Simple homogeneous structures with the Robinson property

Dávid Nyíri
Budapest University of Technology and Economics, Budapest, Hungary

I am going to present a theorem about simple, homogeneous structures of finite relational languages of first-order logic. We call a first-order structure homogeneous if every partial isomorphism between its finite substructures can be extended to an automorphism. Since Fraïsse proved his limit theorem, homogeneous structures were studied exhaustively, especially from a topological and Ramsey-theoretic aspect.

The notion of forking and dividing is a powerful tool to classify complete theories. A formula $\phi(x, a)$ divides over A iff there is an A-indiscernible sequence $(a_i : i \in \omega)$ and $k < \omega$ such that every subset of $\{\phi(x, a_i) : i \in \omega\}$ of cardinality at least k is inconsistent. A formula $\phi(x, y)$ divides ω times iff there is a sequence $(a_i : i \in \omega)$ such that $\phi(x, a_i)$ divides over $a_{<i} = \cup_{j<i} a_j$. A complete theory T is said to be simple iff no formula divides ω times, where consistency in the definition of dividing is taken modulo T. A theory is called supersimple if there are no consistent set of formulas $\{\phi_i(x, a_i) : i \in \omega\}$ such that $\phi_i(x, a_i)$ divides over $a_{<i} = \cup_{j<i} a_j$.

Recently, Koponen in [1] proved that any simple homogeneous structure over a finite relational language that only contains unary or at most binary relations is supersimple. The natural question is whether the restriction of the language can be removed. We say that a structure $\mathcal{M}$ has the Robinson property iff for two subsets A, B of $\mathcal{M}$ and two elements a, b, there is $c \in \mathcal{M}$ such that a and c satisfy the same formulas with parameters from C and b and c satisfies the same formulas with parameters from B. Generalizing Koponen's theorem, I am going to show that the Robinson property is a sufficient condition for a simple homogeneous structure to be supersimple.

[1] V. Koponen, Binary simple homogeneous structures are supersimple with finite rank, *Proc. Amer. Math. Soc.* (2015).