Pairing strategies for k-in-a-row games

Lajos Győrffy, Géza Makay, András Pluhár
University of Szeged, Szeged, Hungary

The positional game k-in-a-row is played by two players on the infinite (chess)board. In the classical version of the game the players alternately put their own marks to previously unmarked squares, and whoever reaches a winning set (k-consecutive squares horizontally, vertically or diagonally) first of his own marks, wins. In the Maker-Breaker game Maker is the one who tries to occupy a winning set, while Breaker only tries to prevent Maker's win. For different values of k, there are several winning strategies either for Maker or for Breaker. One group of those are pairing (paving) strategies. There is a threshold at $k = 9$, that means Breaker have a winning pairing strategy blocks the winning sets of the k-in-a-row game, only if $k \geq 9$.

There are some generalizations of pairs and pairings. We introduce the notion of *t-cakes* such that a pair (of vertices) is a 2-cake and there are t-cakes for $2 \leq t \in \mathbb{N}$ consisting of exactly t vertices of the board. We also introduce *t(-cake)-placements* where a 2-placement is a simple pairing on the board and in a t-placement there are s-cakes where $s \leq t$ for all cakes. A *good* t-placement blocks all k-in-a-row winning sets on the board. There are some thresholds for different values of k and t.

We investigate other similar games e.g. k-in-a-row on the hexagonal board where there are only three direction vectors. We give some results of pairings and t-placements for k-in-a-row type games for the following four cases:

1. The original k-in-a-row game: there are four directions $(1,0),(1,1),(0,1),(1,-1)$. Here Maker wins for $k \leq 5$, Breaker wins for $k \geq 8$, the cases of $6 \leq k \leq 7$ are unsolved yet. For $k = 9$, there exists 194.543 different pairings. There is a good 4-placement for $k \geq 7$, a 6-placement for $k \geq 6$ and an 8-placement for $k \geq 5$.

2. The hexagonal board: it can be represented by the rectangular board on which there are only three directions $(0,1),(1,1),(1,0)$. Here Maker wins for $k \leq 4$, Breaker wins for $k \geq 7$ by pairings. The cases of $5 \leq k \leq 6$ are open. For $k = 7$ there exists 26 different good pairings. There is a good 4-placement for $k \geq 5$.

3. The chessboard with two direction vectors $(1,0),(0,1)$: Here Maker wins for $k \leq 4$, Breaker wins for $k \geq 5$ by one of the 2 different pairings, so the threshold is sharp. There is a good 3-placement for $k = 4$ and a 4-placement for $k = 3$.

4. The simplest case with only one direction $(1,0)$: where Maker wins for $k \leq 2$ and Breaker wins for $k \geq 3$ by a unique pairing.

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