Abstract

Optimal Design of the NURBS Curves and Surfaces Utilizing Multiobjective Optimization and Decision Making Algorithms of RSO

Amir Mosavi[a*], Miklós Hoffmann[b], A. S. Milani[c]

A reliable optimal design process for the Non-uniform rational B-spline (NURBS) curves and surfaces would have a wide and foundational application in CAGD, CAD, image processing, etc. Yet the optimal design and parameter tuning of the NURBS curves and surfaces is a complicated, highly non-linear and multiobjective optimization(MOO) problem. The complexity of the problem is even increased when the criteria of product beauty is included to the design process. More on the problem, applications and previous approaches are available in [2,4], where the use of MOO algorithms enhances the design process by enabling optimization of several design objectives at once.

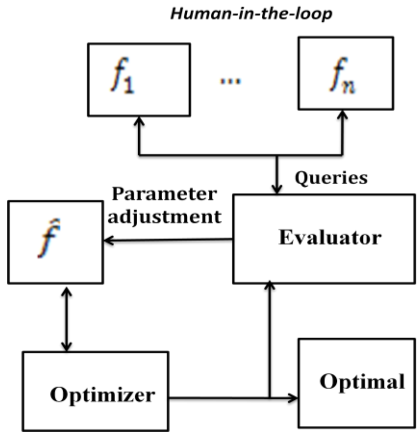
In this article the optimization process of NURBS including four conflicting and highly non-liner design objectives is described. For solving problems as such, with a high level of complexity, modeling the true nature of the problem is of importance and essential. For this reason a considerable amount of efforts is made in modeling the MOO problems in Scilab and the details are described.

Here, as an alternative to the previous approaches the robust and interactive MOO algorithm of RSO[1,3] is proposed in order to efficiently optimize all the design objectives at once including the criteria of beauty in which couldn't be completely considered in the previous attempts. In this framework the quality of the surface, similar to the previous research workflows, is measured using a set of certain functions, then an optimization algorithm is applied in order to optimize the function to improve the quality of the surface.

The problem is modeled in scilab and the model is integrated to the optimizer via advanced interfaces to the RSO algorithm and its brain-computer evolutionary multiobjective optimization implementations and visualization [1,13]. In this framework the application of learning and intelligent optimization and reactive business intelligence approaches in improving the process of such complex optimization problems are described. Furthermore the problem is further reconsidered by reducing the dimensionality and the dataset size[11], multi-dimensional scaling, clustering and visualization tools[3,13].

---

[a] *University of Debrecen Faculty of Informatics, Hungary*
[b] *Department of Computer Graphics and Image Processing University of Debrecen, Hungary*
[c] *School of Engineering, University of British Columbia, Okanagan Campus, Kelowna, Canada*
[*] *a.mosavi [at] math.unideb.hu*



Schematic flowchart of the proposed optimization process [1]; learning the problem definition from the final user in interactive multiobjective optimization [3].

Brief statement of the problem

A tensor product NURBS is defined as; $S(u,v) = \sum_{i=0}^{n-1}\sum_{j=0}^{m-1} \mathbf{P}_{\mathbf{i,j}}\, R_{i,j}(s,t)$, where $\boldsymbol{P}_{i,j}$ are control points of the surface with the orders and the numbers of n *and* m. $R_{i,j}(s,t)$ are the NURBS basis function, depended on the design variables including weights, $\boldsymbol{w}$, the knot vectors, $\boldsymbol{u}$ & $\boldsymbol{v}$, the d_u & d_v orders of the surface and the parameterization, s & t.

Handling the parameterization, knot vectors and NURBS weights is described in [4]. Tuning NURBS weights and knot vector all together dramatically increases the number of DOF which is proportional to $n * m$.

According to the input points, $\boldsymbol{Q}_{i,j}$, and the design variables, the control points, $\boldsymbol{P}_{i,j}$, via utilizing the linear least squares fitting, are calculated and the surface is created [12].

Let $\boldsymbol{M}$ be the collocation matrix used for surface fitting; $\boldsymbol{Q}_x, \boldsymbol{Q}_y, \boldsymbol{Q}_z$ are the coordinates of Q, the data to be fitted; $diag(x)$ a diagonal matrix whose entries are the vector x.

