Statistical properties of the system of two falling balls

András Némedy Varga¹

I would like to talk on joint results with Péter Bálint and Gábor Borbély (see [1]) on a billiard type model with intermittent behavior. The system of two falling balls, introduced by Wojtkowski, describes the motion of two point particles of mass m_1 and m_2 that move along the vertical half-line (bounded from below by the ground), subject to constant gravitational force, and collide elastically with each other and the floor.

We consider the case when the lower ball is heavier (i.e. $m_1 > m_2$) which corresponds to ergodic and hyperbolic dynamics; however, hyperbolicity is not uniform related to arbitrary long series of bounces of the lower ball on the floor before colliding with the upper ball. We present a detailed analysis of the discrete time model and prove that, for an open set of mass ratios, the correlations decay, modulo logarithmic factors, as $\mathcal{O}(1/n^2)$. This rate is summable, accordingly, the central limit theorem is also proved; that is, the system exhibits normal diffusion.

In addition to presenting these results I would like to mention some ongoing work concerning the continuous time model, which can be regarded as a suspension flow, and the structure of the singularity set of the dynamics, which is highly relevant for the statistical properties.

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References

[1] Péter Bálint, Gábor Borbély and András Némedy Varga: Statistical properties of the system of two falling balls, to appear in Chaos, avialable at http://www.math.bme.hu/~nemedy/Research

¹Department of Differential Equations, Institute of Mathematics, Budapest University of Technology and Economics