On the stability regions of the inverted pendulum

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Let a be a step-function defined by

$$a(t) = \begin{cases} A_1, & \text{if } k(T_1+T_2) \le t < (k+1)T_1 + kT_2; \\ -A_2, & \text{if } (k+1)T_1 + kT_2 \le t < (k+1)(T_1+T_2),\ k \in \mathbb{Z}, \end{cases}$$

where $T_1 > 0,\ T_2 > 0,\ A_1 > 0,\ A_2 > g$ with $A_1T_1 = A_2T_2$, and g denotes the constant of the gravity. Consider the second order differential equation

$$\ddot{x} - \frac{g + a(t)}{l} x = 0,$$

which describes the small vibrations of the l-length pendulum around the upper equilibrium $x = 0$, provided that the suspension point of the pendulum is vibrated vertically by the $T_1 + T_2$-periodic acceleration $a(t)$.

We give a sufficient condition guaranteeing stability for the upper equilibrium. We apply this condition to the classical case $T_1 = T_2$, $A_1 = A_2$, and draw a *global* stability map on the $\varepsilon - \mu$ plane, where

$$\varepsilon^2 := \frac{1}{8}\frac{AT^2}{l}, \quad \mu^2 := \frac{g}{A}.$$

The map is global in the sense that ε and μ are not supposed to be small.

We are intrested in the case $T_1 \neq T_2$. If $\gamma := T_1 - T_2,\ T = \frac{T_1+T_2}{2},\ \kappa = \frac{g}{A_2}$ then what can we say about stability? Numerical experiments have been performed with Wolfram Mathematica.