Additive representation functions

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Abstract

Let $\mathbb{N}$ be the set of nonnegative integers. For a given set $\mathcal{A} \subset \mathbb{N}$ the representation functions $R^{(1)}_{h,\mathcal{A}}(n)$, $R^{(2)}_{h,\mathcal{A}}(n)$ and $R^{(3)}_{h,\mathcal{A}}(n)$ are defined as the number of solutions of the equation $a_{i_1}+\cdots+a_{i_h} = n$, $a_{i_1},\ldots,a_{i_h} \in \mathcal{A}$ without any condition and with condition $a_{i_1} < \cdots < a_{i_h}$ and $a_{i_1} \le \cdots \le a_{i_h}$.

In 1978, Nathanson proved, if $\mathcal{A}$ and $\mathcal{B}$ are distinct nonempty sets of integers such that $R^{(1)}_{2,\mathcal{A}}(n) = R^{(1)}_{2,\mathcal{B}}(n)$ for all sufficiently large n then $\mathcal{A} = F_{\mathcal{A}} \cup S$ and $\mathcal{B} = F_{\mathcal{B}} \cup S$, where $F_{\mathcal{A}}, F_{\mathcal{B}}$ and S have some nice properties. We extend this result to the $R^{(1)}_3(n)$ representation function. In this case the construction of the sets $F_{\mathcal{A}}, F_{\mathcal{B}}$ and S are a bit difficult but still nice.

In 2011, Yang posed the next problem in his article: if $p \ge 3$ is a prime and $\mathcal{A}$ is a set of nonnegative integers, then does there exist a set of nonnegative integers $\mathcal{B}$ with $\mathcal{A} \ne \mathcal{B}$ such that $R^{(1)}_{p,\mathcal{A}}(n) = R^{(1)}_{p,\mathcal{B}}(n)$ for all sufficiently large n? We solved this problem in a generalized way. We proved that for arbitrary $h \ge 2$, $h \in \mathbb{Z}$ there exist $\mathcal{A}, \mathcal{B} \subset \mathbb{N}$, such that $R^{(i)}_{h,\mathcal{A}}(n) = R^{(i)}_{h,\mathcal{A}}(n)$ for all sufficiently large n and for $i = 1, 2, 3$.

In the proofs we use generating functions and some other theorem from algebraic and analytic number theory.

In the talk I will try to give a short summarize about the results[1], which is joint work with Sándor ZKiss and Csaba Sándor.

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