Inverse monoids and immersions of 2-complexes

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It is well known that under mild conditions on a connected topological space \( \mathcal{X} \), connected covers of \( \mathcal{X} \) may be classified via conjugacy classes of subgroups of the fundamental group of \( \mathcal{X} \). In [1], we extend these results to the study of immersions into 2-dimensional CW-complexes. An immersion \( f : \mathcal{D} \rightarrow \mathcal{C} \) between CW-complexes is a cellular map such that each point \( y \in \mathcal{D} \) has a neighborhood \( U \) that is mapped homeomorphically onto \( f(U) \) by \( f \). In order to classify immersions into a 2-dimensional CW-complex \( \mathcal{C} \), we need to replace the fundamental group of \( \mathcal{C} \) by an appropriate inverse monoid. We show how conjugacy classes of the closed inverse submonoids of this inverse monoid may be used to classify connected immersions into the complex. We also give a process to construct the 2-complex corresponding to a given conjugacy class given by its generators. We prove that when it is finitely generated, the process ends after a finite number of steps.

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