Computer-aided study of the competition of species for territory

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We consider territory-based population dynamics models, and investigate the spread of species and their competition for territories. In these kinds of models, the change of the occupied territory is examined instead of the number of individuals. The simplest model concerning one species is due to Levins (1969):

$$\frac{dp}{dt} = k(1-p)p - ep,$$

where $p(t)$ is the density of the territory occupied by the given population, k is the rate of colonization, e is the rate of extinction. Assuming that the patches occupied by a species are uniform, the speed of the spread to the patterns in the neighborhood is proportional with $(1-p)p$, meanwhile the extinction does not depend on the neighborhood, hence its speed is ep.

If several species compete for territories, some (or all) of them can occupy patches already being occupied by others. This phenomenon is called overcolonization. A generalization of the Levins's metapopulation model for two-species is:

$$\frac{dp_1}{dt} = k_1 p_1 (1 - p_1 - p_2) - e_1 p_1 + c_1 p_1 p_2 - c_2 p_1 p_2$$

$$\frac{dp_2}{dt} = k_2 p_2 (1 - p_1 - p_2) - e_2 p_2 - c_1 p_1 p_2 + c_2 p_1 p_2,$$

where $p_i(t)$ denotes the density of territory occupied by species $i = 1, 2$; e_i is the rate of extinction, k_i is the rate of colonization of empty patches, c_i is the rate of overcolonization. Special cases are well-known when $c_1 - c_2 = 0$ and $c_1 - c_2 = k_1$. In the first case, there is no interaction between species so only boundary equilibria exist. In the latter case, some hierarchical relations are assumed between populations and interior equilibrium exists which is globally asymptotically stable. The model of general overcolonization is less studied since the analysis is extremely difficult. We show that the output of the species' fight depends on the initial conditions (interior equilibrium exists however it is not globally asymptotically stable).

Finally, we consider even more general models in which the colonization and extinction rates depend on the environment in different ways. In real life, both the coloniziation capability and the extinction can depend on the neighborhood (or density) of the atomic patches. The behavior of such models is much more complicated. We will consider different cases in which neighbors can strengthen or even weaken each other. Our results are illustrated by realtime demonstrations in *Mathematica*.