On different versions of sector conditions

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The theory of central limit theorems for additive functionals of ergodic Markov processes via martingale approximation was initiated in 1986 by Kipnis and Varadhan.

We investigate the following question: given a stationary and ergodic Markov process $\eta(t)$ on the probability space $(\Omega, \mathcal{F}, \pi)$ and a function $f : \Omega \to \mathbb{R}$, what conditions on η and f guarantee central limit theorem/invariance principle for the integral

$$N^{-1/2} \int_0^{Nt} f(\eta(s))\, \mathrm{d}s \tag{1}$$

as $N \to \infty$?

Kipnis and Varadhan originally proved efficient martingale approximation and central limit theorem for the reversible case with no assumptions other than the necessary ones. Since then, the theory have been extended by Varadhan and others to include processes with a varying degree of non-reversibility. Applications include the tagged particle in simple exclusion, persistent random walk in random environment, the myopic self-avoiding walk, self-repellent Brownian polymer etc.

Some of the sufficient conditions for martingale approximation are abstract and not easy to verify for particular applications. So-called sector conditions were introduced in order to provide conditions that can be checked more directly. The (strong) sector condition was introduced by Varadhan in 1996, and was later further generalized by the graded sector condition (by Sethuraman, Varadhan and Yau, 2000) and the relaxed sector condition (by H., Tóth, Vető in 2012).

We state and prove a newer version of the graded sector condition; apart from the conditions being less restrictive than in previous formulations, the proof is also less technical.

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