On doubly biased Maker-Breaker games

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Let V be a finite set and let $\mathcal{F} \subseteq 2^V$ be the family of its subsets. Positional game is a pair $(V, \mathcal{F})$, where V is referred to as *board*, and the sets of $\mathcal{F}$ is referred to as the *winning sets*. In the $(a : b)$ *Maker - Breaker* game, a type of positional game, two players called *Maker* and *Breaker* take turns in claiming previously unclaimed elements of the board. Maker claims a elements in each move and *Breaker* claims b elements in each move. The game ends when all the elements of the board are claimed. Maker aims to claim all the elements of some $F \in \mathcal{F}$ and Breaker wants to prevent him from doing that, i.e. he wants to put at least one element in each winning set. The game is *"Maker's win"* if Maker has a strategy to win against any strategy of Breaker. Otherwise, the game is *"Breaker's win"*.

When $a = b = 1$ the game is called *unbiased*, and in many $(1 : 1)$ Maker - Breaker games, Maker wins quite easily. Thus, to "even out the odds" and increase Breaker's chances to win, the *biased* games are introduced. The question that comes naturally is to determine the winner of $(1 : b)$ Maker - Breaker game, when b is greater than one, and to increase b until the game becomes more balanced.

We study $(a : b)$ Maker - Breaker games played on the edge set of the complete graph on n vertices, $E(K_n)$, where n is sufficiently large integer and both a and b can be greater than one. The winning sets we look at are various graph theoretic properties like spanning tree, Hamilton cycle, etc. For each $a = a(n)$, we want to determine $b_0(a, n) = b_0(a)$, so that for $b < b_0(a)$ the game is Maker's win, and for $b > b_0(a)$ the game is Breaker's win. We refer to $b_0(a)$ as the *threshold bias for a*.

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