Solving the 1-median problem on a network with continuous demand and demand surplus*

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In most location problems on networks demand is concentrated on the nodes of the network, while facilities can be located on the edges. Although several researchers have proposed models where the edges contain demand as well, usually the demand is uniformly distributed along the edge. There are only a few papers dealing with arbitrary distributions, e.g. [1, 2].

We considered a single facility location problem with demand distributed at the nodes and along the edges of the network. The distribution of the demand on the edges is arbitrary.

Let $N = (A, E)$ be a connected and undirected network, with node set $A = \{a_1, a_2, \dots a_n\}$ and edge set $E = \{e_1, e_2, \dots e_m\}$, where $|A| = n$ and $|E| = m$. Furthermore let l_{ij} denote the length of edge $(a_i, a_j) \in E$ and $d(x, y)$ denote the distance between two points on the network $x, y \in N$. The distance is understood as the length of the shortest path from x to y.

We assume the demand is distributed at the nodes as well as along the edges of the network. The demand of node a is denoted by $w_a \geq 0$, while the total demand distributed on a given edge e is $p_e \geq 0$. The distribution of the demand on edge e is given by a random variable with cumulative distribution function (cdf) F_e. Lastly the sum of the demand on the whole network is denoted by $D = \sum_{a\in A} w_a + \sum_{e\in E} p_e$.

Our objective is to minimize the distance of a facility to a set of the costumers weighted by their demand. At least a fraction α of the overall demand has to be covered. The inclusion of edges and nodes is binary, thus, if an edge or node is included, its whole demand has to be served.

$$\min_{A^*\subseteq A, E^*\subseteq E, x\in N} G(x) := \sum_{a\in A^*} w_a d(x, a) + \sum_{e\in E^*} p_e \int_{b\in e} d(x, b) \mathrm{d}F_e(b)$$
$$\text{s.t.} \sum_{a\in A^*} w_a + \sum_{e\in E^*} p_e \geq \alpha D$$

To solve this problem we propose a Branch and Bound algorithm. Branching is done on the location and the cover variables simultaneously, while bounds are computed using the relaxed solution of the cover problem.

[1] Rafael Blanquero and Emilio Carrizosa, Solving the median problem with continuous demand on a network, *Computational Optimization and Applications* **56(3)** (2013), 723–734.

[2] López de los Mozos, M.C. and Mesa, Juan A., The variance location problem on a network with continuously distributed demand, *RAIRO-Oper. Res.* **34(2)** (2000), 155–182.

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