Limit theorems for the one-dimensional myopic self-avoiding walk

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We consider two models of the so-called 'true' (or myopic) self-avoiding random walk on $\mathbb{Z}$. In both cases, we define a self-repelling random walk which is pushed locally in the direction of the negative gradient of its own local time. We investigate the long-time asymptotics of the walks, and we get surprisingly different scaling behaviour of the two models. The scaling limits are also different from the usual diffusion processes. On the other hand, we prove limit theorems for the local time processes using a kind of Ray – Knight approach.

The first model is a nearest neighbour random walk with the following transition probabilities: we define the local time of this walk on the *oriented* edges, and the walker is pushed to areas which have been less visited in the past in terms of this kind of local time. (The model with local time on unoriented edges is well understood in the literature, and the order of the displacement after time t is $t^{2/3}$ with a non-standard scaling limit, see [1].) In the oriented edge-repulsion case, the behaviour is different: the proper scaling of the walk turns out to be $\sqrt{t}$ after time t, but there is no continuous limit process, and the distribution of the displacement is uniform on $[-\sqrt{t}, \sqrt{t}]$. We also prove a limit theorem for the local time process of the walk. After appropriate scaling, it converges to a deterministic triangular limit shape. The results are demonstrated also with computer simulations. See [2].

The second model is a continuous time nearest neighbour jump process on $\mathbb{Z}$. The transition rates are defined in such a way that it is more likely to jump in the direction of the negative gradient of the local time which is given as the total amount of time spent on a particular site. We show that the right scaling of this process is $t^{2/3}$, and that the scaling limit should be the so-called true self-repelling motion which is defined in [4]. The properly rescaled local time process converges to a limit expressed by reflected and absorbed Brownian motions. However the limit objects are the same as in the unoriented edge repulsion case, these are the first mathematically rigorous results in a self-repelling random walk model with site repulsion, which is the original formulation of the problem. See [3].

References

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