$t = M * w \qquad X = diag(Q_x) \quad Y = diag(Q_y) \quad Z = diag(Q_z)$

$v_x = X * t \qquad v_y = Y * t \qquad v_z = Z * t$

The position of the surface's control points $\boldsymbol{p}_x, \boldsymbol{p}_y, \boldsymbol{p}_z$ are given by least solution of the following equations: $d_x = M * v_x \qquad d_y = M * v_y \qquad d_z = M * v_z$

Optimization Objectives

The goal of the optimization process is to produce a set of NURBS surfaces which approximates a set of input points, $\boldsymbol{Q} = \boldsymbol{Q}_{0,0}, \dots, \boldsymbol{Q}_{N-1,M-1} \in \mathbb{R}^d$, $d = 2, 3$, and are optimal with respect to the specified design objectives. Once the surface is created the quality of it could be considered by evaluating a set of specified design objectives, i.e. $O_1(S(s,t))$, . . ., $O_k(S(s,t)$.

---

[a] University of Debrecen Faculty of Informatics, Hungary
[b] Department of Computer Graphics and Image Processing University of Debrecen, Hungary
[c] School of Engineering, University of British Columbia, Okanagan Campus, Kelowna, Canada
*a.mosavi [at] math.unideb.hu

Approximation Error, O_1, the distance between the surface and the points $\boldsymbol{Q}$ measured at the parametrization points $\boldsymbol{s_i}, \boldsymbol{t_i}$, is often subjected to minimization;

$O_1 = \min\left(\sum_{i=0}^{n-1}\sum_{j=0}^{m-1}\left\|\boldsymbol{S}(s_i, t_i) - \boldsymbol{Q}_{i,j}\right\|^2\right)$, under L_2 norm,

$O_1 = \max\left(\left\|\boldsymbol{S}(s_i, t_i) - \boldsymbol{Q}_{i,j}\right\|\right), i = 0, \dots, n-1; j = 0, \dots, m-1$, under L_∞ norm.

Surface Area, O_2, in conflict with approximation error, controls artifacts due to over-fitting;

$O_2 = \int_0^1\int_0^1 \left\|\frac{dS}{ds} \times \frac{dS}{dt}\right\| dsdt$.

Surface Elastic Energy, O_3, as an other conflicting objective is a highly non-linear term;

$O_3 = \int_0^1\int_0^1 \left\|k_{min}^2 + k_{max}^2\right\| dA$, where A is the surface area.

Statement of the general form of the multiobjective optimization problems

According to [3] the general form of the Multiobjective optimization problems is stated as

$$\begin{array}{ll} \text{Minimize} & \mathbf{f}(\mathbf{x}) = \{f_\mathbf{1}(\mathbf{x}), \dots, f_\boldsymbol{m}(\mathbf{x})\} \\ \text{Subjected to} & \mathbf{x} \in \Omega \end{array}$$

where $\mathbf{x} \in \mathbb{R}^n$ is a vector of n decision variables; $\mathbf{x} \subset \mathbb{R}^n$ is the feasible region and is specified as a set of constraints on the decision variables; $\mathbf{f} : \Omega \to \mathbb{R}^m$ is made of m objective functions subjected to be minimization. Objective vectors are images of decision vectors written as $\mathbf{z} = \mathbf{f}(\mathbf{x}) = \{f_\mathbf{1}(\mathbf{x}), \dots, f_\boldsymbol{m}(\mathbf{x})\}$. An objective vector is considered optimal if none of its components can be improved without worsening at least one of the others. An objective vector $\mathbf{z}$ is said to dominate $\mathbf{z}'$, denoted as $\mathbf{z} \prec \mathbf{z}'$, if $z_k \leq z_k'$ for all k and there exist at least one h that $z_h \leq z_h'$. A point $\hat{\mathbf{x}}$ is Pareto optimal if there is no other $\mathbf{x} \in \Omega$ such that $\mathbf{f}(\mathbf{x})$ dominates $\mathbf{f}(\hat{\mathbf{x}})$.

A brief revision on previous approaches to solving the MOO of the NURBS curves and surfaces

The mathematical modeling of the NURBS curves and surfaces design problem results in a multiobjective optimization problem which cannot be handled as such by traditional single objective optimization algorithms. Considering the problem with Conjugate Gradient and Newton based approaches, the optimization process is divided into several phases and each functional is optimized separately [6, 7, 8]. In this approach the multiobjective problem is solved via a single objective optimization algorithm. However the results obtained clearly are not promising.

