Uniform Spacing of Zeros of Orthogonal Polynomials on Intervals with Doubling Property

Let μ is a measure with a compact support on the real line. We are interested in the spacing of the associated orthogonal polynomials $\{p_m = p_m(x, \mu)\}_{m=1}^{\infty}$. It has been known for a long while, that all zeros of p_m are distinct, lie in the support and the zeros of the adjacent orthogonal polynomials alternate. But, if we assume, that the measure has a certain regular property, then we can state more about the post of the zeros. Namely, it was shown recently, if the measure has the doubling property, that is there is a constant L such that for every interval I in the support is true, that $\mu(2I) \leq L\mu(I)$, where $2I$ denotes the twice enlarged I (enlarged from its centre), then the zeros of the associated orthogonal polynomials are uniformly spaced in the sense that if $\cos \theta_{m,k}$ ($\theta_{m,k} \in [0, \pi]$) are the zeros of the m-th orthogonal polynomial associated with μ, then $\frac{1}{Am} \leq \theta_{m,k} - \theta_{m,k+1} \leq \frac{A}{m}$ for a suitable constant A depending on only the doubling constant.

We have generalised the previous result. It shall turn out that for the regular spacing of the zeros, it is a sufficient assumption that the measure has the doubling property on an interval. Then we can give a uniform two-sided estimate for the distance of the consecutive zeros falling in this interval, that is the spacing of the zeros is determined by a local property of the measure in a sense. However, the procession of the estimation is similar to the global case above. We use distinct tools for the two sides under the doubling property. In the case of the upper estimate the main tools are the classical Markoff inequality, the Christoffel function and the fast decreasing polynomials, while to the lower estimate we use the Remez inequality and the fact that the m-th orthogonal polynomial p_m is orthogonal to the every polynomial of degree at most $m - 1$.