot[Range[10]]}
```

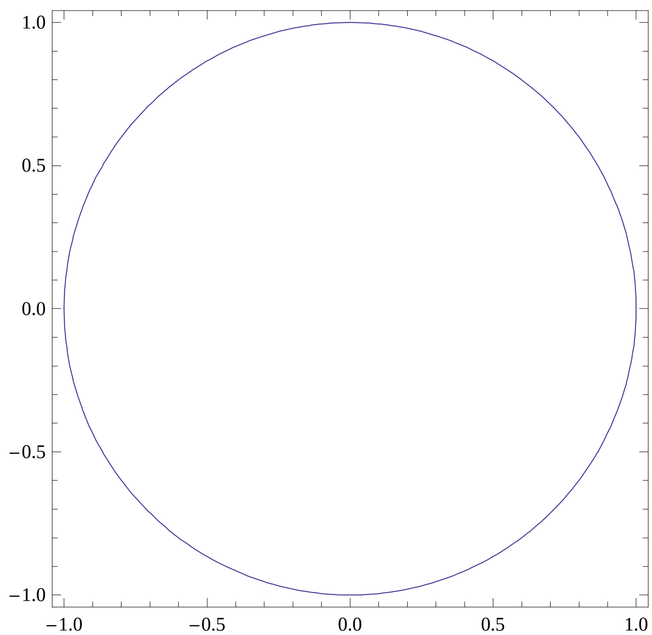
{  ,  }

Variánsok

```
ParametricPlot[{t, t^2}, {t, 0, 1/2}]
```



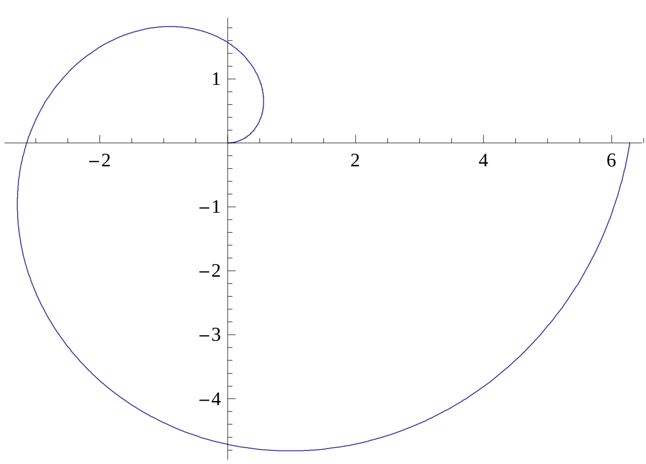

```
ContourPlot[x^2 + y^2 == 1, {x, -1, 1}, {y, -1, 1}]
```



In[4]:=

```
PolarPlot[ϕ, {ϕ, 0, 2π}]
```

Out[4]=



Ábrázoljunk különböző (algebrai) görbéket, pl. kör, hiperbola, lemniszkáta, cusp etc.
Keressünk rá a helpben a térgörbék, felületek ábrázolására! Ábrázoljunk pl. a csavargörbét, síkot, félgömböt stb.

Interaktív prezentációk

In[5]:=

```
Manipulate[Plot[a x + b, {x, -5, 5}, PlotRange → {-10, 10}], {a, -1, 1, .1}, {b, -4, 4, .1}]
```

Out[5]=