Balázs Udvari - The number of convex polygons determined by a point set

Let $\mathcal{P}$ be a set of points in general position in the plane. Erdős and Szekeres proved in 1935 that for every k there exists a number $ESz(k)$ such that when $|\mathcal{P}| \geq ESz(k)$ there is a convex k-gon in $\mathcal{P}$. (That is, there are k points of $\mathcal{P}$ which are the vertex set of a convex k-gon.) For general k, $ESz(k)$ is unknown; Erdős and Szekeres conjectured that it is $2^{k-2}+1$. The conjecture has been validated for $k \leq 6$.

A related question is the following: at least how many convex k-gons are there in a point set with no three collinear points? Even the first non-trivial special case, $k=4$, is unsolved. Let $\square(\mathcal{P})$ be the number of convex quadrilaterals in the general point set $\mathcal{P}$ and $\square(n) = \min_{|\mathcal{P}|=n} \square(\mathcal{P})$. Then the best known bounds are: $0.37553\binom{n}{4} + O(n^3) \leq \square(n) \leq 0.3807\binom{n}{4} + O(n^3)$.

In the talk, we will show classical results on both topics and we will present a new method for determine $\square(n)$.