Shattering extremal set systems

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A basic problem in mathematics is to characterize objects given by local properties. In general it is a hard task to verify such local properties, what makes the study of these structures difficult.

We say that a set system $\mathcal{F}$ on $\{1, 2, ..., n\}$ shatters a given set S if all of its subsets can be obtained by intersecting it with a set from $\mathcal{F}$. It can be proved that a set system $\mathcal{F}$ shatters at least $|\mathcal{F}|$ sets. Here we study shattering-extremal set systems, those, which shatter exactly $|\mathcal{F}|$ sets.

When considering the elements of $\mathcal{F}$ as characteristic vectors, one can define the ideal of polynomials in n variables vanishing on $\mathcal{F}$. It is worth investigating this ideal $I(\mathcal{F})$ instead of $\mathcal{F}$, because several algebraic tools can be of great help, among others Gröbner bases and the standard monomials for different term orders. This point of view yields an efficient algorithm for testing the shattering-extremality of a set system ([1]) and also a possible way to generalize shattering-extremality for any finite sets of vectors.

Another standard way of looking at a set system in general is to embed it into the Hamming graph $\{0, 1\}^n$, and study the properties of the resulting inclusion graph. Several interesting results can be obtain for the inclusion graphs of shattering-extremal families (e.g. [2]), together with my supervisor Lajos Rónyai (BME, SZTAKI) we studied the inclusion graphs of shattering-extremal families of small Vapnik-Chervonenkis dimension.

References

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[2] G. Greco, Embeddings and trace of finite sets, Information Processing Letters, Vol. 67 (1998), 199-203