Previously an evolutionary MOO algorithm [4] is used to handle this case. In this approach the results are reported promising due to the robustness and efficiency of evolutionary algorithms. Evolutionary algorithms [9] are natural choice for multiobjective optimization since at each step the algorithms keeps a population, which is a set of solutions instead of a single, optimal, solution. Because of the robustness and efficient handling of highly non-linear objective functions and constrains the use of evolutionary algorithm in geometrical problem has proved to be a powerful technique [10].

----
[a] *University of Debrecen Faculty of Informatics, Hungary*
[b] *Department of Computer Graphics and Image Processing University of Debrecen, Hungary*
[c] *School of Engineering, University of British Columbia, Okanagan Campus, Kelowna, Canada*
[*] *a.mosavi [at] math.unideb.hu*

Abstract

In the proposed RSO algorithms, in contrast to the evolutionary algorithms, the decision maker guides the optimization in the desirable search locations and the final desirable surface. In this case the computation cost is minimized and the preferences of the decision maker are effectively considered.

References

[1] R. Battiti and P. Campigotto, Reactive search optimization: Learning while optimizing. an experiment in interactive multi-objective optimization. In S. Voss and M. Caserta, editors, Proceedings of MIC 2009, VIII Metaheuristic International Conference, Lecture Notes in Computer Science. Springer Verlag, 2010.

[2] A. Mosavi, On Engineering Optimization the Splined Profiles, in Proceedings of International modeFRONTIER Users' Meeting, Trieste, Italy, 2010.

[3] R. Battiti, M. Brunato, Reactive Business Intelligence. From Data to Models to Reactive Search Srl, Italy, February 2011.

[4] R. Goldenthal, M. Bercovier, Design of Curves and Surfaces by Multiobjective Optimization, *Mathematical Methods for Curves and Surfaces*, 2004.

[5] B. Roberto, A. Passerini (2010). "Brain-Computer Evolutionary Multi-Objective Optimization (BC-EMO): a genetic algorithm adapting to the decision maker." (PDF). *IEEE Transactions on Evolutionary Computation* 14 (15): 671–687.

[6] M.I.G. Bloor, M.J. Wilson, H. Hagen, The smoothing properties of variational schemes for surface design, Computer Aided Geometric Design,Vol. 12, pp. 381–394, 1995.

[7] G. Brunnett, H. Hagen, P. Santarelli, Variational design of curve and surfaces, *Surveys on Mathematics for Industry*, Vol 3. pp. 1–27, 1993.

[8] G. Brunnett, J. Kiefer, Interpolation with minimal-energy splines, *Computer-Aided Design*, Vol. 26, No. 2, pp. 137–144, 1994.

[9] K. Deb, , Evolutionary algorithms for multi-criterion optimization in engineering design, Evolutionary Algorithms in Engineering and Computer Science, Miettinen K., Makela, M. M., Neittaanmaki, P., and Periaux J. (eds.), John Wiley Sons, Ltd, Chichester, UK, pp. 135–161, 1999.

[10] G. Renner, (ed.), *Computer-Aided Design, Genetic Algorithms*, Vol 35, Issue 8. pp. 707–769, 2003.

[11] A. Mosavi(2010). Multiple criteria decision-making preprocessing using data mining tools. *IJCSI International Journal of Computer Science Issues*, V7, Issue 2, No 1.

[12] W. Ma, J. P. Knuth, NURBS curve and surface fitting and interpolation, Mathematical Methods for Curves and Surfaces, M. Dæhlen,T. Lyche, and L. L. Schumaker (eds.), Vanderbilt University Press, Nashville, 1995 pp. 315–322

[13] A.Mosavi, M.Azodinia, Kasun N. Hewage, Abbas S. Milani, M.Yeheyis, Reconsidering the Multiple Criteria Decision Making Problems of Construction Workers Using; Grapheur, Poster presented at EnginSoft International Conference, Verona, Italy 20-21 Oct.

---

[a] *University of Debrecen Faculty of Informatics, Hungary*
[b] *Department of Computer Graphics and Image Processing University of Debrecen, Hungary*
[c] *School of Engineering, University of British Columbia, Okanagan Campus, Kelowna, Canada*
[*] *a.mosavi [at] math.unideb.hu*