Universality limits for generalized Jacobi measures

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Let μ be a finite Borel measure supported on the real line and suppose that $d\mu(x) = w(x)dx$. If the eigenvalue distribution of an $n \times n$ Hermitian ensemble of random matrices invariant under unitary conjugation is given by

$$p(x_1, \dots, x_n) = \frac{1}{Z_n} \prod_{1 \leq i < j \leq n} |x_i - x_j|^2 \prod_{k=1}^{N} w(x_k) dx_k,$$

then the k-point correlation functions describing the density of eigenvalues defined by

$$R_{k,n}(x_1, \dots, x_k) = \frac{n!}{(n-k)!} \int \dots \int p(x_1, \dots, x_n) dx_{k+1} \dots x_n$$

can be expressed as

$$R_{k,n}(x_1, \dots, x_k) = \det \left(\sqrt{w(x_i) w(x_j)} K_n(x_i, x_j) \right)_{i,j=1}^{n}, \tag{1}$$

where $K_n(x, y)$ denotes the so-called Christoffel-Darboux kernel defined by

$$K_n(x, y) = \sum_{k=0}^{n} p_k(x) p_k(y)$$

and $p_k(x)$ denotes the k-th orthonormal polynomial with respect to μ. Because of this reason, determining scaling limits of the type

$$\lim_{n \to \infty} \frac{K_n\left(x_0 + \frac{a}{n}, x_0 + \frac{b}{n}\right)}{K_n(x_0, x_0)}, \quad a, b \in \mathbb{R},$$

which are called universality limits, are especially important. Our goal is to describe universality limits for measures behaving like

$$d\mu(x) = w(x) |x - x_0|^\alpha dx$$

in a neighbourhood of some point x_0. This question has been studied for 30+ years with many partial results, but the complete answer has only been found recently in [1].

[1] T. Danka, Universality limits for generalized Jacobi measures, *submitted for consideration for publication*, available at arXiv with identifier arXiv:1605.04275