Parameter estimation in a spatial linear regression model

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Let $[a,c] \subset (0,\infty)$ and $b_1, b_2 \in (a,c)$, let $\gamma_{1,2} : [a,b_1] \to \mathbb{R}$ and $\gamma_0 : [b_2,c] \to \mathbb{R}$ be continuous, strictly decreasing functions and let $\gamma_1 : [b_1,c] \to \mathbb{R}$ and $\gamma_2 : [a,b_2] \to \mathbb{R}$ be continuous, strictly increasing functions with $\gamma_{1,2}(b_1) = \gamma_1(b_1) > 0$, $\gamma_2(b_2) = \gamma_0(b_2)$, $\gamma_{1,2}(a) = \gamma_2(a)$ and $\gamma_1(c) = \gamma_0(c)$. We consider the problem of estimating the parameters of a linear regression model $Z(s,t) = m_1 g_1(s,t) + \cdots + m_p g_p(s,t) + U(s,t)$ based on observations of Z on the set G which contains the points bounded by the functions γ_0, γ_1, γ_2 and $\gamma_{1,2}$, where the driving process U is a Gaussian random field and $g_1, \ldots, g_p$ are known functions. Using a discrete approximation we obtain explicit forms of the maximum-likelihood estimators of the parameters in the cases when U is either a Wiener or a stationary or nonstationary Ornstein-Uhlenbeck sheet. In the case when U is a standard Wiener sheet we have a generalization of the results of Arató, N. M. [1] and Baran *et al.* [4]. We also consider the cases when the driving process U is a stationary and a zero start Ornstein-Uhlenbeck sheet and generalize the results of Arató, N. M. [2] and Baran *et al.* [3].

Moreover, we present some simulation results to illustrate the theoretical ones where the driving Gaussian random sheets are simulated with the help of their Karhunen-Loève expansions (see e.g. [5]).

References

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