Is it possible to determine a point lying in a simplex if we know the distances from the vertices?

György Pál Gehér
University of Szeged, Szeged, Hungary

It is an elementary fact that if we fix an arbitrary set of $d+1$ affine independent points $\{p_0, \dots p_d\}$ in $\mathbb{R}^d$, then the Euclidean distances $\{|x-p_j|\}_{j=0}^d$ determine the point x in $\mathbb{R}^d$ (and therefore in the simplex $\mathrm{Conv}(p_0, \dots p_d)$) uniquely. In my talk I would like to investigate a similar problem in general normed spaces. Namely, I will present a characterization of those, at least d-dimensional, real normed spaces $(X, \|\cdot\|)$ for which every set of $d+1$ affine independent points $\{p_0, \dots p_d\} \subset X$, the distances $\{\|x-p_j\|\}_{j=0}^d$ determine the point x lying in the simplex $\mathrm{Conv}(p_0, \dots p_d)$ uniquely. Surprisingly, the characterization depends on d.

[1] Gy.P. Gehér, Is it possible to determine a point lying in a simplex if we know the distances from the vertices?, *J. Math. Anal. Appl.* **439** (2016), 651–663.