

CRITICAL RELATIONS OF THE $2k$ -CROWN POSET

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A set of finitary operations on a finite set is called a clone if it contains all projections and is closed under superposition (composition). Clearly, the set of all finitary operations (on a finite set) is a clone. The largest (with respect to inclusion) clones that are smaller than this one are called maximal clones. I. G. Rosenberg, in a classical result, has classified the maximal clones into six classes. For five of the six classes it has been shown that the clones of these classes are finitely generated, that is there is a finite subset of the clone which generates the whole clone. (Generating is just adding all the projections and taking all possible superpositions.) The unsettled class is the class of clones containing monotone operations of bounded partial orders, that is posets having both least and largest elements. Some partial results have already been obtained. Monotone clones of at most seven element posets are proven to be finitely generated and so are posets with at least three variable monotone near unanimity operations. G. Tardos [1] showed that the clone of a particular eight element poset is not finitely generated. This was the first proof showing a maximal clone to be not finitely generated. L. Zádori [2] generalised Tardos's result by describing all series parallel posets having not finitely generated clones. No one has found non-finitely generated maximal clones since, though one may conjecture there are a lot of them. In his proof, Tardos heavily relied on deciding the extendibility of particular partial functions between posets. He had to describe all the so-called obstacles of his eight element poset. The problem is that for other posets, finding the obstacles can get quite difficult. Instead of obstacles, one might use critical relations to handle the problem of deciding the extendibility of partial functions. Critical relations are invariant relations that are not (non-trivial) direct products and not (non-trivial) intersections of invariant relations either. In our talk we describe the critical relations of the so-called crown posets. The description is surprisingly easy compared to how complicated obstacles can get in this case.

- [1] G. TARDOS, A maximal clone of monotone operations which is not finitely generated, *Order* **3** (1986), 211–218.
- [2] L. ZÁDORI, Series parallel posets with nonfinitely generated clones, *Order* **10** (1993), 305–316.