Time-delayed patchy environment model of the capillary migration assay

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The chemotaxis is the movement of the cells induced by chemical compounds. In our joint work the two chamber capillary assay was investigated, which is a frequently used method to measure chemotactic response of the eukaryotic protozoa. The differential equation system (Keller-Segel model) which was developed and successfully applied to describe the chemotactic response of bacteria, was not suited to forecast the behavior of our model cell the eukaryote ciliated protozoa *Tetrahymena pyriformis*.

In the present study the patchy space interpolation was used to simplify the model. We used the time-delayed version of Fick's first law to model the density of the cells during the measurement:

$$\frac{\mathrm{d}u_1(t)}{\mathrm{d}t} = d\int_{-\infty}^{t} ae^{a(\tau-t)}(u_2(\tau)-u_1(\tau))\mathrm{d}\tau \tag{1}$$

$$\frac{\mathrm{d}u_2(t)}{\mathrm{d}t} = d\int_{-\infty}^{t} ae^{a(\tau-t)}(u_1(\tau)-u_2(\tau))\mathrm{d}\tau, \tag{2}$$

where $u_1(t)$, $u_2(t)$ denotes the cell density in the two chamber, $d > 0$ is the diffusion coefficient which is proportional to the area of the interface between the two tanks, and the activity of the cells. The parameter $a > 0$ is the degree of delay.

With an appropriate substitution, we reduced this time-delayed system to a system of ordinary differential equation with one relevant parameter.

It was proved that the solutions have one asymptotic stable equilibrium irrespectively of the parameter value. This equilibrium undergoes a node-focus bifurcation. For feasible values of the parameter, the solutions are positive.

This modified system describes the observed phenomena during measurement correctly, and gives us the opportunity to optimise the capillary assay.