Random matrices and orthogonal polynomials

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In my talk I will present our two results, first the reprove of the convergence of the empirical measure of the roots of Hermite polynomials (limit measure is given again by the semicircle law) and show that the sum of the k^{th} power of its roots is a polynomial in n with leading coefficient same as the k^{th} moment of the semicircle law. The second result is an analogous statement for Laguerre polynomials, but now the limit distribution is given by the Marchenko-Pastur law, and the sum of k^{th} power of the roots are polynomials of the dimension and the parameter, which suggests an intrinsic connection between random matrices and orthogonal polynomials. Again the leading coefficients are given by the corresponding moment of the limit distribution. All of the results were proved using only elementary analytical and combinatorial methods. I will also mention the fact, that expectation of the characteristic polynomial of a Wigner matrix coincides with the appropiate monic Hermite polynomial and the same is true for covariance type random matrices but the corresponding polynomial is the monic Laguerre polynomial (see [3]). The talk will also include our latest work, where we want to prove the semicircle law for Gaussian type random symmetric matrices by showing concentration of the joint density function of the eigenvaluesaround its maximum and by using the fact that the argument of the maximum of the Vandermonde determinant on the unit sphere is given by the roots of the Hermite polynomial (see [4]) .

[1] M. Kornyik, Gy. Michaletzky, Wigner matrices, the moments of roots of Hermite polynomials and the semicircle law, *submitted to J. Approximation Theory., arXiv preprint arXiv:1512.03724*

[2] M. Kornyik, Gy. Michaletzky, On the moments of roots of Laguerre-polynomials and the Marchenko-Pastur law, *submitted to J. Constructive Approximation, arXiv preprint arXiv:1602.05001*

[3] P. J. Forrester, A. Gamburd, Counting formulas associated with some random matrix averages, *J. Combinatorial Theory, Series A 113 (2006) 934-951.*

[4] K. Lundengård, J. Österberg, S. Silvestrov, Extreme points of the Vandermonde determinant on the sphere and some limits involving the generalized Vandermonde determinant, *arXiv preprint arXiv:1312.6193*