

A SIMPLE COMBINATORIAL PROOF OF SHAPIRO'S CATALAN CONVOLUTION

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Using generating functions, Shapiro proved the following elegant convolution formula involving Catalan numbers of even index:

$$\sum_{k=0}^n C_{2k} C_{2n-2k} = 4^n C_n.$$

We give a simple combinatorial proof of this formula. In addition, we show bijectively that it is equivalent with the alternating convolution formula of central binomial coefficients. Our key observation is a non-standard interpretation of the Catalan number C_{2k} .