Iterated limits for aggregation of randomized INAR(1) processes

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The INAR(1) process is an integer-valued modification of the well-known autoregressive (AR(1)) process. This property makes it a good candidate to model the development of a simple population, where the next generation consists of the offsprings of the previous generation, and some individuals joining the population by immigration. The usual INAR(1) process $(X_k)_{k=0,1,\ldots}$ follows the dynamics

$$X_k = \sum_{j=1}^{X_{k-1}} \xi_{k,j} + \varepsilon_k, \qquad k = 1, 2, \ldots,$$

where $(\varepsilon_k)_{k=1,2,\ldots}$ are i.i.d. non-negative integer-valued random variables, $(\xi_{k,j})_{k,j=1,2,\ldots}$ are i.i.d. Bernoulli random variables with mean $\alpha \in [0, 1]$, and X_0 is a non-negative integer-valued random variable such that X_0, $(\xi_{k,j})_{k,j=1,2,\ldots}$ and $(\varepsilon_k)_{k=1,2,\ldots}$ are independent. We randomize the process by assuming that α is a random variable, and the properties above hold conditionally on α. Suppose that conditionally on α the innovations, $(\varepsilon_k)_{k=1,2,\ldots}$, have Poisson distribution, and α has a density function of the form

$$\psi(x)(1 - x)^{\beta}, \qquad x \in [0, 1),$$

where $\beta \in (-1, \infty)$ and ψ is an integrable function on $[0, 1)$ having a limit $\lim_{x\uparrow 1} \psi(x) = \psi_1 > 0$. The distribution of X_0 is chosen such that the randomized INAR(1) process is strictly stationary.

It is interesting to see how the number of all individuals living until a certain time behaves as this time point increases. In this talk we not only examine this behavior, but we also consider many independent, strictly stationary randomized INAR(1) time series with i.i.d. α variables, and define a second summation among these replicas. Limit theorems are presented for the stochastic process

$$\sum_{j=1}^{N} \sum_{k=1}^{\lfloor nt \rfloor} X_k^{(j)}, \qquad t \geq 0,$$

after appropriate centering and scaling, where $(X_k^{(j)})_{k=0,1,2,\ldots}$ stands for the j-th INAR(1) process for every $j = 1, 2, \ldots$ index. We provide limit theorems where N and n converge to infinity in an iterated manner, meaning that first N, and then n converges to infinity, or vica versa. The talk is based on the papers [1] and [2]. We show that the limit distributions depend on the centralization of the sum, the order of the iteration, and the distributions of α and the the innovations.

[1] Barczy, M., Nedényi, F., and Pap, G., Iterated limits for aggregation of randomized INAR(1) processes with Poisson innovations, *arXiv:1509.05149* (2015).

[2] Nedényi, F., and Pap, G., Iterated scaling limits for aggregation of random coefficient AR(1) and INAR(1) processes, *arXiv:1601.04679* (2016).