Abstract

Uniform convergence of double trigonometric series with general monotonic coefficients

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The basic theorem in the theory of uniform convergence of sine series is due to Chaundy and Jolliffe from 1916. They proved that if $\{c_k\}_{k=1}^{\infty}$ is a nonnegative, monotonically decreasing sequence, then the series $\sum_{k=1}^{\infty} c_k \sin kx$ converges uniformly in x if and only if $kc_k \to 0$. Since 1916, a number of papers were published to extend this theorem by enlarging the class of monotonic coefficients, while keeping the condition $kc_k \to 0$ still necessary and sufficient. The largest class defined for the appropriate extension is class SBVS_2 introduced by P. Kórus in 2009. In that year, M. Dyachenko and S. Tikhonov proved a theorem for the uniform convergence of cosine series with β-general monotone coefficients. That theorem leads to the following. If $\{c_k\}_{k=1}^{\infty}$ is from SBVS_2, then the conditions $\sum_{k=1}^{\infty} c_k$ converges and $kc_k \to 0$ are sufficient for the uniform convergence of the series $\sum_{k=1}^{\infty} c_k \cos kx$, and these conditions are necessary in case $\{c_k\}_{k=1}^{\infty}$ is also nonnegative.

For the uniform regular convergence of the double sine series $\sum_{j=1}^{\infty}\sum_{k=1}^{\infty} c_{jk} \sin jx \sin ky$, Žak and Šneider proved in 1966 that if $\{c_{jk}\}_{j,k=1}^{\infty}$ is nonnegative, monotonically decreasing (in a two dimensional sense), then the regular convergence of the previously defined double sine series is uniform if and only if $jkc_{jk} \to 0$ as $j + k \to \infty$. This result was extended in 2011 to the coefficient class SBVDS_1 (the sufficiency part was even proved for a larger class SBVDS_2). Now we give sufficient conditions for the uniform regular convergence of the sine-cosine series $\sum_{j=1}^{\infty}\sum_{k=1}^{\infty} c_{jk} \sin jx \cos ky$ and the double cosine series $\sum_{j=1}^{\infty}\sum_{k=1}^{\infty} c_{jk} \cos jx \cos ky$ with coefficients from SBVDS_1 which are necessary in case $\{c_{jk}\}_{j,k=1}^{\infty}$ is also nonnegative.