ON CRITICAL PERCOLATION PROBABILITY AND UNIMODULAR RANDOM GRAPHS

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We investigate the generalisations of the classical critical percolation probabilities p_c, p_T and the critical probability $\tilde{p}_c$ defined by Duminil-Copin and Tassion [3] to unimodular random graphs and the relations between them. We prove that $p_c = \tilde{p}_c$ holds for bounded degree unimodular graphs and we show by an example that there are unimodular graphs with sub-exponential volume growth and $p_T < p_c$.

We further examine the question of Schramm's conjecture in the case of unimodular random graphs: does $p_c(G_n)$ converge to $p_c(G)$ if $G_n \to G$ in the local weak sense? We give certain conditions which imply the inequality $\liminf p_c(G_n) \geq p_c(\lim G_n)$ and we show by examples that Schramm's conjecture does not hold in general for unimodular graphs: there are sequences such that $G_n \to G$ but $p_c(G) > \lim p_c(G_n)$ or $p_c(G) < \lim p_c(G_n) < 1$.

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