On attacking Zolotarev polynomial approximation problems with quantifier elimination and Gröbner bases

Robert Vajda Bolyai Institute, University of Szeged Hungary vajdar@math.u-szeged.hu

Abstract:

The classical Zolotarev approximation problem is a real polynomial approximation problem. Given a polynomial p of degree n, one asks for the best uniform approximation of p on a closed bounded interval by a polynomial q of degree n-1or less. We show how this minimax problem can be formulated as a real quantifier elimination problem in different ways. Moreover, we explore how additional theoretical knowledge about the optimal approximating polynomial can lead to equational constraints and therefore to more effective solution to the problem. We slightly generalize the problem setting as well. The problem family is also important to benchmark existing quantifier elimination software packages. All the explorations and computations have been done by the author using Wolfram Mathematica.